Cooperative Pursuit with Sensing Limitations

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***Abstract*— We address a discrete-time pursuit-evasion problem involving multiple pursuers and a single evader in an unbounded, planar environment in which each player has limited-range sensing. The evader appears at a random location in a bounded region and moves only when sensed. We propose a *sweep-pursuit-capture* strategy for a group of at least three pursuers and determine a lower bound on the probability of capture for the evader. This bound is a function of the pursuer formation and independent of the initial evader's spatial distribution and the evader strategy. We also provide an upper bound on the time for our pursuit strategy to succeed. These results show that on the basis of maximizing the probability of evader capture per pursuer, the pursuers should search the bounded region as a single group (*conjoin*) rather than to divide the region into smaller parts and search simultaneously in smaller groups (*allocate*).**

I. Introduction

The game of pursuit can be posed as to determine a strategy for a team of pursuers to capture an evader in a given environment. By *capture*, we mean that the evader and some pursuer meet at the same location after a finite time. The aim of the pursuers is to capture an evader for any evader trajectory. The evader wins the game if it can avoid capture indefinitely. All the players have identical motion capabilities.

A. Related Work

The continuous time version of this problem has been studied by Ho *et al.* [1], Lim *et al.* [2] and Pachter [3] to cite a few. Recently, significant attention has been received by the discrete-time version of the game. The paper by Sgall [4] gives sufficient conditions for a single pursuer to capture an evader in a semi-open environment. This strategy has been extended by Kopparty and Ravishankar [5] to the case of multiple pursuers, in an unbounded environment, to capture a single evader which is inside their convex hull. Alonso *et al.* [6] and Alexander *et al.* [7] propose strategies so that the pursuer can reduce the distance between itself and the evader to a finite, non-zero amount after finite time steps. The game has also been studied in different types of bounded environments, e.g., circular environment by Alonso *et al.* [6], curved environments by LaValle *et al.* [8]. Visibility-based pursuit evasion has been studied by Guibas *et al.* [9], Sachs *et al.* [10] and in polygonal environments by Isler *et al.* [11].

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Each of above mentioned works proposes strategies which require that the pursuers have unlimited sensing capacity. A more realistic assumption is to introduce sensing limitations for the pursuers and the evader. In this context, Gerkey *et al.* [12] have studied a version of visibility limited to an angle, instead of the entire region. This is termed as *searching using a flashlight*. Suzuki and Yamashita have studied visibility limited to k-searchers, where the pursuer has k such finite angle search flashlights [13]. Isler *et al.* [14] have considered the problem on a graph, with the visibility of the pursuer limited to nodes adjacent to the current node of a pursuer. A framework which uses probabilistic models for sensing devices for the agents can be found in the works of Hespanha *et al.* [15] and Vidal *et al.* [16].

B. Contributions

We consider the case of sensing capabilities restricted to a closed disc of a given sensing radius, around the current positions of the players. To the best of our knowledge, this is the first paper in pursuit-evasion that uses such a model for limited sensing. The motion of each player is confined to a closed disc of a given stepping radius around its current position. The game is played in an unbounded, planar environment. The evader is assumed to be born in a bounded region known to the pursuers. The evader follows a *reactive rabbit* model [14], i.e., moves only when detected. We propose a strategy for the pursuers, comprising of three phases - sweep, pursuit and capture. In the sweep phase, the pursuers search the bounded region in a proposed formation. They *succeed* when they detect the evader inside a special *capture* region, which we characterize for the pursuer formation. We show that using our sweep strategy, the probability of pursuer success is a function of the pursuer formation and independent of the initial evader distribution. Next, we propose a cooperative pursuit strategy for the pursuers to confine the appropriately-sensed evader within their sensing discs. We show that using this pursuit strategy, the problem is converted into a previously-studied problem of pursuit-evasion with unlimited sensing. We also give an upper bound on the time for our trapping strategy to converge.

Of all proposed pursuer formations, we define a *limiting* formation that gives maximum probability of evader capture for a given number of pursuers. For this limiting formation, we analyze the pursuers' decision to search the bounded region as a single group (*conjoin*) or to divide the region into smaller parts and search simultaneously in smaller groups (*allocate*).

