Dynamic Vehicle Routing for Robotic Networks
Lecture #8: Different Task Models

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Motivation for Team Forming

• Group of vehicles monitoring a region
• Several different sensing modalities:
  • electro-optical,
  • infra-red,
  • synthetic aperture radar,
  • foliage penetrating radar,
  • etc.
• Each event requires a subset of sensing modalities
• Equip each vehicle with a single sensing modality
• Form appropriate team to properly assess each event

How do we create teams in real-time to observe each event (service each request)?

Lecture outline

1 Dynamic Team Forming

2 Three Policies
  • Complete Team
  • Task-Specific Team Policy
  • Scheduled Task-Specific Team Policy

3 Analysis of Policies
  • Throughput vs System Time
  • Comparison of Results

Literature Review

Scaling laws in Robotic Networks

Throughput vs Delay in Wireless Networks

Graph Coloring
T. A. McKee and F. R. McMorris. Topics in Intersection Graph Theory, volume 2 of Monographs on Discrete Mathematics and Applications. SIAM, 1999
### Dynamic Team Forming

Set of services \( \{r_1, \ldots, r_k\} \).

**Vehicle properties:**
- \( k \) different vehicle types.
- Vehicle type \( j \in \{1, \ldots, k\} \), can provide only service \( r_j \).

**Task (demand) model:**
- Poisson and Uniform arrivals
- Each task requires a subset of services in \( \{r_1, \ldots, r_k\} \).
- \( K \) different types of tasks
- Tasks of type \( \alpha \) arrive at rate \( \lambda_\alpha \)
- Task completed when required vehicles *simultaneously* spend on-site service time at location.


### Load Factor and Stability

- \( R_\alpha \in \{0, 1\}^k \) is zero-one column vector recording services required for task-type \( \alpha \).
- On-site service for task-type \( \alpha \) is \( \bar{s}_\alpha \)
- \( m_j \) vehicles provide service \( r_j \).

**Necessary stability condition:**

\[
\begin{bmatrix}
\lambda_1 \bar{s}_1 \\
\vdots \\
\lambda_K \bar{s}_K
\end{bmatrix}
\begin{bmatrix}
m_1 \\
\vdots \\
m_K
\end{bmatrix}
<
\begin{bmatrix}
r_1 \\
\vdots \\
r_K
\end{bmatrix}
\]

Load factor is now a vector

### Example of Team Forming

- \( k = 4 \) different services, \( \{r_1, r_2, r_3, r_4\} \).
- \( m = 8 \) vehicles, two of each type: \( m_j = 2 \) for \( j \in \{1, 2, 3, 4\} \).
- \( K = 6 \) task types, \( \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_3\}, \{r_2, r_4\} \).

Task type \( \alpha = \{r_1, r_3\} \) has on-site service \( \bar{s}_\alpha \), arrival rate \( \lambda_\alpha \), and

\[
R_\alpha = [1 0 1 0]^T.
\]
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3 Analysis of Policies

Task-Type Unbiased Policies

For a policy $\pi$:
- System time of each task-type $T_{\pi,1}, \ldots, T_{\pi,K}$
- Feasible set of system times are subset of $\mathbb{R}^K$
- Optimization space similar to priority queues, but with teaming

To simplify, consider task-type unbiased policies

$$T_{\pi,1} = T_{\pi,2} = \cdots = T_{\pi,K} =: T_\pi$$

and the optimization: $\inf_\pi T_\pi$.

Policy 1: Complete Team Policy

Complete Team Policy
1: Form $\min\{m_1, \ldots, m_k\}$ teams of $k$ vehicles, each team contains one vehicle of each type.
2: Have each team meet and move as a single entity.
3: In each region run UTSP policy (from Lecture 3).

Can also use Divide & Conquer policy for each team.

Two services $y, b$
3 task-types $y, b, \{y, b\}$
4 vehicles
- 2 yellow
- 2 blue
Dynamic Vehicle Routing (Lecture 8/8)

In each region form a queue for each task type.

Divide robots into teams to form required task types.

For each time slot in the schedule:
  - For each team, service corresponding queue with
    - Service each task by one of its m_{TST} vehicles, where:
      \[
      m_{TST} := \left\lfloor \min_j \left\{ \frac{m_j}{e_j^T R \mathbf{1}_K} \right\} \right\rfloor
      \]

**Policy 2: Task-specific Team Policy**

1. For each of the K task types, create m_{TST} teams of vehicles.
2. Service each task by one of its m_{TST} corresponding teams, according to the UTSP policy.

**Policy 3: Scheduled Task-Specific Team Policy**

Partition Q into \( \min_i \{m_i\} \) regions and assign one robot of each type to each region.

