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A notion of passivity for hybrid systems

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Abstract

We propose a notion of passivity for hybrid systems. Our work is motivated by problems in haptics and teleoperation where several computer controlled mechanical systems are connected through a communication channel. To account for time delays and to better react to user actions it is desirable to design controllers that can switch between different operating modes. Each of the interacting systems can be therefore naturally modeled as a hybrid system. A traditional passivity definition requires that a storage function exists that is common to all operating modes. We show that stability of the system can be guaranteed even if different storage function are found for each of the modes, provided appropriate conditions are satisfied when the system switches.

Keywords: hybrid systems, passivity, multiple Lyapunov functions, haptics

1 Introduction

In this paper we are primarily concerned with the stability analysis of systems that involve haptic devices. Figure 1 shows a schematic of such a system.

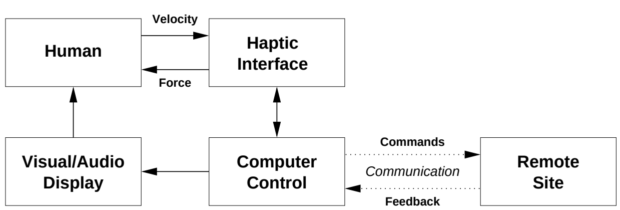

Figure 1: A system with a haptic device.

A human interacts with a computer bi-directionally through a haptic interface and uni-directionally through a visual and audio display. The nature of this interaction depends on the configuration of the remote site. The system will be a virtual-reality display if the remote site consists of a computer that simulates an artificial environment, a telemanipulation system if the remote site consists of a robot interacting with a real environment, or a collaborative haptic environment if the remote site consists of a mirror image of the computer interface shown. All these configurations are characterized by the interaction between a human participant and a computer interface that will attempt to make the communication of haptic information meaningful. Because the interaction between the participant and the computer will be both at the signal level (i.e., motions and forces) and the symbolic level (i.e., meaningful representations of these), the system will necessarily be hybrid.

Stability of bilateral manipulation was investigated by a number of researchers and passivity emerged as an especially powerful paradigm to study stability of both linear and non-linear systems with time-delay [1, 2, 3, 4]. Results on bilateral manipulation proved to be relevant also for the study of devices providing haptic interface to artificial environments. While such virtual displays make the teleoperation predominantly open-loop and thus less sensitive in terms of stability, stability becomes an issue due to the interaction of the human with the virtual environment through a mechanical haptic display. Haptic displays have many similarities with teleoperator systems and it is therefore not surprising that passivity proved to be a useful tool for the stability analysis of haptic displays too. Colgate and his coworkers used passivity to analyze the stability of haptic displays interacting with linear [5] and passive [6] environments. Passivity analysis based on linear circuit theory was used for stability analysis in [7]. The most general stability results for haptic systems are derived in [8] where passivity is shown to be appropriate for the analysis of both passive and non-passive, and linear and non-linear environments than can be implemented using either implicit or explicit numerical methods.

Developments in teleoperation and emergence of haptic displays both call for computer interfaces that implement several different discrete behaviors and can be therefore represented by Figure 1. However, so far the hybrid nature of such systems has not been acknowledged. This can be attributed to the lack of techniques for the design of hybrid controllers, a subject of considerable research [9, 10, 11]. On the other hand, stability of hybrid systems has been well studied. Classical Lyapunov theory is extended for nonsmooth and hybrid systems in [12, 13]. Multiple Lyapunov functions are proposed for stability analysis of hybrid systems in [14, 15, 16]. These works only provide conditions for stability, they do not offer a method for designing controllers that would satisfy such conditions. A controller design methodology based on multiple Lyapunov functions is described in [17] and a generalized framework for stabilization of nonlinear hybrid systems was proposed in [18]. A practical method for designing controllers for piecewise-linear systems using multiple Lyapunov functions is proposed in [19, 20, 21]. There, the problem of finding a set of Lyapunov functions is transformed into a (numerically tractable) problem of solving a system of linear matrix inequalities. In [22], these ideas are used to derive a simplified test for stability of a hybrid system modeled with a Petri net.