C. Biological Motivation

The inspirations for the strategies proposed in this paper have been derived from aspects of animal behavior. It is well known that predators hunt as a conjoined group, when it is less efficient to hunt alone. This behavior is also observed when the prey is large or can move as fast as the predators [17]. Further, predators show an inclination towards specialized behavior by maintaining a fixed formation during search and capture of preys [18]. Such specializations suggest that there may be configurations that are preferred during group hunting. Also, in presence of sensing limitations, groups tend to maintain spacing between each other that is regulated by their sensory capabilities [19]. These facts give us some additional hints towards selecting capture-conducive predator formations.

D. Organization

The problem assumptions and its mathematical model are presented in Section II. Pursuer formations are defined in Section III. In Section IV, we describe the *search-pursuit-capture* strategies and give the corresponding results. The respective proofs are given in Section V. Section VI deals with the analysis for determining whether the pursuers should *allocate* or *conjoin*. In figures, circles around the agents denote their sensing ranges.

II. Problem Set-up

We assume that there exists a finite, region, $\mathcal{G} \subset \mathbb{R}^2$, where an evader appears with a uniform spatial density. The motion and sensing abilities of the evader are restricted to closed discs of radii r_{step} and r_{sens} respectively, around the current evader position. We have a total of n pursuers, who have the same motion and sensing capabilities as those for the evader. We assume a discrete-time model with alternate motion of the evader and the pursuers. The pursuers have complete communication between themselves, i.e., they can communicate the location of a sensed evader as well as their own position with respect to a fixed, global reference frame, among themselves. The evader wins if it can avoid being captured indefinitely. We seek pursuer strategies to capture the evader.

Define ${\mathbb{R}^2}_{\text{e}} = \mathbb{R}^2 \cup \phi$, where $\mathbb{R}^2$ is the unbounded, planar environment and ϕ is the null element. Here, the null element refers to the fact that during sensing, the measurement of the position of an evader may not be available to all pursuers. Let $e[t]$ and $p_k[t]$ denote the absolute positions of the evader and the k^{th} pursuer respectively, at time t. Here, k takes all values in $\{1,\dots,n\}$. The equations of motion, in discrete-time, can be written as,

$$\begin{aligned} e[t+1] &= e[t] + u^e(e[t], \{y^{p_k}[t]\}), \\ p_k[t+1] &= p_k[t] + u^{p_k}(e[t], y^e[t+1], p[t]), \end{aligned} \tag{1}$$

where $y^{p_k}[t] \in {\mathbb{R}^2}_{\text{e}}$ is the measurement of the k^{th} pursuer position taken by the evader at the t^{th} time instant and $y^e[t+1] \in {\mathbb{R}^2}_{\text{e}}$ is the measurement of the evader position taken by some pursuer at the $(t+1)^{th}$ time instant. These are given by

$$y^{p_k}[t] = \begin{cases} p_k[t], & \text{if } \|p_k[t] - e[t]\| \leq r_{\text{sens}}, \\ \phi, & \text{otherwise.} \end{cases}$$

Similarly,

$$y^e[t+1] = \begin{cases} e[t+1], & \text{if for some } k \in \{1,\dots,n\}, \\ & \|p_k[t] - e[t+1]\| \leq r_{\text{sens}}, \\ \phi, & \text{otherwise.} \end{cases}$$

The functions $u^e : {\mathbb{R}^2}_{\text{e}} \times \mathbb{R}^2 \to \mathbb{R}^2$ and $u^{p_k} : {\mathbb{R}^2}_{\text{e}} \times {\mathbb{R}^2}_{\text{e}} \times \mathbb{R}^2 \to \mathbb{R}^2$ are termed as *strategies* for the evader and pursuer respectively. Due to the *reactive rabbit* model for the evader, $u^e = 0$ until the evader is sensed by the pursuers for the first time. The constraint on the maximum step size gives,

$$\|u^e\|, \|u^{p_k}\| \leq r_{\text{step}}.$$

Let t_0 denote the time at which the evader is detected by the pursuers. We seek sufficient conditions on $e[t_0]$ relative to positions of the pursuers $p_k[t_0]$ and a corresponding pursuers strategy u^{p_k} so that the pursuers will capture the evader for any evader strategy u^e.

III. Some Preliminaries

In this section, we define certain preliminaries which we propose to use in our solution. We define two notions of *capture* as follows.

