On the Dynamics of Influence and Appraisal Networks

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New text “Lectures on Network Systems”

Lectures on Network Systems, 1 edition
ISBN 978-1-986425-64-3

For students: free PDF for download
For instructors: slides and answer keys
http://motion.me.ucsb.edu/book-lns
https://www.amazon.com/dp/1986425649
300 pages (plus 200 pages solution manual)
3K downloads since Jun 2016
150 exercises with solutions

Linear Systems:
- social, sensor, robotic & compartmental examples,
- matrix and graph theory, with an emphasis on
  Perron–Frobenius theory and algebraic graph theory,
- averaging algorithms in discrete and continuous time,
  described by static and time-varying matrices, and
- positive & compartmental systems, dynamical flow systems, Metzler matrices.

Nonlinear Systems:
- nonlinear consensus models,
- population dynamic models in multi-species systems,
- coupled oscillators, with an emphasis on the
  Kuramoto model and models of power networks

Dynamics and learning in social systems

Dynamic phenomena on dynamic social networks
- opinion formation, information propagation, collective learning,
  task decomposition/allocation/execution
- interpersonal network structures, e.g., influences & appraisals

Questions on collective intelligence, rationality & performance:
- wisdom of crowds vs. group think
- influence centrality (democracy versus autocracy)
- collective learning or lack thereof
opinion dynamics over influence networks
- seminal works: French '56, Harary '59, DeGroot '74, Friedkin '90
- recently: bounded confidence, learning, social power
- key object: row stochastic matrix

dynamics of appraisal networks and balance theory
- seminal works: Heider '46, Cartwright '56, Davis/Leinhardt '72
- recently: dynamic balance, empirical studies
- key object: signed matrix

Not considered today:
- other dynamic phenomena (epidemics)
- static network science (clustering)
- game theory and strategic behavior (network formation)

Selected literature on opinion dynamics
J. R. P. French. A formal theory of social power. 
doi:10.1037/h0046123

M. H. DeGroot. Reaching a consensus. 

N. E. Friedkin and E. C. Johnsen. Social influence and opinions. 
doi:10.1080/0022250X.1990.9990069

doi:10.1016/j.arcontrol.2017.03.002

Selected literature on social power & reflected appraisal

URL http://www.jstor.org/stable/3033844

N. E. Friedkin. A formal theory of reflected appraisals in the evolution of power. 
doi:10.1177/0001839212441349
Influence systems: statistical results on empirical data

doi:10.15195/v3.a20

doi:10.1073/pnas.1710603114

Influence systems: the mathematics of social power

Appraisal systems and collective learning

Postulated mechanisms for opinion dynamics 1/2

French-DeGroot averaging model

\[ y_i^{+} := \text{average}(y_i, \{y_j, j \text{ is neighbor of } i\}) \]

\[ y(k + 1) = Ay(k) \]

where \( A \) is nonnegative and row-stochastic

Consensus under mild connectivity assumptions:

\[ \lim_{k \to \infty} y(k) = (c^T y(0)) \mathbb{1}_n \]

self-weight = level of closure: \( a_{ii} \) diagonal entries of influence matrix
social power: \( c_i \) entries of dominant left eigenvector

Postulated mechanisms for opinion dynamics 2/2

Averaging (French-DeGroot model)

\[ y(k + 1) = Ay(k) \quad \lim_{k \to \infty} y(k) = (c^T y(0)) \mathbb{1}_n \]

Averaging + attachment to initial opinion (F-J model)

\[ y(k + 1) = (I_n - \Lambda)Ay(k) + \Lambda y(0), \]
\[ \Lambda = \text{diag}(A) \]

Convergence under mild connectivity+stubborness assumptions:

\[ \lim_{k \to \infty} y(k) = V \cdot y(0), \quad \text{for} \ V = (I_n - (I_n - \Lambda)^{-1} \Lambda \mathbb{1}_n)^{-1} \mathbb{1}_n/n = \text{average contribution of each agent} \]

self-weight = level of closure: \( a_{ii} \) diagonal entries of influence matrix
social power: \( c_i \) entries of centrality vector

Deliberative groups in social organization

- government: juries, panels, committees
- corporations: board of directors
- universities: faculty meetings

Natural social processes along sequences

- opinion dynamics for single issue?
- levels of openness and closure along sequence?
- influence accorded to others? emergence of leaders?

