Distributed Minimax Adaptive Control For Uncertain Networked Systems

Joint work with Anders Rantzer & Olle Kjellqvist

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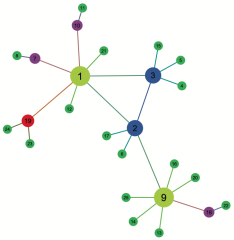
Adaptive Control for Uncertain Dynamical Networks

Control of Uncertain Networks

- Control of networked systems rely crucially on the availability of accurate local dynamics.
- Uncertain local dynamics along with additive disturbance adds complexity
- Limited access to global information calls for distributed control
- Problem: Need adaptive & robust distributed control that can handle uncertainty in local dynamics.



Research - Contributions

1. Design history dependent adaptive control to handle uncertainty in local dynamics.
2. Design of minimax adaptive control facilitating distributed implementation.

[1] **Papers:**

- C. Lidström & A. Rantzer, "H-infinity optimal distributed control in discrete time", *IEEE CDC*, 2017.
- V. Renganathan, A. Rantzer, & O. Kjellqvist, "Distributed Implementation of Minimax Adaptive Controller For Finite Set of Linear Systems", 2023.

Problem Statement

Uncertain LTI Network Dynamics

- Application: Controlling linear models of transportation & buffer (irrigation) networks
- Network Model: $\mathcal{G}(\mathcal{V}, \mathcal{E})$, with $|\mathcal{V}| = N$ and $|\mathcal{E}| = E$. Every subsystem $i \in \mathcal{V}$ evolves as

$$x_i(t+1) = a_i x_i(t) + b \sum \underbrace{(u_i(t) - u_j(t))}_{:=u_{ij}(t)} + w_i(t).$$

- Communication Condition: $a_i^2 + 2b^2|\mathcal{N}_i| < a_i, \quad \forall i \in \mathcal{V}$.
- Uncertainty in Local Dynamics: $a_i \in \mathbf{A_i} := \{a_i \in (0,1) \mid a_i^2 + 2b^2|\mathcal{N}_i| < a_i\}$, $|\mathbf{A_i}| = M$.
- Compact Notation: $x(t+1) = Ax(t) + Bu(t) + w(t)$ with $B = b\mathcal{I}$ and $A^2 + BB^\top \prec A$

Main Problem

Given $\gamma > 0$, design distributed control input for each node $i \in \mathcal{V}$ satisfying following cost

$$\sum_{\tau=0}^{T} \sum_{i \in \mathcal{V}} \left(|x_i(\tau)|^2 + |u_i(\tau)|^2 \right) \leq \gamma^2 \sum_{\tau=0}^{T} \sum_{i \in \mathcal{V}} |w_i(\tau)|^2 .$$

Distributed H_∞ Network Control - With Known Dynamics

Distributed H_∞ Control

Given a known (A, B) pair, with A being symmetric and Schur stable, the controller with gain $K = B^\top (A - I)^{-1}$ achieves the minimum value of $\left\|((A-I)^2 + BB^\top)^{-1}\right\|^{\frac{1}{2}}$ for the ℓ_2 gain from the disturbance to the error.

Problems with Unknown Dynamics

- Unknown dynamics calls for an adaptive control that learns dynamics from the past data
- Note that Main Problem has a finite solution if and only if

$$\gamma > \gamma^\dagger > \max_{i \in \{1,\ldots,N^M\}} \underbrace{\left\|((A_i - I)^2 + BB^\top)^{-1}\right\|^{\frac{1}{2}}}_{:=\gamma_i^\star},$$

- $\gamma_i^\star$ denotes the minimum ℓ_2 gain from the disturbance to the error with controller $K^\star = B^\top (A_i - I)^{-1}$ for matrix A_i or equivalently $u_i = \frac{bx_i(t)}{a_i(t)-1}$ for all nodes $i \in \mathcal{V}$.
- $\gamma^\dagger$ is the ℓ_2 gain achieved by the optimal minimax adaptive control.
- We need to search over the space of exponential number (N^M) of corner matrices to get a lower bound for $\gamma^\dagger$ of the centralized minimax adaptive controller.