Definition III.1 (Capture notions) *The evader is said to be* captured *by the pursuers if for any evader strategy u^e, some pursuer is at the same position as the evader after a finite time.*

Similarly, the evader is said to be trapped within the sensing radii *of the pursuers if for any evader strategy u^e, the motion disc of the evader is completely contained within the union of the sensing discs of the pursuers after a finite time. We define the* trapping time t^* *as the time taken by the pursuers to trap the evader within their sensing radii.*

We would like to point out here that the time needed to capture as well as the trapping time could possibly depend on the initial relative locations of the pursuers and the evader.

Let the sensing radius be κ times the stepping radius. We assume throughout our problem that κ is large enough: typically greater than $2\sqrt{2}$. We define the following pursuer formation.

Definition III.2 (Trapping chain) *A group of $n \geq 3$ pursuers $\{p_1,\dots,p_n\}$ are said to be in a* trapping chain *formation if*

(i) *$p_1,\dots,p_n$ are at the vertices of a convex polygon, and*
(ii) *for all $k \in \{1,\dots,n-1\}$,*

$$\|p_k - p_{k+1}\| \leq 2r_{\text{step}}\sqrt{\kappa^2 - 4}.$$

An example of a trapping chain is shown in Figure 1. We define the *capture region* for a trapping chain as under.

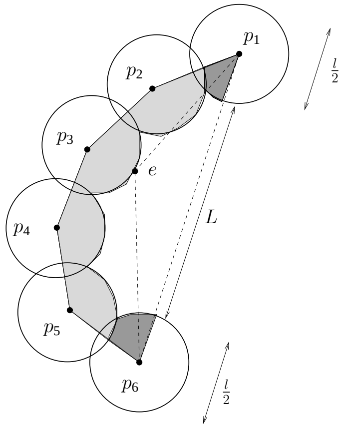

Fig. 1. A trapping chain

$$\mathcal{S}[t] = \bigcup_{k\in\{2,\ldots,n-1\}} \mathcal{B}_{p_k}(r_{\text{sens}}) \cap \mathring{\mathrm{Co}}\{p_1,\ldots,p_n\}[t].$$

The lightly shaded region in Figure 1 is the proposed capture region, $\mathcal{S}$, for the trapping chain.

There is a chance that the evader can step into a region such that the pursuers trap it within their sensing radii. So we define an *extended capture region* for the trapping chain as under,

$$\mathcal{S}^e[t] = \bigcup_{k\in\{1,\ldots,n\}} \mathcal{B}_{p_k}(r_{\text{sens}}) \cap \mathring{\mathrm{Co}}\{p_1,\ldots,p_n\}[t].$$

The darkly shaded region along with the lightly shaded region in Figure 1 is the extended capture region, $\mathcal{S}^e$, for the trapping chain.

IV. The Algorithms and Main Results

We have three phases of sweep, pursuit and capture. In this section, we describe the pursuer strategies in each phase and the corresponding results. The proofs of the main results are given in Section V. Throughout the three phases, the pursuers maintain a trapping chain formation. The following are the strategies in each phase.

A. *Sweep Phase*

In this phase, the aim of the pursuers is to sense an evader within the capture region of the trapping chain. For this purpose, we propose that the pursuers sweep $\mathcal{G}$ in the direction of the outward normal to p_1p_n, with respect to the convex hull of the pursuers. We demonstrate our result for a square region $\mathcal{G}$ of length b. But it would be clear from our approach that the result is valid for any bounded environment. For a trapping chain shown in Figure 1, we define,

$$L = \|p_1 - p_n\| - 2r_{\text{sens}},$$
$$l = 4r_{\text{sens}}.$$

As the pursuers move in the direction described earlier, they clear a rectangular strip of length b and width of at most $L + l$, of which the favorable length is L. The *sweeping policy* for the pursuers is as follows.

(i) Choose the first rectangular strip at a random distance, l_0, from one edge of $\mathcal{G}$ and sweep it length-wise. The distance l_0 is a uniform random variable taking values in the interval $[-\frac{l}{2}, L + \frac{l}{2}]$. Here, negative l_0 implies that some of the pursuers may begin sweeping from outside the region $\mathcal{G}$.

(ii) Form a sweeping path for $\mathcal{G}$ and sweep along adjacent strips as shown in Figure 2.

The shaded region in Figure 2 refers to the area that would fall in the proposed capture region, $\mathcal{S}$. Now we are interested in determining the probability that an evader falls in the shaded region in Figure 2. That is given by the following result.