The aim of this paper is to develop a framework for the passivity analysis of hybrid systems. We argue that the classical notion of passivity is too restrictive in the hybrid systems setting. We propose a more general notion of passivity for hybrid systems and show that several classical results can be generalized using stability criteria for hybrid systems.

The paper is organized as follows. We first define our model for a hybrid systems. We then briefly review the definition of passivity for continuous systems and show why it is desirable to generalize this notion. Section 4 contains the main results: a definition of passive hybrid systems (PHS) and the relation between this notion of passivity and stability of hybrid systems. We conclude the paper with an example of a haptic display interacting with a virtual environment where the notion of PHS can be used to show that the interaction will be stable.

2 Hybrid system model

Several formal models for hybrid systems have been proposed in the past [23, 24, 25, 26]. Typically, a model is selected according to problem to be addressed. We will mainly follow the approach in [27].

Intuitively, a hybrid system can be described as a finite set of discrete states, with each discrete state corresponding to a different continuous dynamics. The state of a hybrid system is therefore composed of discrete and continuous components. The evolution of the continuous state can be described by a vector field that is a function of the continuous control. In general it might be possible to force the system to switch from one discrete state to a different discrete state. We will assume that the continuous state does not change during such switches. A selection of a discrete state can be modeled by a set of discrete inputs controlling the evolution of the discrete dynamics. The formal model of a hybrid system can be thus given as:

A **hybrid system** is a tuple:

$$\mathrm{HS} = (\Xi, \mathcal{M}, \Gamma, \mathcal{U}, \Sigma, \mathcal{F}, \mathcal{H}) \tag{1}$$

where

1. $\Xi \subset \mathbb{Z}$ is a (finite) set of discrete states.
2. $\mathcal{M} = \{M_i\}_{i \in \Xi}$ is a collection of (differentiable, connected) manifolds. For simplicity we assume $M_i \subseteq \mathbb{R}^n$.
3. $\Gamma \subset \mathbb{Z}$ is a set of discrete inputs.
4. $\mathcal{U} \subset \mathbb{R}^m$ is a set of continuous inputs.
5. $\mathcal{F} = \{f_i\}_{i \in \Xi}$ is a set of ($\mathcal{C}^1$) controlled vector fields:
$$\begin{aligned} f_i : \quad & M_i \times \mathcal{U} \to TM_i \\ & (x,u) \mapsto f_i(x,u) \in T_x M \end{aligned}$$
6. $\Sigma : \Xi \times \mathbb{R}^n \times \Gamma \times \mathcal{U} \to \Xi$ is a function describing the discrete evolution of the system.
7. $\mathcal{H} = \{h_i\}_{i \in \Xi}$ is a set of ($\mathcal{C}^1$) output maps $h_i : M_i \times \mathcal{U} \to \mathbb{R}^m$.

The evolution of a hybrid system can be described as follows. The system evolves on M_i following the vector field f_i as long as $\Sigma(i,x,\eta,u) = i$. When $\Sigma(i,x,\eta,u)$ becomes equal to $j \neq i$, the system dynamics switches to (M_j, f_j). In this paper we assume that there are finitely many switches in any finite time interval. We therefore exclude phenomena like chattering. The value of $\Sigma(i,x,\eta,u)$ can change either because the trajectory of the system leaves the manifold M_i and enters M_j, or because the discrete input η changes. In general, the vector fields in $\mathcal{F}$ will be different, reflecting changes in the dynamics of the system. Also the dimensions of the manifolds in $\mathcal{M}$ might be different.

