Coverage control for mobile sensing networks

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***Abstract*— This paper describes decentralized control laws for the coordination of multiple vehicles performing spatially distributed tasks. The control laws are based on a gradient descent scheme applied to a class of decentralized utility functions that encode optimal coverage and sensing policies. These utility functions are studied in geographical optimization problems and they arise naturally in vector quantization and in sensor allocation tasks. The approach exploits the computational geometry of spatial structures such as Voronoi diagrams.**

I. Introduction

Technological advances in wireless networking and in miniaturization of electro-mechanical systems are leading to the design and deployment of swarms of interconnected robotic systems. Communicating through ad-hoc networks, large numbers of coordinated autonomous vehicles will perform a variety of challenging tasks in aerial, underwater, space, or land environments. In scientific and commercial domains, coordinated vehicles will perform search and recovery operations, manipulation in hazardous environments, exploration, surveillance and reconnaissance, distributed data collection and fusion, and environmental monitoring for pollution detection and estimation.

Our central motivation is provided by distributed sensing networks in scientific exploration or surveillance missions. The motion coordination problem is to maximize the information provided by a swarm of vehicles taking measurements of some process. A similar problem arises when the sensors are either mobile or reconfigurable, e.g., range and focus or pan and tilt of an active camera system.

Working prototypes of such sensing networks have already been developed; see [1], [2], [3], [4]. In [4], launchable miniature mobile robots communicate through a wireless network. The vehicles are equipped with various micro electro-mechanical devices including sensors for vibrations, acoustic, magnetic, and IR signals as well as an active video module (i.e., the camera or micro-radar is controlled via a pan-tilt unit). A related system is suggested in [5] under the name of Autonomous Oceanographic Sampling Network; see also [6], [7], [8]. In this case, underwater vehicles are envisioned measuring temperature, currents, and other distributed oceanographic quantities. The vehicles communicate via an acoustic local area network and coordinate their motion in response to local sensing information and to evolving global data. This distributed sensing network would provide the novel ability to sample the environment adaptively in space and time. By identifying evolving temperature and current gradients with higher accuracy and resolution than current static sensors, this technology could lead to the development and validation of improved oceanographic models.

Literature Review: Recent years have witnessed a large research effort focused on motion planning and motion control problems for multi-vehicle systems. Issues include formation control [9], [10], [11], [12], cooperative motion planning [13], [14], cooperative manipulation [15], conflict avoidance [16], [17], and architectures for distributed control [18]. Motivated by applications in the context of distributed sensing networks, we identify a novel "coverage" control problem for multi-vehicle systems and we strive to design decentralized control laws that optimize the vehicles' locations for sensing purposes. Our starting point is the survey [19] on centroidal Voronoi tessellations and the treatment of locational optimization problems in the textbook [20]. Furthermore, our approach is related to a number of methods in (i) vector quantization for image processing, (ii) design optimal quadrature rules, (iii) clustering analysis and the k-means problem, (iv) optimal resource placement, and (v) mesh optimization methods. For example, we refer the reader interested in algorithms for mesh optimization to the surveys [21], [22].

Statement of Contributions: Our technical approach is based on decentralized gradient methods for geographic cost functions called *locational optimization problems*; see [20]. Decentralized control laws in robotics have traditionally been the subject of behavior-based robotics [9], [18], [23] and have been designed mainly on the basis of heuristics. In this paper, we propose a formal definition of decentralized utility function. We notice how a class of geographic optimization problems called *locational optimization* precisely enjoys the required properties. We present our treatment for general manifold spaces, we provide a coordinate-free version of the differential of the locational optimization formula (and of its proof), and we collect a number of elementary facts about area, centroid, and polar moment of inertia for planar Voronoi regions. Finally, we present some ideas on how to include formation constraints in the coverage problem.

The paper is organized as follows. Section II presents some basic ideas and tools. Section II-B contains the definition of decentralized utility function and the locational optimization problem is discussed in Section III. A variety of simplifications take place when dealing with Euclidean spaces and metrics, as shown in Section IV.