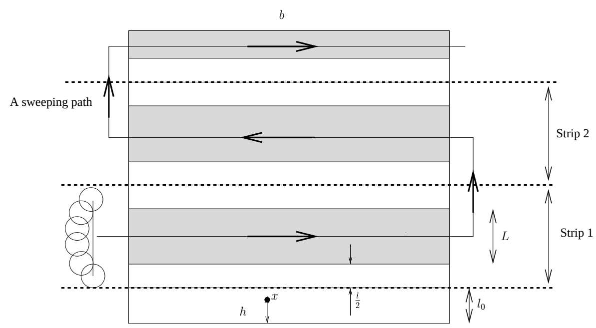

Fig. 2. A sweeping path

Theorem IV.1 (Sweep property) *For an evader located anywhere in $\mathcal{G}$, the probability, P, of detecting it inside $\mathcal{S}$ for a group of pursuers in a trapping chain, using the* sweeping policy*, is given by*

$$P = \frac{L}{L + l},$$

where L and l are defined in Figure 1.

Notice that the probability of pursuer success, P, depends only on the number of pursuers n and the relative pursuer locations. We shall refer to it as $P(n)$ from now on. We will use the result in Theorem IV.1 in Section VI. The pursuers win when the evader is detected in $\mathcal{S}$ by the pursuers. Otherwise, the evader is scared away and lost forever.

B. *Pursuit phase: algorithm* TRAP

Once an evader has been detected within the proposed capture region at time t_0, the pursuers need to ensure that the evader is trapped within their sensing ranges. For this purpose, we propose the following algorithm,

At each time step $t \geq t_0$,

(i) While $e[t+1] \notin \mathcal{S}^e[t]$, the pursuers move towards the circumcenter O of $\triangle p_1[t_0]e[t_0]p_n[t_0]$ with maximum step size.

(ii) Otherwise, one of the pursuers which senses the evader directly, moves with maximum step towards the evader and the others move parallel to that pursuer with the maximum step size.

The notation $\triangle XYZ$ denotes the triangle formed by points X, Y and Z. One such move is shown in Figure 3. In case (i) of the algorithm, note that the pursuers may not sense the evader in all the subsequent moves. But the idea is that the pursuers will encircle the evader by completing the chain around it and trap it within the overlapping chain. Then the pursuers can shrink the chain around the evader. Thus, we propose the result,

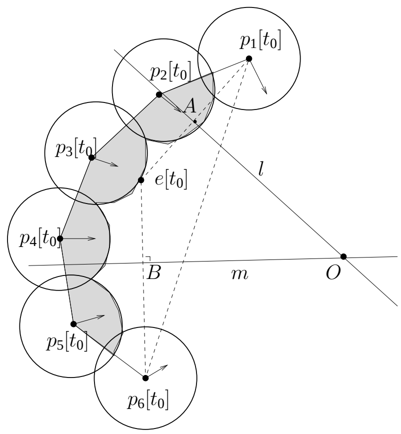

Fig. 3. Algorithm TRAP

Theorem IV.2 (TRAP) *Starting from a trapping chain formation, the pursuers trap the evader within their sensing radii using algorithm* TRAP*, if* $e[t_0] \in \mathcal{S}[t_0]$*. The* trapping time t^* *using algorithm* TRAP *satisfies,*

$$t^* \leq \max_{k \in \{1,\ldots,n\}} \left\lceil \frac{\|p_k[0] - O\|}{r_{\text{step}}} \right\rceil, \tag{2}$$

where O is the circumcenter of $\triangle p_1[t_0]e[t_0]p_n[t_0]$.

The following corollary helps us to link the end of the pursuit phase to the commencement of the final capture phase.

Corollary IV.3 *At the end of algorithm* TRAP*, the evader is inside the convex hull of the pursuers.*

C. The Capture phase

Once an evader is captured within the sensing ranges of the pursuers, the pursuers now have access to the next position of the evader at the present time instant. So the problem reduces to one having unlimited sensing capabilities for the pursuers. A capture strategy for this phase is algorithm SPHERES proposed by Kopparty and Ravishankar [5], which is being reproduced here for clarity and completeness. Let the time at the end of the pursuit phase be $(t_0 + t^*)$.