3 Passivity

We follow the development in [28]. A system defined by:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned} \tag{2}$$

where $f(0,0) = 0$ and $h(0,0) = 0$ is *passive* if there exists a $\mathcal{C}^1$ positive semidefinite function $V(x)$ (called the storage function) such that:

$$u^T y \geq \frac{dV}{dt} + \varepsilon u^T u + \delta y^T y + \rho \psi(x) \quad \forall (x,u) \tag{3}$$

where $\varepsilon, \delta, \rho$ are nonnegative constants, and $\psi(x)$ is a positive semidefinite function of x such that

$$\psi(x(t)) \equiv 0 \Rightarrow x(t) \equiv 0.$$

The intuitive interpretation of this definition is that passive systems can not generate energy on their own. It can be shown that if the system is state strictly passive the origin is an asymptotically stable equilibrium point, and the storage function V becomes a Lyapunov function. But what makes passivity so useful for stability analysis is that, loosely speaking, an interconnection of passive systems is

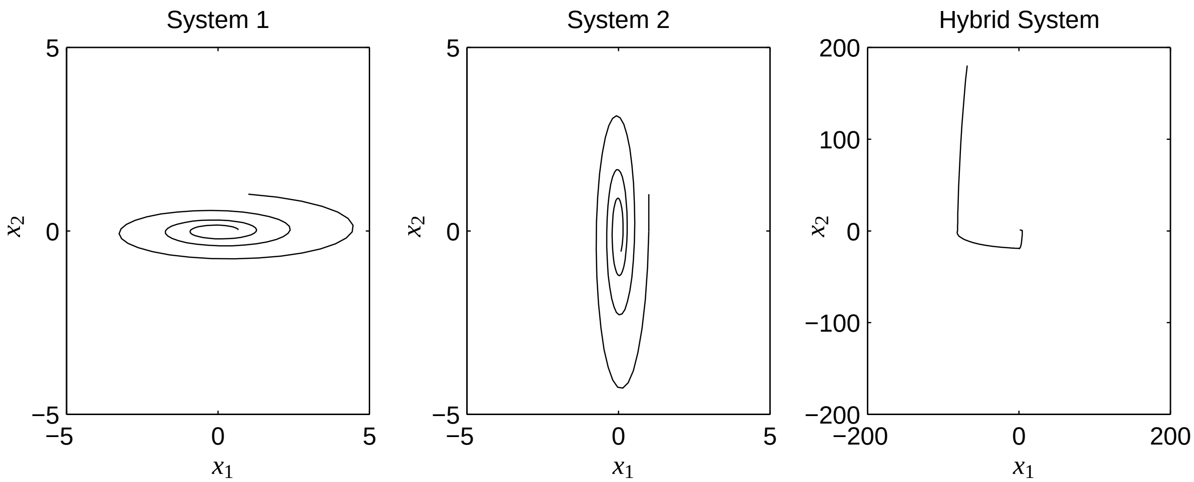

Figure 2: A hybrid system composed of two subsystems. (a) Trajectories of system 1. (b) Trajectories of system 2. (c) Trajectories of the hybrid system.

again passive. This observation has been the basis of the stability proofs in [1, 2, 5, 6, 7, 8].

However, even if the concept of passivity and the energy considerations that lead to stability are intuitive and therefore appealing, the concept might be quite misleading when dealing with hybrid systems. It would seem reasonable to conclude that if the system can switch between two sets of state equations and if each set of equations defines a passive system, the resulting hybrid system must also be passive. The following simple example demonstrates that such conclusion would be wrong.