Groupthink = “deterioration of mental efficiency . . . from in-group pressures,” by I. Janis, 1972
Wisdom of crowds = “group aggregation of information results in better decisions than individual’s” by J. Surowiecki, 2005
Today we skip these proofs

Mathematical analysis of French-DeGroot and F-J models is well understood:
- Jordan normal form
- Perron-Frobenius theory
- algebraic graph theory (connectivity, periodicity, etc)

Experiments on opinion formation and influence networks
domains: risk/reward choice, analytical reliability, resource allocation

- 30 groups of 4 subjects in a face-to-face discussion
- sequence of 15 issues
- each issue is risk/reward choice:
  - what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff?
  - e.g.: medical, financial, professional, etc
- “please, reach consensus” pressure

On each issue, each subject recorded (privately/chronologically):
- an initial opinion prior to the-group discussion,
- a final opinion after the group-discussion (3-27 mins),
- an allocation of “100 influence units”
  - (“these allocations represent your appraisal of the relative influence of each group member’s opinion on yours”).

(1/3) Prediction of individual final opinions

Balanced random-intercept multilevel longitudinal regression

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-J prediction</td>
<td>0.897***</td>
<td>1.157***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.032)</td>
<td></td>
</tr>
<tr>
<td>initial opinions</td>
<td></td>
<td>-0.282***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.031)</td>
<td></td>
</tr>
<tr>
<td>log likelihood</td>
<td>-8579.835</td>
<td>-7329.003</td>
<td>-7241.097</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses; ** p ≤ 0.01, *** p ≤ 0.001; maximum likelihood estimation with robust standard errors; n = 1,800.

FJ averaging model is predictive for risk/reward choice issues

Extensions to: intellective and resource allocation issues


Intellective issue
Two medical teams are working independently to achieve a cure for a disease. Team A succeeds if problems $A_1$ and $A_2$ with $P[A_1] = 0.60$ and $P[A_2] = 0.45$.

Team B succeeds if problems $B_1$, $B_2$, and $B_3$, with $P[B_1] = 0.80$, $P[B_2] = 0.85$, $P[B_3] = 0.95$.

What is your estimate of the probability that the disease will be cured?

Multidimensional resource allocation

What do you recommend as min and max percent of food consumption in terms of (1) Fruits or Vegetables, (2) Grains, and (3) Meats?

What are your ideal percentages in your preferred min/max ranges?
Recent empirical and theoretical results

Averaging models are predictive


Empirical evidence that (1) FJ model substantially clarifies how truth wins in groups engaged in sequences of intellective issues (2) learning and reflected appraisal take place

N. E. Friedkin, W. Mei, A. V. Proskurnikov, and F. Bullo. Mathematical structures in group decision-making on resource allocation distributions. Submitted

Empirical evidence that (1) FJ model provides quantitative mechanistic explanation for uncertain multi-objective decision making problem and (2) FJ provides detailed explanation for group satisficing solutions

Opinion dynamics along sequences

Postulated mechanism for network evolution

From Wikipedia

1. Reflected appraisal = a person’s perception of how others see and evaluate him or her.

2. This process has been deemed important to the development of a person’s self-esteem, because it includes interaction with people outside oneself.

3. The reflected appraisal process concludes that people come to think of themselves in the way they believe others think of them.