II. Preliminaries

A. Setting up the coverage control

In this section we investigate decentralized control laws that achieve "uniform coverage" of a certain space. The problem is loosely stated as follows: given an area A and n vehicles, design a decentralized control law such that the overall vehicles' distribution over A is uniform. For $i \in \{1, \ldots, n\}$, let $p_i(t) \in \mathbb{R}^2$ denote the position of the ith vehicle at time t, and let

$$\dot{p}_i(t) = u_i \,, \qquad (1)$$

where the control u_i can depend only on local information, i.e., the location of p_i and of its neighbors. Since the control law depends only on neighbors, we refer to it as an interaction law between vehicles.

B. Decentralized utility functions

Consider a multi-vehicle system where each agent evolves on three dimensional Euclidean space or on more general spaces such as matrix Lie groups and symmetric spaces. Let the configuration space of each vehicle be the manifold with boundaries Q. A Riemannian metric $\langle\!\langle \cdot\,, \cdot \rangle\!\rangle$ on Q defines a metric tensor $\mathbb{G}$, a distance notion between points and boundaries on Q, nearest neighbor N_i to the point p_i, and gradient vector fields of scalar functions. Let Σ_n be the discrete group of permutations with the natural action on Q^n and let Q^n/Σ_n be the shape space of Q^n. We call $U : Q^n/\Sigma_n \mapsto \mathbb{R}_+$ a *decentralized utility function* if the *gradient control law*

$$u_i(p_1, \ldots, p_n) = -\operatorname{grad}_i U(p_1, \ldots, p_n), \qquad (2)$$

depends only on the location p_i and its nearest neighbor N_i. The notation $\operatorname{grad}_i U$ refers to the gradient of the function U with respect to the argument p_i. We shall also consider control laws that depend on a finite number of neighbors of the point p_i.

C. Abstract Voronoi diagrams

An overview of Voronoi diagrams is presented in [24], [25], concepts and applications are discussed in [26] and abstract Voronoi diagrams are discussed in [27]. Centroidal Voronoi tessellations are discussed in [19].

Let $\{p_1, \ldots, p_n\}$ be a collection of points in a metric space Q. Let the Voronoi region $V_i = V(p_i)$ be the set of all points $q \in Q$ such that $\operatorname{dist}(q, p_i) \leq d(q, p_j)$ for all $j \neq i$. If Q is a finite dimensional Euclidean space, the boundary of each V_i is a convex polygon. The set of regions $\{V_1, \ldots, V_n\}$ is called the Voronoi diagram for the generators $\{p_1, \ldots, p_n\}$. When the two Voronoi regions V_i and V_j are adjacent, p_i is called a *(Voronoi) neighbor* of p_j (and vice-versa). We also define the (i, j)-edge as $\Delta_{ij} = V_i \cap V_j$.

Voronoi diagrams can be defined with respect to various distance functions, for example with respect to the 1-, 2-, s-, and ∞-norm over $Q = \mathbb{R}^m$. Voronoi diagrams can be defined over Riemannian manifolds such as spheres and matrix Lie groups; see [28]. When $Q = \mathbb{R}^2$ and the distance function is Euclidean distance, it is known [20] that (i) the nearest vehicle p_j to p_i is a neighbor, (ii) the average number of neighbors is six.

III. Locational optimization

We present a utility function that measures the ability of a collection of vehicles to provide accurate distributed sensing. We rely on a class of "geographic optimization problems" known within the context of geographical information science; see [20], [26], [29].

Let $\phi : Q \mapsto \mathbb{R}_+$ be a *distribution density function*, that is a scalar function on Q. The measure ϕ plays the role of an "information density" or of a probability density function. In a uniform environment, one might set $\phi(q) = \operatorname{Volume}(Q)^{-1}$, whereas a non-uniform ϕ would be appropriate to monitor targets that navigate over pre-identified areas with high likelihood.

Assume each vehicle has a sensor that provides accurate local measurements and whose performance degrades with distance. Formally, let $f\left(\operatorname{dist}(q, p_i)\right)$ describe the performance degradation, e.g., noise, loss of resolution, etc, of the measurement at the point $q \in Q$ taken from the ith sensor at position p_i. The function $f : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is monotone increasing, one example being a Gaussian-shaped dependency $f(x) = 1 - \exp(-x)$.

