Stability of a Cosserat-Rod Theoretic Boundary Observer

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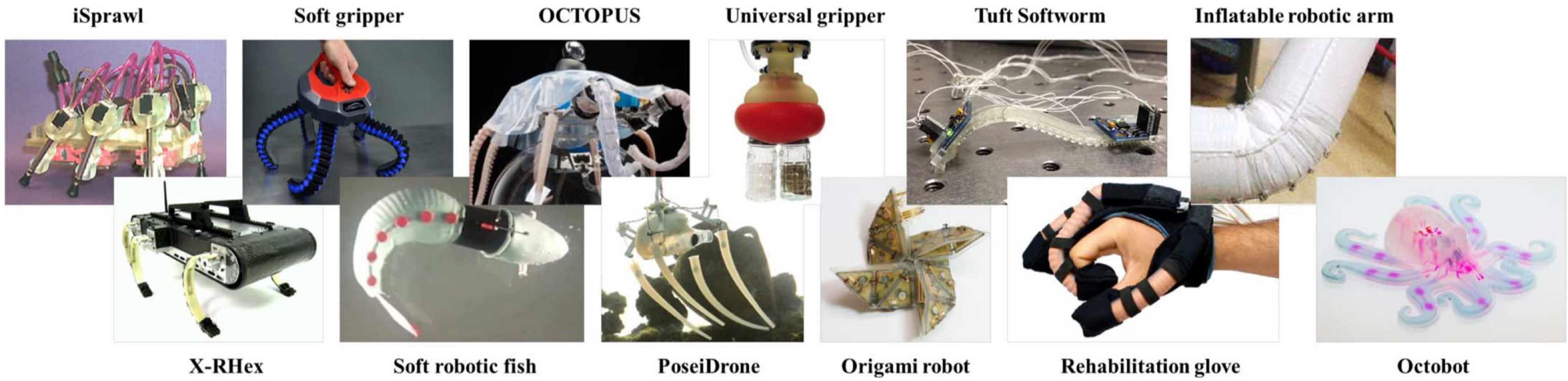
My Research

- PDE control: swarm robots (large number), soft robots (infinite Dof)

Swarm Robotics

Soft Robotics



Mean-field PDEs

Continuum mechanics PDEs

Soft Robot Arm: Continuum Mechanics Modeling

- Configuration: p – position, R – rotation,

$$\text{pose } g = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \in SE(3)$$

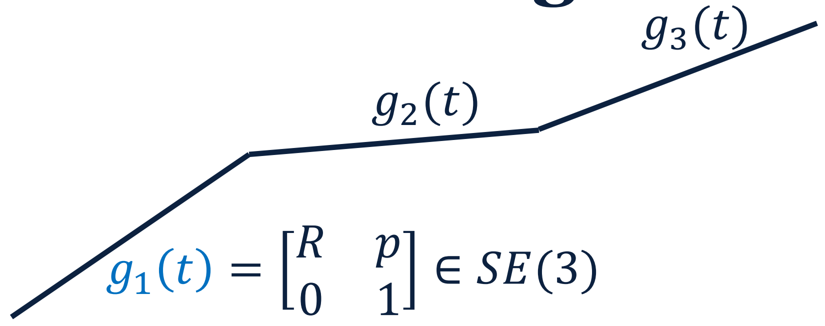
- Kinematics: η – body velocity, ξ – body strain