Reflected appraisal process (Cooley 1902 and Friedkin 2011)

Along issues \( s = 1, 2, \ldots \), individual dampens/elevates self-weight according to prior influence centrality

\[
\text{self-weights} := \text{relative control on prior issues} = \text{social power}
\]

**Balanced random-intercept multilevel longitudinal regression**

individual’s “closure to influence” as predicted by:

- individual’s prior centrality \( c_i(s) \)
- individual’s time-averaged centrality \( \bar{c}_i(s) = \frac{1}{s} \sum_{t=1}^{s} c_i(t) \)

| \( c_i(s) \) | 0.336*** |
| \( \bar{c}_i(s) \) | 0.404*** |
| \( s \) | 0.002 |
| \( s \times c_i(s) \) | 0.171 |
| \( s \times \bar{c}_i(s) \) | 0.095*** |
| log likelihood | -367.331, -327.051, -293.656 |

prior and cumulative prior centrality predicts individual closure
Outline

1. Influence systems: statistical results on empirical data

Influence systems: the mathematics of social power

doi:10.1137/130913250

doi:10.1109/TAC.2018.2822182

2. Appraisal systems and collective learning

Opinion dynamics and social power along issue sequences

French-DeGroot averaging model

\[ y(k+1) = Ay(k) \]

Consensus under mild assumptions:

\[ \lim_{k \to \infty} y(k) = (v_{\text{left}}(A) \cdot y(0))1_n \]

where \( v_{\text{left}}(A) \) is social power

- \( A_{ij} =: x_i \) are self-weights / self-appraisal = level of closure
- let \( W_{ij} \) be relative interpersonal accorded weights define \( A_{ij} =: (1 - x_i)W_{ij} \) so that \( A(x) = \text{diag}(x) + \text{diag}(1_n - x)W \)
- \( v_{\text{left}}(W) = (w_1, \ldots, w_n) = \text{dominant eigenvector for } W \)

Dynamics of the influence network

Reflected appraisal phenomenon (Cooley 1902 and Friedkin 2011)

along issues \( s = 1, 2, \ldots \), individual dampens/elevates self-weight according to prior influence centrality

self-weights \( \rightarrow \) relative control on prior issues = social power

Theorem: For strongly connected \( W \) and non-trivial initial conditions

1. unique fixed point \( x^* = x^*(w_1, \ldots, w_n) \)
2. convergence = forgets initial condition

\[ \lim_{s \to \infty} x(s) = \lim_{s \to \infty} v_{\text{left}}(A(x(s))) = x^* \]

3. accumulation of social power and self-appraisal

- fixed point \( x^* \) has same ordering of \( (w_1, \ldots, w_n) \)
- \( x^* \) is an extreme version of \( (w_1, \ldots, w_n) \)
**Emergence of democracy**

If $W$ is doubly-stochastic:

1. the non-trivial fixed point is $\frac{1}{n}
2. \lim_{s \to \infty} x(s) = \lim_{s \to \infty} v_{\text{left}}(A(x(s))) = \frac{1}{n}

- Uniform social power
- No power accumulation = evolution to democracy

**Emergence of autocracy**

If $W$ has star topology with center $j$:

1. there are no non-trivial fixed points
2. \lim_{s \to \infty} x(s) = \lim_{s \to \infty} v_{\text{left}}(A(x(s))) = e_j

- Autocrat appears in center node of star topology
- Extreme power accumulation = evolution to autocracy

**Analysis methods**

1. existence of $x^*$ via Brower fixed point theorem
2. monotonicity: $i_{\text{max}}$ and $i_{\text{min}}$ are forward-invariant
   \[ i_{\text{max}} = \arg\max_j \frac{x_j(0)}{x_j^*} \]
   \[ \implies i_{\text{max}} = \arg\max_j \frac{x_j(s)}{x_j^*}, \text{ for all subsequent } s \]
3. convergence via variation on classic “max-min” Lyapunov function:
   \[ V(x) = \max_j \left( \ln \frac{x_j}{x_j^*} \right) - \min_j \left( \ln \frac{x_j}{x_j^*} \right) \]
   strictly decreasing for $x \neq x^*$

**Stochastic models with cumulative memory**

Other extensions: modified models, reducible $W$, periodic $W$...