- Each pursuer p_k initially selects (by communication) a point C_k such that,
  - $p_k[t_0+t^*]$ lies on the line segment $C_k e[t_0+t^*]$ and
  - The connected component of

    $$\mathbb{R}^2 \setminus \cup_{k=1}^{n} \mathcal{B}_{C_k}(\|C_k - p_k[t_0 + t^*]\|)$$

    that contains $e[t_0 + t^*]$ is bounded. Here $\mathbb{R}^2$ refers to the entire unbounded environment. This is illustrated in Figure 4.
- For every pursuer, choose $p_k[t+1]$ on line joining $e[t+1]$ and C_k such that $\|p_k[t+1] - e[t+1]\|$ is minimized, subject to $\|p_k[t+1] - p_k[t]\| \leq r_{\text{step}}$. This move is shown in Figure 5.

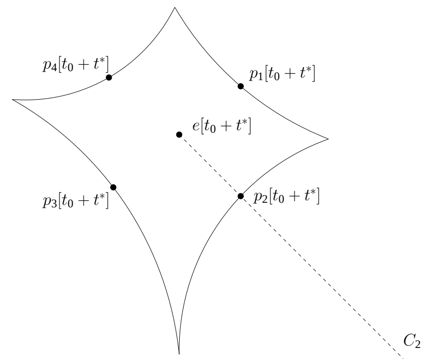

Fig. 4. Algorithm SPHERES: Illustrating selection of points C_k.

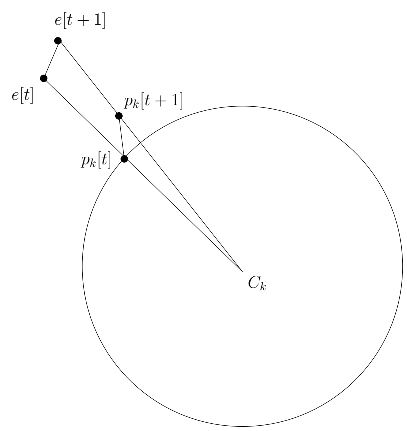

Fig. 5. Algorithm SPHERES: Illustrating a pursuer move.

The algorithm TRAP ensures that until convergence, after every pursuer move, the evader is inside the convex hull of the pursuers. Thus, final capture follows from the following theorem, the proof of which can be found in [5].

Theorem IV.4 (SPHERES [5]) *Assume the evader lies within the convex hull of the pursuers. If every pursuer follows the algorithm* SPHERES*, then the evader will be captured in finite number of steps.*

We would like to point out an important property of algorithm SPHERES: The distance between every pursuer and the evader never increases at the end of every pursuer move [5]. Thus, once the evader is trapped within the sensing ranges of the pursuers, it would remain trapped within their sensing ranges at the end of every move using algorithm SPHERES. The capture phase is now complete.

V. Proofs of the Main Results

The main results in Section IV are proved in this section.

A. The Sweep phase

Proof of Theorem IV.1:

Let the evader be located at any point $x \in \mathcal{G}$ as shown in Figure 2. Let its distance from the lower edge be h. The event that x would lie in the shaded region is given by $l_0 + \frac{l}{2} < h$ or $l_0 - \frac{l}{2} > h$. Thus, if we consider the interval $[-\frac{l}{2}, L + \frac{l}{2}]$, where l_0 takes values, the favorable interval is of length L. Thus, the probability of success for the pursuers is equivalent to determining the ratio of the lengths of the favorable interval, i.e., L to the total interval, i.e., $L + l$. Hence, the result follows. ■

Remark V.1 It is worthwhile to mention here that the probability of success for the proposed sweeping policy for the pursuers is *independent* of the evader's location x. Thus, the optimal policy for the evader in the present framework is to have a uniform spatial probability density of arrival in $\mathcal{G}$. This justifies our problem assumption about evader arrival probability.

B. The Pursuit phase

To prove Theorem IV.2, we first state the following properties of a trapping chain. These properties follow from the definitions of trapping within sensing radii and of the trapping chain.

Lemma V.2 (Trapping chain properties) *If* $e[t] \in \mathcal{S}[t]$, *then the following statements hold.*

(i) *If* $\operatorname{dist}(e[t], \overline{p_k[t]p_{k+1}[t]}) > r_{\text{step}}$, *for all* $k \in \{1, \dots, n-1\}$, *then the evader cannot step outside* $\operatorname{Co}\{p_1[t], \dots, p_n[t]\}$ *by crossing* $\overline{p_k[t]p_{k+1}[t]}$.