The overall "sensing performance" or coverage measure is an integral over Q. To avoid all sensors monitoring the same area, we weigh the relative contributions of each sensor through a max operation, i.e., we define:

$$U(p_1, \ldots, p_n) = \int_Q \min_{i \in \{1,\ldots,n\}} f\left(\operatorname{dist}(q, p_i)\right) \phi(q) dq. \qquad (3)$$

The *locational optimization problem* is to minimize U; in network optimization, vector quantization, and the equivalent discrete problem is known as the n-means clustering problem. Using the notion of Voronoi diagram and denoting the measure element as $d\phi(q) = \phi(q) dq$, one can rewrite the locational optimization function as:

$$U(p_1, \ldots, p_n) = \sum_{i=1}^{n} \int_{V_i} f\left(\operatorname{dist}(q, p_i)\right) d\phi(q). \qquad (4)$$

Remark 3.1: The integral defining the locational optimization function is well defined over manifolds whenever a volume element is available. This is the case when the metric space Q is an oriented Riemannian manifold with a volume n-form. Examples include $\mathbb{R}^n$, sphere, and any Lie group.

A. Examples

We illustrate the locational optimization function via two examples.

First, let χ be a random variable over Q with probability density function ϕ. Given sensors at n locations $p_1, \ldots, p_n$, minimize the expected value of the distance of χ from the closest sensor, i.e., the expected value of the function

$$\min_{i \in \{1,\ldots,n\}} \operatorname{dist}(\chi, p_i).$$

This cost objective is equal to the cost function in equation (3) with $f(x) = x$ since

$$\mathrm{E}\left[\min_i \operatorname{dist}(\chi, p_i)\right] = \int_Q \min_i f\left(\operatorname{dist}(\chi, p_i)\right) \phi(q) dq.$$

Second, consider the problem of estimating an unknown parameter determining the evolution of a distributed quantity; see [30], [31], [32], [33], [34]. Specifically, let θ be a parameter to be identified, and assume a sensor at position $q \in Q$ acquires a measurement $y = y(\theta, q)$. Define a normalized version of the Fisher information value as $M(q, \theta) = (\partial y/\partial\theta)^2$, and recall from Cramer-Rao theorem that the covariance of any estimation algorithm based on the measurement y is lower-bounded by $1/M$. In other words, the location q is a good position to observe the parameter θ if the sensitivity $(\partial y/\partial \theta)$ is "large." The approach in [31], [33] can be described in our setting by the selection of density functions $\phi_1(q) = E\left[M\left(q, \theta\right)\right]$, or $\phi_2(q) = M(q, \widehat{\theta})$, where $\widehat{\theta}$ is the current estimate of θ.

B. The differential of the locational optimization function

We start with a preliminary result that is related to the integral form of the conservation of mass lemma in fluids [35] and to classic divergence theorems; see [36, Chapter I].

Lemma 3.2: Let $\Omega = \Omega(x) \subset Q$ be a region that depends smoothly on a real parameter $x \in \mathbb{R}$ and that has a well-defined boundary $\partial\Omega(x)$ for all x. Let ϕ be a density function over Q. Then

$$\frac{d}{dx}\int_{\Omega(x)} d\phi(q) = \int_{\partial\Omega(x)} \left\langle\!\left\langle \frac{dq}{dx}\,,\, n(q) \right\rangle\!\right\rangle d\phi(q)\,,$$

where n is the unit outward normal to $\partial\Omega(x)$, and where dq/dx denotes the derivative of the boundary points with respect to x.

The differential of the locational optimization function is presented in the following lemma. The proof is an extension to Riemannian manifolds of the procedure in [19]. An alternative proof for the Euclidean case is described in [37].

Lemma 3.3: The partial derivative of the locational optimization function is:

$$\frac{\partial U}{\partial p_i} = \int_{V_i} \frac{\partial}{\partial p_i} f\left(\mathrm{dist}(q, p_i)\right) d\phi(q).$$

Proof: The Voronoi regions $\mathcal{V} = \{V_i\}$ generated by $P = \{p_1, \dots, p_n\}$ provide a tessellation of the manifold Q. We let $P \mapsto \mathcal{V}(P)$ denote the mapping that associates a Voronoi tessellation to a collection of generators P. In what follows, we let $\mathcal{W} = \{W_i\}$ be a generic tessellation of the manifold Q, and we define

$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} f(\mathrm{dist}(q, p_i)) d\phi(q)\,.$$