Scalable & Distributed Implementation

Main Result (Sub-Optimal Distributed Minimax Adaptive Controller)

If Main Problem is solvable, then it has a solution for every node $i \in \mathcal{V}$ of the form

$$u_i(t) = \frac{b x_i(t)}{a_i^\dagger(t) - 1}, \quad \text{where,}$$

$$a_i^\dagger(t) = \arg\min_{a_i \in \mathbf{A_i}} \left\{ \sum_{\tau=0}^{t-1} \left| a_i x_i(\tau) + b \sum_{j \in \mathcal{N}_i} (u_i(\tau) - u_j(\tau)) - x_i(\tau+1) \right|^2 \right\}.$$

Remarks

- Every node $i \in \mathcal{V}$ selects the model that best fits the disturbance trajectory modelled using the collected history in a least-square sense.
- Implementation does not require the knowledge of the ℓ_2 gain bound $\gamma^\dagger$.
- Above result can be verified for values of γ that satisfies certain Riccati type inequality that quantifies cost due to learning & cost due to applying a wrong controller to a model.

Distributed Implementation Details

- To hedge against the uncertainty in (A, B), it is natural for the controller to consider collecting historical data
- Control vector of node $i \in \mathcal{V}$ is $u_{\mathcal{N}_i}(t) := \begin{bmatrix} u_{ij_1}(t) & u_{ij_2}(t) & \cdots & u_{ij_{|\mathcal{N}_i|}}(t) \end{bmatrix}$
- Every node $i \in \mathcal{V}$ collects the data in the form of sample covariance matrix as

$$x_i(t+1) = v_i(t)$$

$$Z^{(i)}(t+1) = Z^{(i)}(t) + \begin{bmatrix} -v_i(t) \\ x_i(t) \\ u_{\mathcal{N}_i}(t) \end{bmatrix} \begin{bmatrix} -v_i(t) \\ x_i(t) \\ u_{\mathcal{N}_i}(t) \end{bmatrix}^\top, \quad \text{with } t \in \mathbb{N} \text{ and } Z^{(i)}(0) = 0.$$

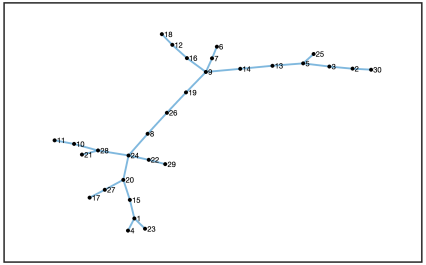
- Distributed minimax adaptive control input between node $i \in \mathcal{V}$ & its neighbor $j \in \mathcal{N}_i$ is

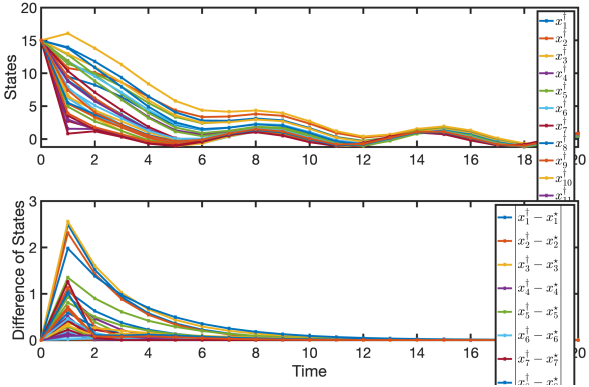
$$u_{ij}^\dagger(t) = \eta^\dagger(x_i(t), x_j(t), Z^{(i)}(t)) = K_{k_t}^{(ij)} \begin{bmatrix} x_i(t) \\ x_j(t) \end{bmatrix}, \quad \text{where,}$$

$$k_t = \underset{a_i \in \mathbf{A}_i}{\arg\min} \left\| \begin{bmatrix} 1 & a_i & b\mathbf{1}_{|\mathcal{N}_i|}^\top \end{bmatrix} \right\|_{Z^{(i)}(t)}^2.$$

Simulation Results

Consider buffer network of size $N = 30$. Random data generated for each node with $M = 2$ different possible values satisfying incidence matrix and communication conditions.





Observation

Once each node $i \in \mathcal{V}$ figures out the true $a_i \in \mathbf{A}_i$, the distributed minimax controller behaves very similar to that of the distributed H_∞ controller.

TEASER FOR FUTURE

Takeaway Message

- Distributed implementation of minimax adaptive control needs only local information exchange and does not require the knowledge of the ℓ_2 gain bound $\gamma^\dagger$

Ideas for Future Work

1. Derive the gain bound, $\gamma^\dagger$ for the optimal distributed minimax adaptive control.
2. Analyse regret both on a node level & on a global network level.
3. Design distributed adaptive control law that reduces regret.
4. If dynamic programming can help us obtain LQR controller, can we design distributed LQR controller using distributed dynamic programming? For reference, see Bertsekas' paper. Any guidance to existing works is appreciated.

Thank You

Questions

Any questions ?

Hope you all enjoyed my presentation ! ☺

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Contributors