Example 1 *Consider the following linear time invariant system:*

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{cases} \begin{bmatrix} -1 & 50 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + I_{2\times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, & x_1x_2 \geq 0 \\ \begin{bmatrix} -1 & 2 \\ -50 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + I_{2\times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, & x_1x_2 < 0 \end{cases}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{cases} \begin{bmatrix} 0.26 & 0.12 \\ 0.12 & 6.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, & x_1x_2 \geq 0 \\ \begin{bmatrix} 6.4 & -0.12 \\ -0.12 & 0.26 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, & x_1x_2 < 0 \end{cases} \tag{4}$$

We can show using the Kalman-Yakubovich-Popov lemma [28] that individually, both systems are passive (and therefore stable). The plots of the trajectories of the two systems are shown in Fig. 2.a and 2.b. The trajectory for the hybrid system (4), shown in Fig. 2.c, clearly shows that the system is not stable (and therefore not passive). One explanation is that to conclude that the resulting system is passive, the storage function in (3) would have to be the same for both systems, which is clearly not necessarily the case.

4 Passivity of hybrid systems

It has been shown in [14, 29, 15, 16] that it is not necessary to find a global Lyapunov function in order to guarantee that a hybrid system is stable; it suffices to analyze the stability in each dynamic regime (M_j, f_j) and the switching behavior of the system. Furthermore, it is known that the storage function of a passive systems is an excellent candidate Lyapunov function for stability analysis. This suggests that the passivity for hybrid systems should be defined in terms of storage functions of the individual discrete regimes; requiring that a single global storage function exists would be too strict.

In the case of smooth systems, the definition in Equation (3) is related to the Lyapunov method for stability analysis. Similarly, a test for stability analysis of hybrid systems can be used to extend the notion of passivity to hybrid systems. In this work we focus on the following stability test:

Proposition 1 ([14]) *Given a hybrid system (1), assume that for every regime i: $0 \in M_i$, $f_i(0,0) = 0$, and that each vector field f_i has an associated Lyapunov function V_i which is defined over M_i. Let $\xi(t) \in \Xi$ denote the switching sequence such that $\xi(t) = i \Rightarrow x(t) \in M_i$, and in addition*

$$V_i(x(t_{i,k})) \leq V_i(x(t_{i,k-1})) \tag{5}$$

where $t_{i,k}$ denotes the k-th time that the vector field f_i becomes "active", i.e., $\xi(t_{i,k}^-) \neq \xi(t_{i,k}^+) = i$. Then the system (1) is (Lyapunov) stable.

With this stability test in mind, we propose the following definition of passivity:

Definition 1 *Take a hybrid system (1) such that for every regime i, $0 \in M_i$ and $f_i(0,0) = 0$. Such a system will be called a* ***passive hybrid system (PHS)*** *if the following two conditions hold:*

1. *Each discrete regime (M_i, f_i) is passive. That is, there exists a storage function V_i and $\varepsilon_i, \delta_i, \rho_i \geq 0$ such that*

$$u^T y \geq \frac{dV_i}{dt} + \varepsilon_i u^T u + \delta_i y^T y + \rho_i \psi_i(x) \quad \forall (x,u) \tag{6}$$

   where (x,u) is a trajectory of (M_i, f_i).

2. *The storage functions have the property that:*

$$V_i(x(t_{i,k-1})) + \int_{t_{i,k-1}}^{t_{i,k}} u^T y dt \geq V_i(x(t_{i,k})). \tag{7}$$

Note that Equation (6) has to hold anytime the system is in the regime i, whereas the integral in Equation (7) runs over the regimes that the system traverses before switching back to i. This definition allows us to state the following result:

Proposition 2 *Consider a PHS according to Definition 1. If the storage functions $V_i(x)$ are positive definite then the origin $x = 0$ of the zero-input system ($u(t) = 0$) is stable.*

Proof: Since the system is PHS, $\frac{dV_i}{dt} \leq u^T y$ according to Equation (6). Therefore, if $u = 0$ the storage functions $V_i(x)$ are Lyapunov functions. For $u = 0$ they also satisfy Equation (5) because of property (7). The system is therefore stable according to Proposition 1. ■

It is thus possible to extend the notion of passivity to hybrid systems in such a way that passivity guarantees stability. However, an extension of passivity will only be useful if it is possible to show that an interconnection of passive systems such as shown in Figure 3 will be passive. The following proposition shows that this is indeed the case.