Since $U(p_1, \dots, p_n) = \mathcal{H}(P, \mathcal{V}(P))$, we have

$$\frac{\partial U}{\partial p_i} = \frac{\partial \mathcal{H}}{\partial p_i}(P, \mathcal{V}(P)) = \frac{\partial \mathcal{H}}{\partial p_i} + \frac{\partial \mathcal{H}}{\partial \mathcal{W}}\Big|_{\mathcal{W}=\mathcal{V}} \frac{\partial \mathcal{V}}{\partial p_i}\,,$$

and since

$$\frac{\partial \mathcal{H}}{\partial p_i}(P, \mathcal{W}) = \int_{W_i} \frac{\partial}{\partial p_i} f(\mathrm{dist}(q, p_i)) d\phi(q)\,,$$

it suffices to show that $(\partial\mathcal{H}/\partial\mathcal{W})(\partial\mathcal{V}/\partial p_i)$ vanishes at $\mathcal{W} = \mathcal{V}$. We therefore focus on computing

$$\frac{\partial \mathcal{H}}{\partial \mathcal{W}} \frac{\partial \mathcal{V}}{\partial p_i} = \frac{\partial}{\partial p_i} \sum_{k=1}^{n} \int_{V_k(p_1, \dots, p_n)} \phi_k(q) dq \Big|_{\phi_k(q) = f(\mathrm{dist}(q, p_k))\phi(q)}$$

where we regard the functions $\phi_k(q) = f(\mathrm{dist}(q, p_k))\phi(q)$ independent of p_i. Since the motion of p_i affects the Voronoi region V_i and its neighboring regions V_j for $j \in \{j_1, \dots, j_{k_i}\}$, we have

$$\frac{\partial \mathcal{H}}{\partial \mathcal{W}} \frac{\partial \mathcal{V}}{\partial p_i} = \frac{\partial}{\partial p_i} \int_{V_i} d\phi_i(q) + \sum_{j \in \{j_1, \dots, j_{k_i}\}} \frac{\partial}{\partial p_i} \int_{V_j} d\phi_j(q).$$

Now, Lemma 3.2 provides the means to analyze the variation of an integral function due to a domain change. Since the boundary of V_i satisfies $\partial V_i = \bigcup_j \Delta_{ij}$, where $\Delta_{ij} = \Delta_{ji}$ is the edge between V_i and V_j, we have

$$\frac{\partial}{\partial p_i} \int_{V_i(p_i)} \phi_i(q) dq = \sum_{j \in \{j_1, \dots, j_{k_i}\}} \int_{\Delta_{ij}(p_i)} \left\langle\!\left\langle \frac{dq}{dp_i}\,,\, n_{ij}(q) \right\rangle\!\right\rangle d\phi_i(q)$$

$$\frac{\partial}{\partial p_i} \int_{V_j(p_i)} \phi_j(q) dq = \int_{\Delta_{ij}(p_i)} \left\langle\!\left\langle \frac{dq}{dp_i}\,,\, n_{ji}(q) \right\rangle\!\right\rangle d\phi_j(q),$$

where we define n_{ij} as the unit normal along Δ_{ij} outward of V_i, and where therefore we have $n_{ji} = -n_{ij}$. Collecting these results we write

$$\frac{\partial \mathcal{H}}{\partial \mathcal{W}} \frac{\partial \mathcal{V}}{\partial p_i} = \sum_{j \in \{j_1, \dots, j_{k_i}\}} \int_{\Delta_{ij}} \left\langle\!\left\langle \frac{dq}{dp_i}\,,\, n_{ij}(q) \right\rangle\!\right\rangle \Big(\phi_i(q) - \phi_j(q)\Big) dq.$$

When $\mathcal{W} = \mathcal{V} = \mathcal{V}(P)$, we have that $f(\mathrm{dist}(q, p_i)) = f(\mathrm{dist}(q, p_j))$ and therefore $\phi_i(q) - \phi_j(q) = 0$ for any q belonging to the edge Δ_{ij}. This concludes the proof. ∎

We summarize the discussion above as follows.

Proposition 3.1: The control law in equation (2) becomes

$$\begin{aligned} u_i(p_1, \dots, p_n) &= -\,\mathrm{grad}_i\, U(p_1, \dots, p_n) \\ &= -\mathbb{G}^{-1} \int_{V_i} \frac{\partial}{\partial p_i} f\left(\mathrm{dist}(q, p_i)\right) d\phi(q) \qquad (5) \end{aligned}$$

and makes the vehicles converge to an extremum point of the locational optimization function.

