Dynamics of delayed car-following models: human vs. robotic drivers

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Summary. A general class of car-following models is studied with different driver reaction time configurations. In particular the stability of traffic flow is investigated in case of human-driven vehicles and computer-controlled (robotic) cars. It is shown that in both cases, time delays can change the frequency of arising oscillations and, consequently, the wavelength of the emerging traveling waves, leading to high-frequency / short-wavelength oscillations. Furthermore, interacting the with the nonlinearities in the system, time delays can make the dynamics excitable, such that waves may be triggered when the uniform flow is linearly stable. Finally, low-frequency and high-frequency oscillations can interact at the nonlinear level leading to very complex dynamics.

Modeling car following

Car following has been studied for six decades and there exist about a hundred different models which can reproduce traffic behavior for chosen sets of driver parameters. Still, no fundamental modeling principles has been established to guide scientists and engineers how to construct car-following models and what kind of qualitative requirements these models must satisfy. In order to change this tendency, a new direction has been identified in transportation science: instead of studying the dynamics of individual models quantitatively, the qualitative dynamics of classes of models shall be studied [1, 3]. Such investigations are useful for classifying the existing models and can also provide guidance for future models.

Here, we investigate a class of models where each driver monitors the kinematic properties of the driver ahead and reacts to such stimuli with reaction time delays. These models can describe the behavior of human drivers fairly accurately, and may also be applied when programming Autonomous Cruise Control (ACC) devices (that consist of a radar sensor and an on board computer and can actuate the vehicle based on information collected about the environment).

To obtain tractable models, usually identical vehicles are considered, so the acceleration of the ith vehicle is given by

$$\dot{v}_i(t) = f\big(h_i(t-\tau), \dot{h}_i(t-\sigma), v_i(t-\kappa)\big)\,, \tag{1}$$

where the dot stands for differentiation with respect to time t, v_i is the velocity of the ith vehicle, h_i is the distance between the ith and the $i+1$st vehicles, that is, $\dot{h}_i(t) = v_{i+1}(t) - v_i(t)$. The delays τ, σ, κ represent driver reaction times to different stimuli. Depending on whether human-driven or computer-controlled vehicles are considered, the following approximations can be made.

1. *Human driver setup:* $\tau = \sigma > 0, \kappa = 0$. This setup represents that drivers react to the distance and to the velocity difference with (the same) delay but they are aware of their own velocity immediately [3].
2. *Robotic driver setup:* $\tau = \sigma = \kappa > 0$. Here the delay accounts for the time needed for sensing, computation and actuation in a computer controlled vehicle [2].

Note that these time delays differ from the desired time gap (a time distance the driver wishes to keeps from the leader) and from the relaxation time (which is related to the inertia of the vehicle); such parameters are incorporated in the nonlinear function f.

In order to obtain a feasible model one must assume that the traffic system (1) possesses a one-parameter set of uniform flow equilibria:

$$h_i(t) \equiv h^*\,, \quad \dot{h}_i(t) \equiv 0\,, \quad v_i(t) \equiv v^*\,, \tag{2}$$

and that there exists a functional relationship between the equilibrium headway h^* and the equilibrium velocity v^*:

$$0 = f(h^*, 0, v^*) \quad \Rightarrow \quad v^* = V(h^*)\,. \tag{3}$$

Here the monotonically increasing non-negative function V expresses that the more sparse traffic is, the faster drivers want to go and the quantity $1/V'(h^*)$ corresponds to the desired time gap. For simplicity, we assume periodic boundary conditions, that is, N vehicles are placed on a circular road of length L, which yields the algebraic equation $\sum_{i=1}^{N} h_i(t) = L$. This determines the equilibrium headway $h^* = L/N$ and the equilibrium velocity v^* through (3). Analytical results can be obtained for arbitrarily large N, but here we present the diagrams for $N = 33$ vehicles, which is small enough to keep the illustrations readable but is large enough to represent the limit $N \to \infty$ for realistic traffic scenarios.