$$\partial_t g = g\eta^\wedge$$

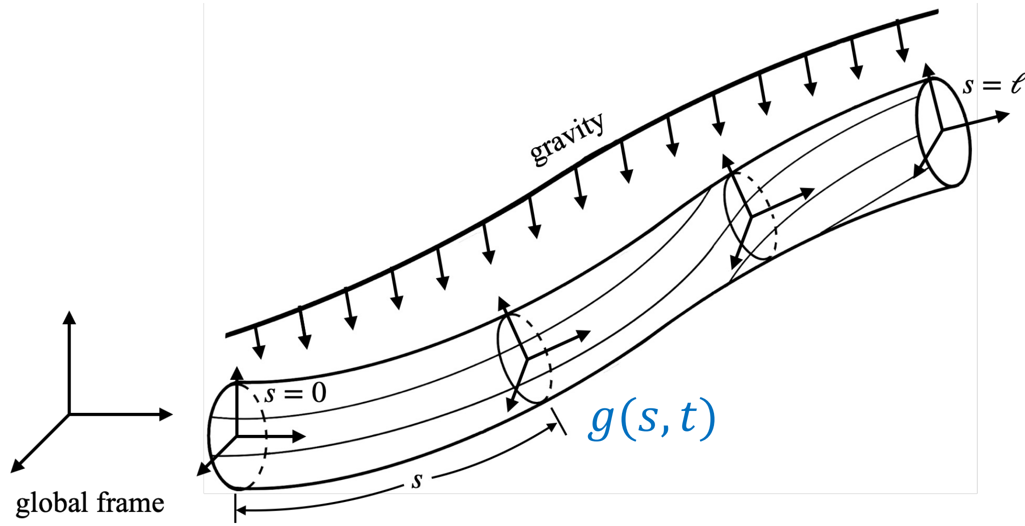
$$\partial_s g = g\xi^\wedge$$

- Dynamics: ϕ – internal wrench, ψ – external wrench

$$J\partial_t\eta - ad_\eta^T J\eta = \partial_s\phi - ad_\xi^T\phi + \psi$$



Rigid robot



Continuum/Soft robot

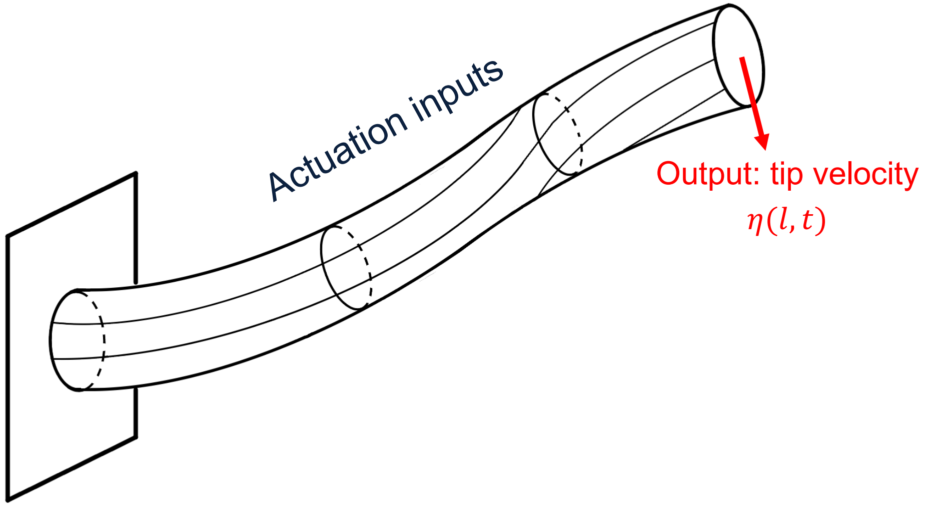
[1] Renda, Federico, et al. "Dynamic model of a multibending soft robot arm driven by cables." *IEEE Transactions on Robotics* 30.5 (2014): 1109-1122.