C. Formation constraints

Formation and distance constraints might arise for a variety of reasons including communication constraints in environment with obstacles. The following treatment is inspired by the presentation in [12].

A formation constraint function is a differentiable, positive definite, strictly convex function $F : Q \times \cdots \times Q \to \mathbb{R}_+$. The shape and orientation of the robot formation is uniquely determined by $(p_1, \dots, p_n) = F^{-1}(0)$. A semidefinite function F allows for a free orientation and location of the formation. Consider for example

$$F(p_1, \dots, p_n) = \sum_{i \neq j} \tau_{ij} \left(\mathrm{dist}(p_i, p_j) - d_{ij}\right)^2$$

where $\tau_{ij} = \tau_{ji} \geq 0$. Only relative distances appear, therefore the formation is maintained under rigid displacements.

To maximize coverage while maintaining formation, the vehicles need to solve the constrained nonlinear minimization problem

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \int_{V_i} f\left(\mathrm{dist}(q, p_i)\right) \phi(q) dq \\ \text{subject to} \quad & \sum_{i \neq j} \tau_{ij} \left(\mathrm{dist}(p_i, p_j) - d_{ij}\right)^2 = 0 \end{aligned}$$

Algorithms for this optimization problem can be designed in various manners. If the formation is to be maintained accurately as the agents move, one could employ Lagrange multipliers. If instead the formation constraint is to be regarded as a performance measure to be optimized together with the coverage measure, one could employ a penalty function method. In other words, a penalty function methods corresponds to a gradient descent control for the function $U(p_1, \ldots, p_n) + \lambda F(p_1, \ldots, p_n)$, for some scalar $\lambda > 0$.

IV. Euclidean Setting

In this section we start by reviewing definitions and expressions for the center of mass and the polar moment of inertia of planar regions and in particular of convex polygons. We later show the connection of these concepts with the treatment in the previous section.

Let V be a connected subset of the plane $\mathbb{R}^m$ with density function $\rho(q)$. The mass $M_V \in \mathbb{R}_+$, the centroid $C_V = (C_{V,x}, C_{V,y}) \in \mathbb{R}^m$, and the polar moment of inertia $J_{V,p} \in \mathbb{R}_+$ about the point p of the region V are defined as

$$
\begin{aligned}
M_V &= \int_V \rho(q)\, dq \\
C_V &= \frac{1}{M_V} \int_V q\, \rho(q)\, dq \\
J_{V,p} &= \int_V \|q - p\|^2 \rho(q)\, dq\, .
\end{aligned}
$$

Additionally, by the parallel axis theorem, one can write,

$$J_{V,p} = J_{V,C_V} + M_V \|p - C_V\|^2 \tag{6}$$

where $J_{V,C_V} \in \mathbb{R}_+$ is defined as the polar moment of inertia of the region V about its centroid.

Next, we show how, under certain hypothesis, the integration step necessary to compute the control law (5) can be avoided by taking into account the problem geometry. Indeed, we obtain an *algebraic* expression of the gradient control law in terms of the vertices of the Voronoi regions.

A. Voronoi Regions in $\mathbb{R}^m$

We make the following four assumptions in the locational optimization problem. Assume the n sensors live on a compact polyhedra in $\mathbb{R}^m$, and the distance function is $\mathrm{dist}(q, p_i) = \|q - p_i\|$. Furthermore, assume that $f(x) = x^2$ and $\phi(q) = \rho(q)$. Then the locational optimization function in equation (4) becomes

$$
\begin{aligned}
U(p_1, \ldots, p_n) &= \sum_{i=1}^{n} \int_{V_i} \|q - p_i\|^2 \rho(q)\, dq \\
&\equiv \sum_{i=1}^{n} J_{V_i, p_i} \\
&= \sum_{i=1}^{n} J_{V_i, C_{V_i}} + \sum_{i=1}^{n} M_{V_i} \|p_i - C_{V_i}\|^2
\end{aligned}
$$

where J_{V_i,p_i} is the polar moment of inertia of the Voronoi region V_i about the point p_i, and M_{V_i} is the mass of the Voronoi region V_i.

