**Motivations**

- **Drivers**
  - "big data" increasingly available
  - Quantitative methods in social sciences
  - Applications in marketing and (in)-security

- Dynamical processes over social networks
  - Opinion dynamics, info propagation
  - Network formation and evolution
  - Co-evolutionary processes

- Key novelty: sequence of issues

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**Small deliberative groups**

- Small deliberative groups are assembled in most social organization to deal with sequences of issues in particular domains:
  - Judicial, legislative and executive branches: grand juries, federal panels of judges, Supreme Court – standing policy bodies, congressional committees – advisory boards
  - Corporations: board of directors/trustees
  - Universities: faculty meetings

- Group properties may evolve over its issue sequence according to natural social processes that modify its internal social structure

- Possible systematic changes:
  1. A stabilization of individuals’ levels of openness and closure to interpersonal influences on their initial preferences,
  2. A stabilization of individuals’ ranking of, and influence accorded to, other members’

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**Opinions, influence networks and centrality**

- Dynamics and formation of opinions
  - Convex combinations of opinions
  - Model by French (’56), Harary (’65), and DeGroot (’74)

- Dynamics of influence networks and social power
  - Reflected appraisal hypothesis by Cooley, 1902
    - Individual’ self-appraisal (e.g., self-confidence, self-esteem, self-worth) is influenced by the appraisal of other individuals of her
  - Mathematization by Friedkin, 2012:
    - Varying social power and self-confidence
    - Constant relative interpersonal relations

- Network centrality
  - Centrality measure of network nodes, e.g., eigenvector centrality by Bonacich, 1972
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The dynamics of opinions

DeGroot opinion dynamics model
\[ y(t + 1) = W y(t) \]

- Opinions \( y \in \mathbb{R}^n \)
- Influence network = row-stochastic \( W \)
- by P-F: \( \lim_{t \to \infty} y(t) = (w^T y(0))1_n \)
  where \( w \) is dominant left eigenvector of \( W \)

  - Self-weights \( W_{ii} =: x_i \)
  - Interpersonal accorded weights \( W_{ij} \)
  - Relative interpersonal accorded weights \( C_{ij} \), where \( W_{ij} = (1 - x_i)C_{ij} \)

\[ W(x) = \text{diag}(x)I_n + \text{diag}(1_n - x)C \]
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The dynamics of social power and self-confidence

Reflected appraisal hypothesis by Cooley, 1902:

individual’ self-appraisal (e.g., self-confidence, self-esteem, self-worth) is influenced by the appraisal held by others of her

Mathematization by Friedkin, 2012:

along a sequence of issues, individual dampens/elevates self-weight \( x_i \) according to her relative prior control

\[ x(s + 1) = w(x(s)) \]

self-appraisal = self-weights relative control = social power

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along a sequence of issues, individual dampens/elevates self-weight \( x_i \) according to her relative prior control

\[ x(s) \xrightarrow{W(x(s))} \xrightarrow{w(x(s))} \]

reflected appraisal mechanism

\[ x(s + 1) = w(x(s)) \]
Influence Networks

For any $C$ row-stochastic, irreducible with zero diagonal and $c \in \Delta_n$,
- $\max\{c_i\} \leq 0.5$
- $c_i = 0.5 \iff G(C)$ is with star topology and $i$ is the center

DeGroot-Friedkin dynamics

$$F(x) = \begin{cases} e_i, & \text{if } x = e_i \text{ for all } i \\ \left( \frac{c_1}{1-x_1}, \ldots, \frac{c_n}{1-x_n} \right)/\sum_{i=1}^{n} c_i, & \text{otherwise} \end{cases} \text{ if } x = e_i \text{ for all } i$$

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$$F : \Delta_n \to \Delta_n$$ locally Lipschitz
The vertices $\{e_i\}$ are fixed points under $F$
relative interpersonal weights $C$ play role only through $c$
$c = \text{appropriate eigenvector centrality (dominant left eigenvector)}$
Main results for generic “relative interpersonal accorded weights”

- unique non-trivial fixed point: $x^* = x^*(c)$ in interior of $\Delta_n$
- convergence = forgetting initial conditions for all non-trivial initial conditions,
  $\lim_{k \to \infty} x(k) = \lim_{k \to \infty} w(x(k)) = x^*$
- accumulation of social power and self-appraisal
  - fixed point $x^* > 0$ has same ordering of $c$
  - social power threshold $T$ such that: $x^*_i \geq c_i \geq T$ or $x^*_i \leq c_i \leq T$

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**Doubly-stochastic $C$: emergency of democracy**

**Lemma (Convergence to democracy)**

If $C$ is doubly-stochastic:
1. the non-trivial fixed point of $F$ is $\frac{1}{n} e_n$.
2. for all non-trivial initial conditions,
   $\lim_{k \to \infty} x(k) = \lim_{k \to \infty} w(x(k)) = \frac{1}{n} e_n$.

- Uniform social power
- No power accumulation

**Star topology: emergency of autocracy**

**Lemma (Convergence to autocracy)**

If graph has star topology with center $j$:
1. there are no non-trivial fixed points of $F$ for all initial non-trivial conditions,
2. $\lim_{k \to \infty} x(k) = \lim_{x \to \infty} w(x(k)) = e_j$.

- Autocrat appears in star center
- Extreme power accumulation

**Proof methods**

1. existence via Brower fixed point theorem ($F$ continuous on compact)
2. ranking and uniqueness: elementary steps and contradictions
3. monotonicity: $i_{\max}$ and $i_{\min}$ are invariant
   $$i_{\max} = \arg \max_j x_j(0) x_j^- \implies i_{\max} = \arg \max_j x_j(s) x_j^- \quad \forall s$$
4. convergence: Lyapunov function decreasing everywhere $x \neq x^*$
   $$V(x) = \max_j \left( \ln \frac{x_j}{x_j^*} \right) - \min_j \left( \ln \frac{x_j}{x_j^*} \right)$$
Ongoing experiment

- 30 groups of 4 subjects in a face-to-face discussion
- opinion formation on a sequence of 15 issues
- issues in the domain of choice dilemmas:
  - what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff
- 15 groups under pressure to reach consensus, other 15 no
- On each issue, each subject privately recorded (in following temporal order):
  1. an initial opinion on the issue prior to the group-discussion,
  2. a final opinion on the issue upon completion of the group-discussion (which ranged from 3-27 minutes), and
  3. an allocation of 100 influence units (under the instruction that these allocations should represent their appraisals of the relative influence of each group member’s opinion on their own opinion).

Contributions and future work

Contributions
- a new perspective and a novel dynamical model for social power, self-appraisal, influence networks
- dynamics and feedback in sociology
- a new potential explanation for the emergence of autocracy see “iron law of oligarchy” by Michels 1911

Future work
- Robustness of results for distinct models of opinion dynamics
- Robustness of results for higher-order models of reflected appraisal


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