Boundary Observer

- Boundary observer = model + boundary correction [1]

$$\begin{cases} \partial_t \hat{\xi} = \partial_s \hat{\eta} + ad_{\hat{\xi}} \hat{\eta}, \\ J \partial_t \hat{\eta} = \partial_s \hat{\phi} - ad_{\hat{\xi}}^T \hat{\phi} + ad_{\hat{\eta}}^T J \hat{\eta} + \psi, \\ \hat{\eta}(0,t) = 0, \\ \hat{\phi}(\ell,t) = -\Gamma(\hat{\eta} - \eta)(\ell,t) \end{cases}$$

Boundary dissipation



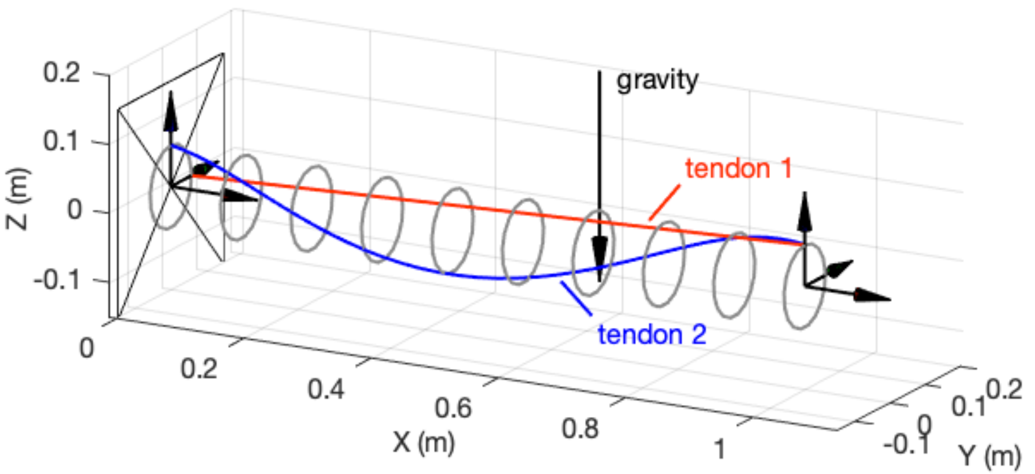
Problem setup

Theorem

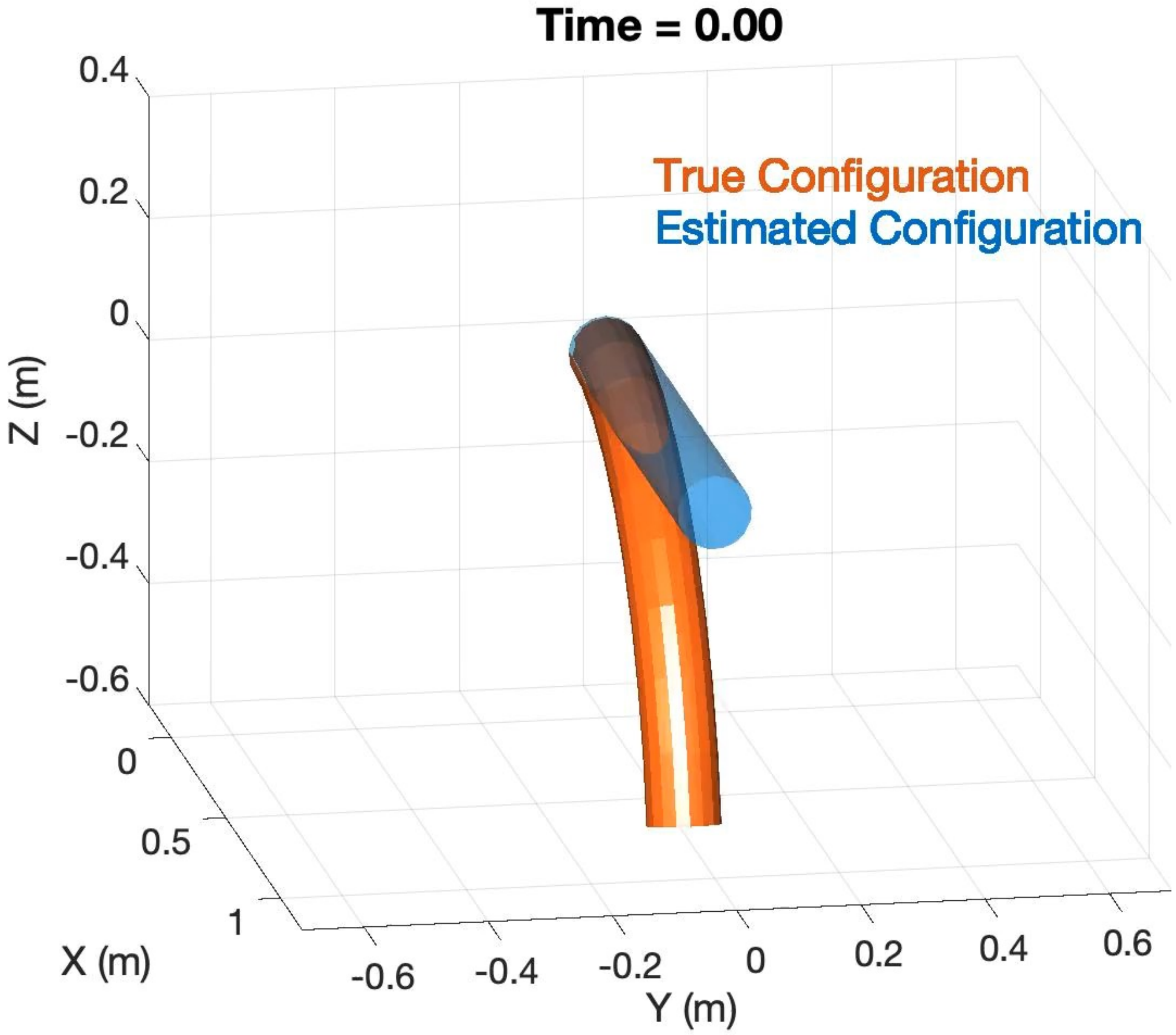
Let $y = [\xi, \eta]^T$. Estimation error $\| \hat{y} - y \|_{H_x^1}$ is locally ISS to $\| y \|_{H_x^1}$ (true states).