(ii) *If* $\operatorname{dist}(e[t], \overline{p_k[t]p_{k+1}[t]}) \leq r_{\text{step}}$, *for some* $k \in \{1, \dots, n-1\}$, *then the evader is* trapped within the sensing radii *of pursuers* p_k *and* p_{k+1}.

Proof of Theorem IV.2: If $\operatorname{dist}(e[0], \overline{p_k[t_0]p_{k+1}[t_0]}) \leq r_{\text{step}}$ for some $k \in \{1, \dots, n-1\}$, then the evader is already trapped within the sensing ranges of the pursuers, from part (ii) of Lemma V.2. So let $\operatorname{dist}(e[t_0], \overline{p_k[t_0]p_{k+1}[t_0]}) > r_{\text{step}}$, for every $k \in \{1, \dots, n-1\}$. If $e[t+1] \in \mathcal{S}^e[t]$, for any $t \geq t_0$, then the pursuers would use part (i) of the algorithm TRAP and the result follows.

Finally, if $e[t_0+1] \notin \mathcal{S}[t_0]$, then the pursuers compute the circumcenter O of $\triangle p_1[t_0]e[t_0]p_n[t_0]$. Construct the perpendicular bisectors, l and m of $\overline{e[t_0]p_1[t_0]}$ and $\overline{e[t_0]p_n[t_0]}$ respectively, as shown in Figure 3. Any point on the side opposite to $e[t_0]$ of the lines l and m can be reached faster by p_1 and p_n respectively. Since all the other pursuers are moving towards O, the overlap between their sensing discs increases at each step. Thus, the motion of the evader is confined to the quadrilateral $e[t_0]AOB$, which is cleared by the pursuers in finite time. The best path for the evader is to move along $e[t_0] - O$ with maximum step size. Since $r_{\text{sens}} > r_{\text{step}}$, the sensing discs of pursuers p_1 and p_n overlap before the evader can reach O, thus closing the trapping chain around the evader. Note that the evader is within the convex hull of the pursuers at the end of every pursuer move.

The bound in equation (2) is the time taken by the furthest pursuer (and hence all the pursuers) to reach point O. Thus, clearly at the end of this time, the sensing discs of the pursuers would have covered the interior of their convex hull. Thus, after at most $\max_{k\in\{1,\dots,n\}} \left\lceil \frac{\|p_k[t_0]-O\|}{r_{\text{step}}} \right\rceil$ steps, the evader's motion circle would be contained within the sensing radii of the pursuers. ■

C. The Capture phase

The proof of Theorem IV.4 can be found in [5].

VI. Allocate or Conjoin?

Our analysis in the previous sections sheds some light onto the trade-offs that agents evaluate when deciding between *allocating* a task by dividing into smaller groups and performing the task as a *conjoined* group. We explore some of these trade-offs in what follows.

Given a total of kn pursuers and an environment large enough to avoid trivial cases, which of the following options is advantageous,

(i) Divide the environment $\mathcal{G}$ into k identical parts. Form k groups of n pursuers each and assign each group to a part of the environment. Run the sweep strategy independently on each group.

(ii) Form a single chain of kn pursuers and search $\mathcal{G}$ using the sweep strategy.

Since the pursuit and capture phases imply capture, the probability of successful evader detection in the sweep phase is equal to the probability of capture of the evader for our sweeping strategy. From the results in Section IV, it is evident that larger the total width of the sweep, i.e., $L + l$, the higher is the probability P. But, beyond a certain configuration, the convergence time for algorithm TRAP may become arbitrary large. Such a configuration results when $\overline{p_1p_n}$ is tangent to the sensing discs of all the other pursuers. The separation between the pursuers is $2r_{\text{step}}\sqrt{\kappa^2-4}$.

To obtain finite upper bound on time, we define a limiting formation as under,

Definition VI.1 (Limiting trapping chain) *Given a* $\delta > 0$, n *pursuers are said to be in the limiting trapping chain if,*

(i) *for all* $k \in \{1, \dots, n-1\}$,

$$\|p_k - p_{k+1}\| = 2r_{\text{step}}\sqrt{\kappa^2-4} \triangleq d,$$

(ii) *for all* $k \in \{2, \dots, n-1\}$,

$$\operatorname{dist}(p_k, \overline{p_1p_n}) = r_{\text{sens}} + \delta.$$

Such a limiting configuration is shown in Figure 6. Theorem IV.1 leads to the following result for a limiting chain.