Additionally, the control law in equation (5) becomes

$$
\begin{aligned}
\frac{\partial U}{\partial p_i} &= \int_{V_i} \frac{\partial}{\partial p_i} \left(\|q - p_i\|^2\right) \rho(q) dq \qquad (7)\\
&= 2 \int_{V_i} (p_i - q) \rho(q) dq \\
&= 2 \left(p_i \int_{V_i} \rho(q) dq - \int_{V_i} q \rho(q) dq \right) \\
&= 2 M_{V_i} (p_i - C_{V_i})\, . \qquad (8)
\end{aligned}
$$

It is worth noting that the control law $\dot{p_i} = -\partial U / \partial p_i = 2M_{V_i}(C_{V_i} - p_i)$ has the geometric interpretation that each vertex goes toward the centroid of its Voronoi region. In other words, the equilibrium state is reached when all vertices are in the centroid of their respective Voronoi polygons. Furthermore, the function U and its partial derivative depend uniquely on the Voronoi polygon V_i and the position p_i, which makes the control law decentralized. Similar arguments are at the basis of the Lloyd algorithm for vector quantization described in [19].

B. Voronoi Regions in $\mathbb{R}^2$ with Uniform Density

In this section, we assume the Voronoi region V_i is a convex polygon on a plane with N_i vertices labeled $\{(x_0, y_0), \ldots, (x_{N_i-1}, y_{N_i-1})\}$ such as in Figure 1. It is convenient to define $(x_{N_i}, y_{N_i}) = (x_0, y_0)$. Furthermore, we assume that the density function is unity, i.e. $\phi(q) = \rho(q) = 1$. By evaluating the integrals over the polygon, one can obtain

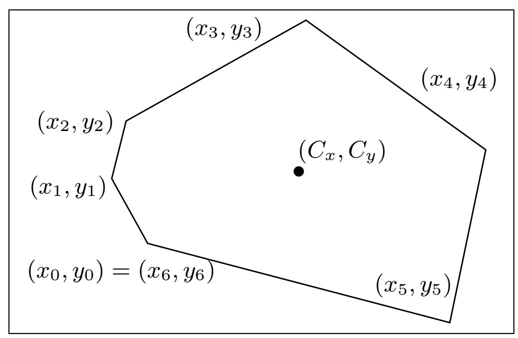

Fig. 1. Notation conventions for a convex polygon.

the following closed form expressions

$$
\begin{aligned}
M_{V_i} &= \frac{1}{2} \sum_{k=0}^{N_i-1} (x_k y_{k+1} - x_{k+1} y_k) \\
C_{V_i,x} &= \frac{1}{6M_{V_i}} \sum_{k=0}^{N_i-1} (x_k + x_{k+1})(x_k y_{k+1} - x_{k+1} y_k) \\
C_{V_i,y} &= \frac{1}{6M_{V_i}} \sum_{k=0}^{N_i-1} (y_k + y_{k+1})(x_k y_{k+1} - x_{k+1} y_k)\, .
\end{aligned}
$$

To present a simple formula for the polar moment of inertia, let $\bar{x}_k = x_k - C_{V_i,x}$ and $\bar{y}_k = y_k - C_{V_i,y}$, for $k \in \{0, \ldots, N_i - 1\}$. Then one can show that polar moment of inertia of a polygon about its centroid, $J_{V_i,C}$ becomes

$$
\begin{aligned}
J_{V_i,C_{V_i}} = \frac{1}{12} \sum_{k=0}^{N_i-1} &(\bar{x}_k \bar{y}_{k+1} - \bar{x}_{k+1} \bar{y}_k) \cdot \\
&(\bar{x}_k^2 + \bar{x}_k x_{k+1} + \bar{x}_{k+1}^2 + \bar{y}_k^2 + \bar{y}_k \bar{y}_{k+1} + \bar{y}_{k+1}^2)\, .
\end{aligned}
$$

To compute the polar moment of inertia J_{V_i,p_i} of the Voronoi polygon about an arbitrary point p_i, one can use equation (6) as,

$$J_{V_i,p_i} = J_{V_i,C_{V_i}} + M_{V_i} \|p_i - C_{V_i}\|^2.$$

The proof of some of these formulas can be found in [38]; they are all based on decomposing the polygon V_i into the union of disjoint triangles.