[1] **Zheng, Tongjia**, Qing Han, and Hai Lin. "Full State Estimation of Soft Robots From Tip Velocities: A Cosserat-Theoretic Boundary Observer." *submitted to IEEE Transactions on Robotics.*

Simulation



- Two tendons actuated alternatively
- IMU sampling rate: 200 Hz
- Hypothesis: globally ISS



Simulation using SoRoSim

https://www.youtube.com/watch?v=UhkzhJGT7GQ

Discussion: stability of conservation laws

- **Hypothesis:** (1) is globally asymp. stable (in L^2)

$$\begin{cases} \partial_t \phi = \partial_s \eta + ad_\xi \eta, \\ \partial_t \eta = \partial_s \phi - ad_\xi^T \phi + ad_\eta^T J\eta, \\ \eta(0,t) = 0, \\ \phi(\ell,t) = -\Gamma\eta(\ell,t) \end{cases} \quad (1)$$

Conservation laws on SE(3)

Boundary dissipation

- Facts:
    - (1) is stable in L^2, total energy $\dot{\mathcal{E}}(t) = -\Gamma\eta^2(\ell,t) \leq 0$ (boundary value)
    - (1) is locally exp. stable in H^1 [1]

- Fact [2]: (2) is globally exp. stable in L^2 using $V = \int R_1^2 \exp(-\mu s) + R_2^2 \exp(\mu s) ds$.

$$\begin{cases} \partial_t \phi = \partial_s \eta, \\ \partial_t \eta = \partial_s \phi, \\ \eta(0,t) = 0, \\ \phi(\ell,t) = -\Gamma\eta(\ell,t) \end{cases} \quad (2)$$

- Contraction theory for PDEs?

[1] Rodriguez, Charlotte, and Günter Leugering. "Boundary feedback stabilization for the intrinsic geometrically exact beam model." *SIAM Journal on Control and Optimization* 58.6 (2020): 3533-3558.

[2] Bastin, Georges, and Jean-Michel Coron. *Stability and boundary stabilization of 1-d hyperbolic systems*. Vol. 88. Basel: Birkhäuser, 2016.