Proposition VI.2 (Limiting trapping chain property) *For a limiting trapping chain,*

$$P(n) = \frac{nd - (3d - 2c)}{nd - (3d - l - 2c)}.$$

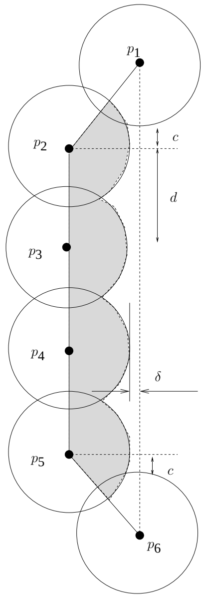

Fig. 6. A limiting trapping chain

We define a performance metric as the probability of capturing the evader per pursuer, i.e., $\frac{P(N)}{N}$, where $N \triangleq kn$ is the total number of pursuers. Intuitively, this metric refers to the amount of benefit per pursuer. Using option (i) for a limiting trapping chain,

$$\frac{P(N,k)}{N} = \frac{Nd - (3d - 2c)k}{N(Nd - (3d - l - 2c)k)}. \tag{3}$$

Equation (3) follows from the fact that when we divide $\mathcal{G}$ into k identical parts, the probability that the evader would be in any one of the parts is $\frac{1}{k}$ and the fact that the capture of the evader by one group implies capture by no other group. The quantity, $\frac{P(N,k)}{N}$, is a maximum when $k = 1$. Thus, searching the environment as a conjoined chain is the better option. This fact is supported by a "strong pack adhesive behavior" in wolves, refer Section 14.5 from [20].

Let us examine the effect of increasing the size of a single group. Consider a single limiting trapping chain, i.e., $k = 1$ and $N = n$. Equation (3) gives us,

$$\frac{P(n)}{n} = \frac{nd - (3d - 2c)}{n(nd - (3d - 2c - l))}.$$

The plot of $\frac{P(n)}{n}$ versus n reveals that the measure increases initially, reaches a maximum and then decreases as shown in Figure 7. Thus, there exists an optimal number of pursuers in a single chain. This result is analogous to the results in Model 3 of Packer and Ruttan [17]. This fact is also observed in sizes of wolf-packs which are noted to be ranging from 3 to 15, refer Section 14.1 from [20]. □

VII. Conclusions and Future Directions

We have investigated the problem of capturing an evader with multiple pursuers in an unbounded environment. Our approach is novel in that it considers the case of sensing capabilities limited to a finite disc. We propose a *sweep-pursuit-capture* strategy for a group of pursuers to capture an evader placed randomly inside a bounded environment. We give an expression for the probability of success in the sweep phase which is a function of the pursuer formation and *independent* of the environment and the location of the evader inside it. We then give a deterministic policy using a novel algorithm TRAP in the *pursuit* phase followed by an existing algorithm SPHERES [5], to complete the *capture*. Thus, the probability of success in the sweep phase is also the probability of capturing an evader. We then evaluate a decision to be made by a group of pursuers to either *conjoin* or *allocate*, during the sweep phase. Using the measure of success probability per pursuer, we conclude that it is advantageous for the pursuers to sweep the region as a *conjoined* unit. This result has similar analogies in the behavior of wolves during hunting operations.

In this paper, we consider any arbitrary motion for the evader, once it is detected. In reality, evader motion can be specialized or predictable. We have assumed that the pursuers group together as a chain. Interesting future directions would be to determine and characterize pursuer formations and possibly more efficient strategies for specialized evader behaviors. Additional information on the total time to capture would shed more light on the tendencies to *allocate* or *conjoin*.

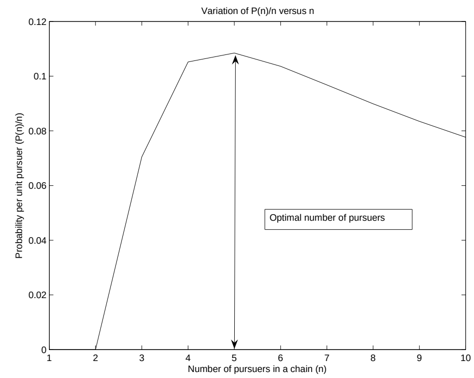

Fig. 7. Plot of $\frac{P(n)}{n}$ versus n

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