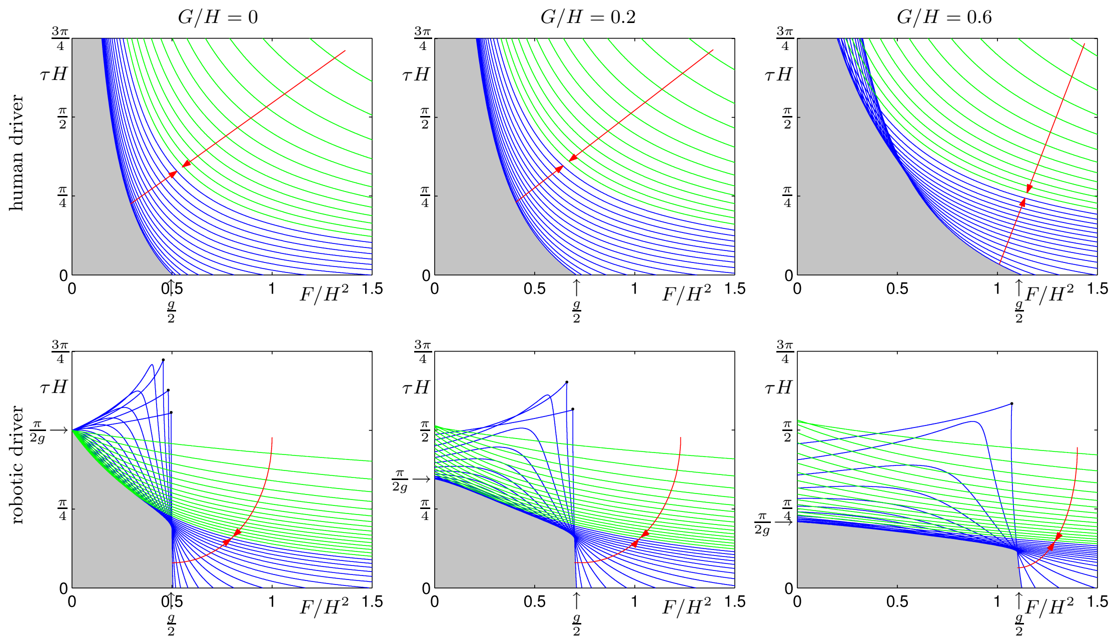

Figure 1: Linear stability diagrams for the general model (1) in case of human and robotic drivers. The stable regions are shaded and Hopf bifurcations take place when crossing curves as "going away" from the origin. The red arrows show the decrease of the discrete wavelength from L to $2L/N$. We use the notation $g = 2\frac{G}{H} + 1$ when indicating special points.

Linear instabilities and traveling waves

The equilibrium (2,3) may loose its stability via Hopf bifurcations which results in oscillations in the form of traveling waves that propagate along the chain of vehicles. To analyze this behavior we linearize (1) about the uniform flow (2), that is, using the variables $\tilde{v}_i(t) = v_i(t) - v^*$ and $\tilde{h}_i(t) = h_i(t) - h^*$ we obtain

$$\dot{\tilde{v}}_i(t) = F\tilde{h}_i(t-\tau) + G\dot{\tilde{h}}_i(t-\sigma) - H\tilde{v}_i(t-\kappa)\,, \tag{4}$$

where $\dot{\tilde{h}}_i(t) = \tilde{v}_{i+1}(t) - \tilde{v}_i(t)$. For physically realistic models, the derivatives of f must satisfy

$$F = \tfrac{\partial}{\partial h} f(h^*, 0, v^*) \geq 0\,, \quad G = \tfrac{\partial}{\partial \dot{h}} f(h^*, 0, v^*) \geq 0\,, \quad H = -\tfrac{\partial}{\partial v} f(h^*, 0, v^*) \geq 0\,. \tag{5}$$

When analyzing the stability of the trivial solution of (4), one may find the location of Hopf bifurcation curves in the space of non-dimensional parameters $(F/H^2, G/H, \tau H)$ for both the human driver setup and the robotic driver setup as shown in Fig. 1; see caption for description. One may observe that for small delay the uniform flow loses stability to long-wavelength ($\approx L$) oscillations, while for larger delays short-wavelength ($\approx 2L/N$) oscillations appear first. For the human driver setup such change only happens for sufficiently large G/H, while for the robotic driver setup this occurs any G/H. Furthermore, a trade-off may be observed: when increasing G/H the stable area increases in the F/H^2 direction, while it shrink in the τH direction. That is, the systems becomes more robust against long-wavelength excitations, but in the same time becomes more fragile against short-wavelength excitations.

Nonlinear effects in traffic dynamics

When investigating the dynamics at the nonlinear level (by normal from analysis and numerical continuation) one may find that time delays make the Hopf bifurcations robustly subcritical, that is, traveling waves may appear even when the uniform flow equilibrium is stable. In this case sufficiently large excitations can trigger traffic jams, while without these the flow remains smooth. This means that the behavior of an individual driver has an important role in determining the state of the whole traffic system. Studying this behavior can help one to identify the characteristics of undesired human behavior and also allow one to shape the response of ACC driven vehicles such that they can mitigate the effect of waves triggered by human drivers. This is a very challenging task because one must "damp down" low-frequency / long-wavelength as well as high-frequency / short-wavelength oscillations. Moreover, oscillations of different frequencies can "mix" at the nonlinear level since the nonlinear response of drivers yield multiple excitation frequencies. Consequently, an originally low-frequency excitation, when propagating along the chain of vehicles, eventually can lead to high-frequency oscillations or vice versa.

References

[1] Gasser I., Werner B. (2010) Dynamical phenomena induced by bottleneck *Phil. Trans. R. Soc. A* **368**:4543-4562.

[2] Orosz G., Moehlis J., Bullo F. (2010) Robotic reactions: delay-induced patterns in autonomous vehicle systems. *Phys. Rev. E* **81**:025204(R).

[3] Orosz G., Wilson R. E., Stépán G. (2010) Traffic jams: dynamics and control. *Phil. Trans. R. Soc. A* **368**:4455-4479.