1. assume noisy interpersonal weights $W(s) = W_0 + N(s)$
2. assume noisy perception of social power
   \[ x(s + 1) = v_{\text{left}}(A(x(s))) + n(s) \]
   **Thm:** practical stability of $x^*$

3. assume self-weight := cumulative average of prior social power
   \[ x(s + 1) = (1 - \alpha(s))x(s) + \alpha(s) \left( v_{\text{left}}(A(x(s))) + n(s) \right) \]
   **Thm:** a.s. convergence to $x^*$ (under technical conditions)
Recent extensions on social power evolution

doi:10.1109/TAC.2016.2554280

doi:10.1016/j.ifacol.2017.08.1426

doi:10.1137/16M1065677


Summary (Social Influence)

New perspective on influence networks and social power
- designed/executed/analyzed experiments on group discussions
- proposed/analyzed/validated dynamical models with feedback
- novel mechanism for power accumulation / emergence of autocracy

Open directions
- model robustness
- dynamics of interpersonal appraisals
- larger-scale online experiments
- intervention strategies for optimal group discussions

No one speaks twice, until everyone speaks once
Robert’s Rules of Order & parliamentary procedures

Outline

1 Influence systems: statistical results on empirical data
2 Influence systems: the mathematics of social power
3 Appraisal systems and collective learning

doi:10.1109/TAC.2017.2775963

Appraisal systems and collective learning

Teams and tasks
- individuals with skills
- executing a sequence of tasks
- related through networks of interpersonal appraisals and influence

Natural social processes along sequences
- how is task decomposed, assigned and executed?
- how do individuals learn about each other?
- how does group performance evolve?

models/conditions for learning correct appraisals and achieving optimal assignments
model/conditions for failure to learn and correctly assign
**Selected literature on learning in appraisal systems**


**Collective Learning Model**

- **Transactive memory system (TMS)**
  - collective knowledge on who knows what
  - task execution & observation lead group to increasingly accurate knowledge & consensus

- 
  - $n$ individuals with skill levels $x \in \Delta_n$
  - TMS = appraisal matrix row stochastic
  - workload assignment $w \in \Delta_n$
  - optimal assignment: $w^* = x$

**Collective Learning Model: Key Assumptions**

- **task**
  - assignment rule based on appraisal matrix
  - task execution based on skills
  - appraisal matrix
  - appraise/influence dynamics
  - individual relative performance

**Collective Learning Model: Result 1/3**

1. TMS is akin to a manager
   - Along assign/appraise dynamics: generalized replicator equation
     
     \[
     \dot{w}_i = w_i \left( p_i - \sum_k w_k p_k \right) \implies w(t) \to x
     \]
   - akin to replicator equation modeling a “manager dynamics”

**Key assumptions of assign/appraise dynamics**

- Static assignment by appraisal centrality or appraisal average
- Static individual performance $p_i = x_i/w_i$
- Observation of own vs average performance
  
  \[ \phi_i = \text{own performance} - \text{average of observed subgroup performance} \]
- appraise/influence: elevate/dampen self-appraisal + opinion exchange
1. TMS is akin to a manager
2. Opinion exchange compensates for lack of observation.

**Conditions for asymptotic optimization**

- assign/appraise dynamics:
  
  \[ w(t) \rightarrow w^* = x \]

- additionally with influence dynamics:
  
  \[ w(t) \rightarrow w^* = x \]

moreover: consensus on correct appraisals

**Summary**

**Contributions**

- dynamics and feedback in sociology and organization science
- a new perspective on social power, self-appraisal, influence networks
- a new explanation of team learning and rationality

**Next steps**

1. theoretical analysis of increasingly realistic models
2. validation via human subject experiments on larger networks
3. Outreach/collaboration for control community with sociologists, psychologists, organization scientists on problems from Mathematical Sociology and Network Science