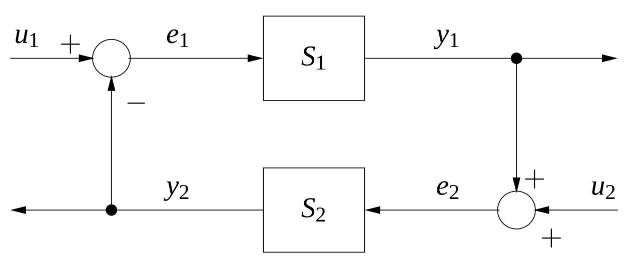

Figure 3: A feedback interconnection of control systems.

Proposition 3 *Let $S_1 = (\Xi_1, \mathcal{M}_1, \Gamma_1, \mathcal{U}_1, \Sigma_1, \mathcal{F}_1, \mathcal{H}_1)$ and $S_2 = (\Xi_2, \mathcal{M}_2, \Gamma_2, \mathcal{U}_2, \Sigma_2, \mathcal{F}_2, \mathcal{H}_2)$ be two PHS that are interconnected as in Figure 3. Suppose that the feedback system has a well-defined model $S = (\Xi_1 \times \Xi_2, \mathcal{M}_1 \times \mathcal{M}_2, \Gamma_1 \times \Gamma_2, \mathcal{U}_1 \times \mathcal{U}_2, \mathcal{F}, \Sigma)$ with the continuous state $x = \left[x_1^T x_2^T\right]^T$, input $u = \left[{u_1}^T {u_2}^T\right]^T$, and output $h = \left[h_1^T h_2^T\right]^T$. Then S is a PHS.*

Proof: We will use superscripts or subscripts 1 and 2 to refer to S_1 and S_2, respectively. To show that S is a PHS we need to show that (6) and (7) hold. Let us first show that (6) holds. Since S_1 and S_2 are PHS, we have for $a = \{1,2\}$:

$$\begin{aligned} e_a^T y_a &\geq \frac{dV_i^a}{dt} + \varepsilon_i^a e_a^T e_a + \delta_i^a y_a^T y_a + \rho_i^a \psi_i^a(x_a) \\ &\geq \frac{dV_i^a}{dt} + \delta_i^k y_a^T y_a \quad \forall (x_a, e_a) \end{aligned} \tag{8}$$

From $e_1 = u_1 - y_2$ and $e_2 = u_2 + y_1$, we have:

$${e_1}^T y_1 + {e_2}^T y_2 = {u_1}^T y_1 + {u_2}^T y_2 \tag{9}$$

Now define $V_{(i,j)}(x) = V_i^1(x_1) + V_j^2(x_2)$. From (8) and (9) we obtain:

$$u^T y \geq \frac{\partial V_{(i,j)}}{\partial x} \left[\begin{array}{c} f_i^1(x_1) \\ f_j^2(x_2) \end{array} \right] + y^T \left[\begin{array}{cc} \delta_i^1 I & 0 \\ 0 & \delta_j^2 I \end{array} \right] y \tag{10}$$

which shows (6).

To show (7), consider $t_{(i,j),k}$ and $t_{(i,j),k-1}$, where $t_{(i,j),k}$ denotes the k-th time that the vector field $f_{(i,j)}$ becomes "active". Note that Equations (6) and (7) imply that if $t_1 < t_2$ and $\xi_a(t_1) = \xi_a(t_2) = i$, $V_i^a(x_a(t_1)) + \int_{t_1}^{t_2} u_a^T y_a dt \geq V_i^a(x_a(t_2))$. This is true regardless of the number and location of switches on the interval (t_1, t_2). But then it follows that

$$\begin{aligned} V_{(i,j)}(x(t_{(i,j),k})) &= V_i^1(x_1(t_{(i,j),k})) + V_j^2(x_2(t_{(i,j),k})) \\ &\leq V_i^1(x_1(t_{(i,j),k-1})) + \int_{t_{(i,j),k-1}}^{t_{(i,j),k}} {u_1}^T y_1 dt + \\ &\quad + V_j^2(x_2(t_{(i,j),k-1})) + \int_{t_{(i,j),k-1}}^{t_{(i,j),k}} {u_2}^T y_2 dt \\ &= V_{(i,j)}(x(t_{(i,j),k-1})) + \int_{t_{(i,j),k-1}}^{t_{(i,j),k}} u^T y dt \end{aligned}$$