1. In each region form a queue for each task type.
2. For each time slot in the schedule:
   - Divide robots into teams to form required task types.
   - For each team, service corresponding queue with TSP tour.
Policy 3: Scheduled Task-Specific Team Policy

Service schedule:
- two time slots \( L = 2 \)
- slot one: \( \{y\}, \{b\} \)
- slot two: \( \{y, b\} \)

Assumptions for Analysis

Assumptions:
- \( m_i = \frac{m}{k} \) for each vehicle type \( i \).
- \( \lambda_\alpha = \frac{\lambda}{K} \) for each task-type \( \alpha \).
- on-site service has mean \( \bar{s} \) and is upper bounded by \( s_{\text{max}} \).
- \( pK \) of the \( K \) task-types require service \( r_j \), where \( p \in [1/k, 1] \).

- With assumptions, necessary stability condition becomes
  \[ \frac{\lambda}{m} < \frac{1}{pk\bar{s}}. \]

- Define per-vehicle throughput as \( B_m := \frac{\lambda}{m} \).

Throughput vs System Time Profile

\[
B_m \mapsto \begin{cases} 
\max \left\{ \frac{T_{\text{ord}}(B_m/B_{\text{crit}})}{(1-B_m/B_{\text{crit}})^2}, \frac{T_{\text{min}}}{(1-B_m/B_{\text{crit}})^2} \right\}, & \text{if } B_m < B_{\text{crit}}, \\
+\infty, & \text{if } B_m \geq B_{\text{crit}}. 
\end{cases}
\]

- \( T_{\text{min}} \) = minimum achievable system time for positive throughput.
- \( B_{\text{crit}} \) = maximum achievable throughput (or capacity).
- \( T_{\text{ord}} \) = system time at a constant fraction of capacity 
  \( (3 - \sqrt{5})/2 \approx 38\% \) of capacity \( B_{\text{crit}} \).

Example (Single vehicle DVR)

\[
B_{\text{crit}} = \frac{1}{\bar{s}} \\
T_{\text{min}} = E_{\varphi}[\|X - p^*\|]/v + \bar{s} \quad \text{(light load)} \]
\[
T_{\text{ord}} \approx C(\int_Q \varphi^{1/2}(x)dx)^2/v^2 \quad \text{(heavy load numerator)}
\]
Throughput vs System Time Profile

System Time for each Policy

<table>
<thead>
<tr>
<th></th>
<th>$T_{\text{min}}$</th>
<th>$T_{\text{ord}}$</th>
<th>$B_{\text{crit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower bound ($\mathcal{T}^*$)</td>
<td>$\sqrt{k}$</td>
<td>$k$</td>
<td>$\frac{1}{pk^3}$</td>
</tr>
<tr>
<td>Complete Team</td>
<td>$\sqrt{k}$</td>
<td>$k$</td>
<td>$\frac{1}{k^3}$</td>
</tr>
<tr>
<td>Task-Specific</td>
<td>$\sqrt{pk\mathcal{K}}$</td>
<td>$pk\mathcal{K}$</td>
<td>$\frac{1}{pk^3}$</td>
</tr>
<tr>
<td>Scheduled Task-Specific</td>
<td>$L\sqrt{k}$</td>
<td>$Lk$</td>
<td>$\frac{\mathcal{K}}{S_{\text{max}}Lk}$</td>
</tr>
</tbody>
</table>

where $L \in [p\mathcal{K}, \mathcal{K}]$

Best policies for different scenarios:
- If throughput is low, then use complete team
- If $p$ is close to 1, then use complete team
- If $p$ is close to $1/k$, then for best capacity use
  - Task-Specific if enough vehicles
  - Scheduled Task-Specific otherwise

Lecture outline

1. Dynamic Team Forming
2. Three Policies
   - Complete Team
   - Task-Specific Team Policy
   - Scheduled Task-Specific Team Policy
3. Analysis of Policies
   - Throughput vs System Time
   - Comparison of Results

Workshop Structure and Schedule

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00-8:30am</td>
<td>Coffee Break</td>
</tr>
<tr>
<td>8:30-9:00am</td>
<td>Lecture #1: Intro to dynamic vehicle routing</td>
</tr>
<tr>
<td>9:05-9:50am</td>
<td>Lecture #2: Prelims: graphs, TSPs and queues</td>
</tr>
<tr>
<td>9:55-10:40am</td>
<td>Lecture #3: The single-vehicle DVR problem</td>
</tr>
<tr>
<td>10:40-11:00am</td>
<td>Break</td>
</tr>
<tr>
<td>11:00-11:45pm</td>
<td>Lecture #4: The multi-vehicle DVR problem</td>
</tr>
<tr>
<td>11:45-1:10pm</td>
<td>Lunch Break</td>
</tr>
<tr>
<td>1:10-2:10pm</td>
<td>Lecture #5: Extensions to vehicle networks</td>
</tr>
<tr>
<td>2:15-3:00pm</td>
<td>Lecture #6: Extensions to different demand models</td>
</tr>
<tr>
<td>3:00-3:20pm</td>
<td>Coffee Break</td>
</tr>
<tr>
<td>3:20-4:20pm</td>
<td>Lecture #7: Extensions to different vehicle models</td>
</tr>
<tr>
<td>4:25-4:40pm</td>
<td>Lecture #8: Extensions to different task models</td>
</tr>
<tr>
<td>4:45-5:00pm</td>
<td>Final open-floor discussion</td>
</tr>
</tbody>
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