C. Simulations

In this section we provide a simulation for the control laws described in Section IV for the planar Euclidean setting with uniform density. The results are shown in the four illustrations in Figure 2. The vehicles' initial locations are in a tight group in the lower left corner of the admissible region; see the bottom-left figure. The vehicles' final locations are illustrated in the bottom-right figure. The bottom left and right figure also illustrate the initial and final Voronoi diagrams. The reduction in the cost function shown in the top-right figure provides a measure of the uniform coverage the vehicles provide. The paths of the vehicles are also included in the top-left figure: the initial locations are shown as small diameter black circles and final locations are shown as larger diameter red circles.

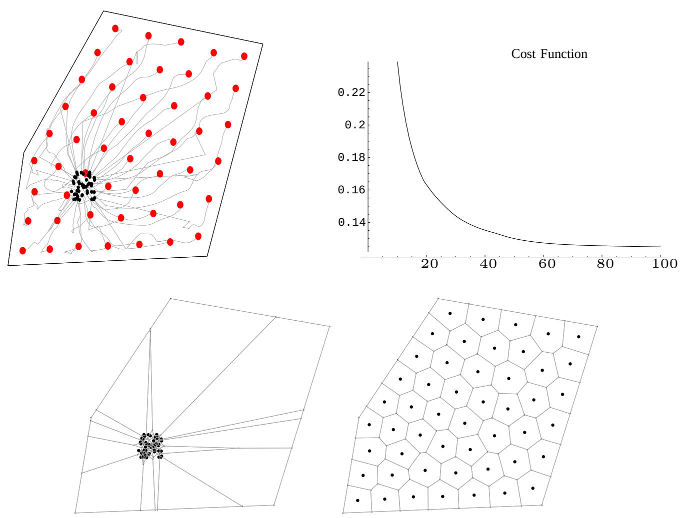

Fig. 2. Uniform distribution of sensors obtained by 16 vehicles in a polygonal environment. The vehicles' initial positions are in a tight group in the lower left corner and their final positions are optimally distributed.

V. Conclusions

We have presented some new control laws for networks of mobile agents performing a spatially distributed sensing task. The technical approach relies on ideas from locational optimization and centroidal Voronoi diagrams. The approach in this note leads to a variety of interesting avenues of research that seem amenable to technical progress.

Future research directions include extending the control laws to the setting of time-varying environments (e.g., consider a time-varying distribution density function), non-isotropic sensors (e.g., such as cameras and directional antennas), and nonlinear dynamics (e.g., nonholonomic vehicles). Additionally, we plan to implement our algorithms on a group of all-terrain vehicles.

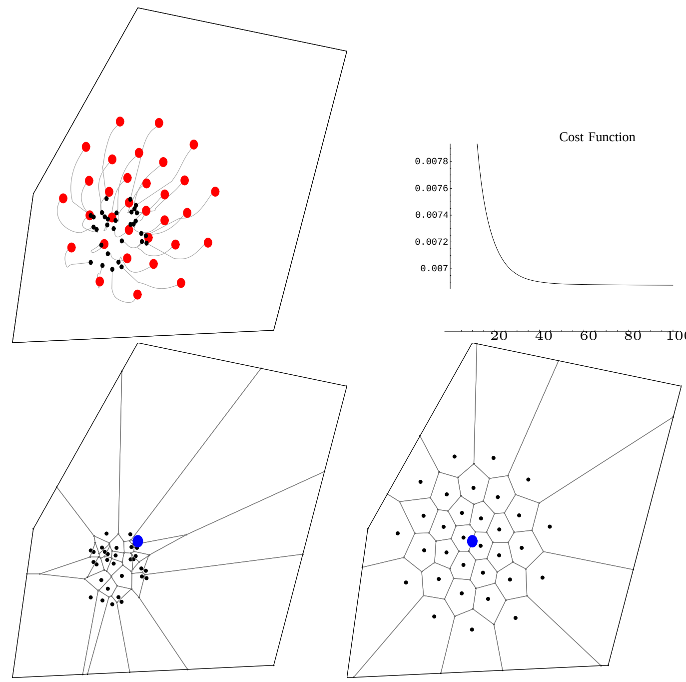

Fig. 3. Non-uniform setting. The distribution density function has an inverse exponential about the location shown by the large circle in the bottom left and right figures.

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