■

The last two propositions lead to the following result:

Corollary 1 *If S_1 and S_2 are PHS, then S is stable.*

5 Example

Consider a feedback connection of two systems S_1 and S_2 as shown in Figure 3. Let S_1 be a planar Cartesian haptic display (a 2 DOF gantry mechanism) and S_2 a model of a virtual environment. Let the model of S_1 be:

$$m\ddot{q}_1 + b\dot{q}_1 + kq_1 = u_1$$

where q_1 denotes the x and y coordinates of the haptic display, u_1 are the actuator forces, m is the mass , b is the friction in the linear bearings (the same in both directions) and k is the stiffness of the mechanism. By setting $x_1 = \left[q_1^T \;\; \dot{q}_1^T\right]^T$,

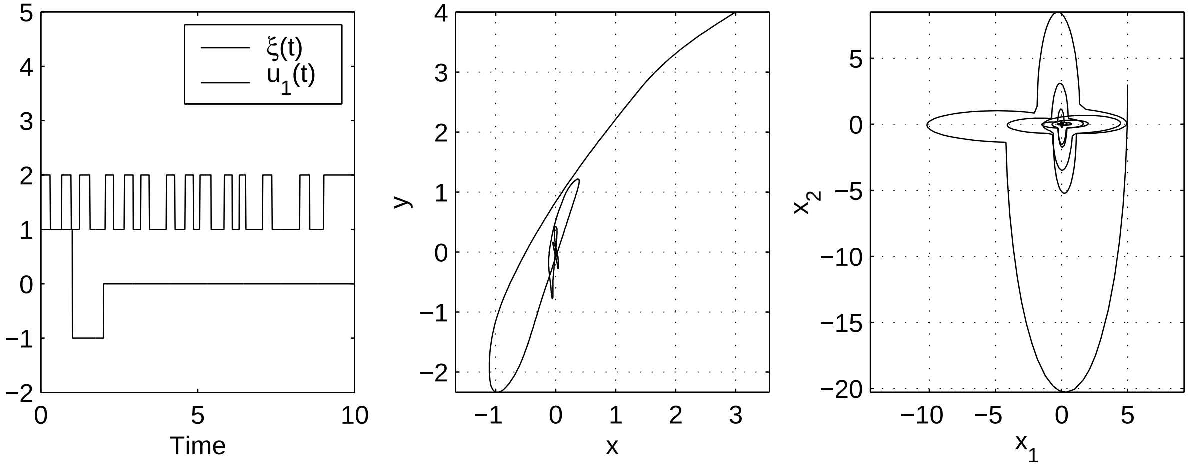

Figure 4: A haptic device interacting with a hybrid virtual environment.

we can write the following state equations:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} x_1 + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u_1$$

$$y_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_1$$

The outputs of S_1 are the velocities in x and y directions. Since S_1 is a mechanical system with dissipation it is easy to see that it is passive where the storage function is the mechanical energy.

*** It appears to me that in the next paragraph we mean to write ξ not ξ_2. Am I correct?

To illustrate the approach we let S_2 be the hybrid system from Example 1, but we will modify the switching rules to make the system a PHS. Assuming that $\Xi = \{1,2\}$ we set:

$$\xi_2(t^+) = \begin{cases} \xi_2(t^-) & t - t_s < T \text{ or } V^{3-\xi_2(t_s^-)} + \\ & + \int_{t_s}^{t} u_2{}^T y_2 dt < V^{3-\xi_2(t^-)} \\ 3 - \xi_2(t^-) & \text{otherwise} \end{cases}$$

We have used the expression $3 - \xi_2$ to flip the discrete state between 1 and 2 and t_s to denote the time when the system last switched between the two regimes. The switching rule therefore has a built in hysteresis (the system evolves in every regime at least for time T) and we have explicitly enforced the condition (7). Since each of the regimes is passive by itself so that (6) holds, the resulting system is PHS. Such behavior might model for example a particle moving in a potential field that can switch between two configurations. If the human interacts with the system, the human input can be modeled as the input u_1 in Figure 3, with u_2 set to 0.

We would like to know whether the interaction between the haptic display and the environment will be stable. Since S_1 is a passive continuous mechanical system (and therefore trivially a PHS), and since S_2 was designed to be a PHS, the overall system is a PHS and therefore stable according to Corollary 1. Figure 4 shows a trajectory of the system. The first panel shows the evolution of the discrete state $\xi(t)$ and the input. The next panel shows the trajectory of the haptic device S_1. The third panel shows the trajectory of the system S_2. The initial states were $x_1 = \begin{bmatrix} 3 & 0 & 4 & 0 \end{bmatrix}^T$ and $x_2 = \begin{bmatrix} 5 & 3 \end{bmatrix}^T$.

6 Conclusion

We developed a framework for passivity analysis of hybrid systems. We showed that the classical notion of passivity is too restrictive in the hybrid systems setting and proposed a more general notion of passivity for hybrid systems. Several classical results linking passivity and stability were then generalized using stability criteria for hybrid systems. The work was motivated by problems in haptics and teleoperation where passivity is extensively used for stability analysis. An example demonstrates the applicability of the method.

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References

[1] R. J. Anderson and M. W. Spong, “Bilateral control of teleoperators with time delay,” *IEEE Transactions on Automatic Control*, vol. 34, pp. 494–501, May 1989.

[2] R. J. Anderson and M. W. Spong, "Asymptotic stability for force reflecting teleoperators with time delay," *The International Journal of Robotics Research*, vol. 11, pp. 135–149, April 1992.

[3] G. Niemeyer and J.-J. E. Slotine, "Towards force-reflecting teleoperation over the internet," in *Proc. 1998 Int. Conf. on Robotics and Automation*, (Leuven, Belgium), pp. 1909–1915, 1998.

[4] S. Stramigioli, A. v.d. Schaft, and B. Maschke, "Geometric scattering in tele-manipulation of port controlled Hamiltonian systems," in *Proceedings of the 39th IEEE Conference on Decision and Control*, (Sydney, Australia), 2000.

[5] J. E. Colgate and G. G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," *Journal of Robotic Systems*, vol. 14, no. 1, pp. 37–47, 1997.

[6] J. M. Brown and J. E. Colgate, "Passive implementation of multibody simulations for haptic display," in *Proceedings of the 1997 ASME International Mechanical Engineering Congress and Exhibition*, vol. DSC-61, pp. 85–92, 1997.

[7] R. J. Adams and B. Hannaford, "Stable haptic interaction with virtual environments," *IEEE Transactions on Robotics and Automation*, vol. 15, pp. 465–474, June 1999.

[8] B. E. Miller, J. E. Colgate, and R. A. Freeman, "Guaranteed stability of haptic systems with nonlinear virtual environments." Submitted to IEEE Transactions on Robotics and Automation, September 1999.

[9] R. Alur, T. A. Henzinger, and E. D. Sontag, eds., *Hybrid systems III*, vol. 1066 of *Lecture notes in computer science*, (New York, NY), Springer Verlag, 1996.

[10] P. Antsaklis, W. Kohn, A. Nerode, and S. S. Sastry, eds., *Hybrid systems II*, vol. 999 of *Lecture notes in computer science*, (New York, NY), Springer Verlag, 1995.

[11] R. L. Grossman, A. Nerode, A. P. Ravn, and H. Rischel, eds., *Hybrid systems*, vol. 736 of *Lecture notes in computer science*, (New York, NY), Springer Verlag, 1993.

[12] L. Hou and A. N. Michel, "Stability analysis of a general class of hybrid dynamical systems," in *Proceedings of the 1997 ACC*, (Albuquerque, NM), pp. 2805–2809, 1997.

[13] E. Sontag and H. J. Sussmann, "Nonsmooth control-lyapunov functions," in *Proceedings of the 34th IEEE Conference on Decision and Control*, (New Orleans, LA), 1995.

[14] M. S. Branicky, "Stability of switched and hybrid systems," in *Proceedings of the 33rd IEEE Conference on Decision and Control*, (Lake Buena Vista, FL), pp. 3498–3503, 1994.

[15] G. A. Lafferriere, "Discontinuous stabilizing feedback using partially defined Lyapunov functions," in *Proceedings of the 33rd IEEE Conference on Decision and Control*, (Orlando, FL), 1994.

[16] P. Peleties and R. DeCarlo, "Asymptotic stability of m-switched systems using Lyapunov-like functions," in *American Control Conf.*, (Boston), pp. 1679–1684, 1991.

[17] J. Malmborg, B. M. Bernhardsson, and K. J. Åström, "A stabilizing switching scheme for multi-controller systems," in *13th IFAC World Congress*, (San Francisco, CA), 1996.

[18] M. Žefran and J. W. Burdick, "Stabilization of systems with changing dynamics," in *Hybrid Systems: Computation and Control*, LNCS 1386, Springer, 1998.

[19] M. Johansson and A. Rantzer, "On the computation of piecewise quadratic Lyapunov functions," in *Proceedings of the 36th IEEE Conference on Decision and Control*, (San Diego, CA), 1997.

[20] S. Pettersson and B. Lennartson, "Stability and robustness for hybrid systems," in *Proceedings of the 35th IEEE Conference on Decision and Control*, (Kobe, Japan), 1996.

[21] A. Hassibi and S. P. Boyd, "Quadratic stabilization and control of piecewise-linear systems," in *Proceedings of American Control Conference*, vol. 6, (Philadelphia, PA), pp. 3659–64, June 1998.

[22] K. X. He and M. D. Lemmon, "Lyapunov stability of continuous-valued systems under the supervision of discrete-event transition systems," in *Hybrid Systems: Computation and control*, LNCS 1386, pp. 175–189, Springer, 1998.

[23] A. Back, J. Guckenheimer, and M. Myers, "A dynamical simulation facility for hybrid systems," in *LNCS 736*, pp. 255–267, Springer-Verlag, 1993.

[24] M. S. Branicky, V. S. Borkar, and S. K. Mitter, "A unified framework for hybrid control," in *Proceedings of the 33rd IEEE Conference on Decision and Control*, (Lake Buena Vista, FL), pp. 4228–4234, 1994.

[25] R. W. Brockett, "Hybrid models for motion control systems," in *Essays in Control: Perspectives in the Theory and its Applications*, pp. 29–53, Boston: Birkhäuser, 1993.

[26] X. Nicollin, A. Olivero, J. Sifakis, and S. Yovine, "An approach to the description and analysis of hybrid systems," in *LNCS 736*, pp. 149–178, Springer-Verlag, 1993.

[27] R. Alur, C. Courcoubetis, T. Henzinger, and P. H. Ho, "Hybrid automata: an algorithmic approach to the specification and verification of hybrid systems," in *LNCS 736*, pp. 209–229, Springer-Verlag, 1993.

[28] H. K. Khalil, *Nonlinear systems*. New York: Macmillan Publishing Company, 1992.

[29] R. A. Decarlo, M. S. Branicky, S. Pettersson, and B. Lennartson, "Perspectives and results on the stability and stabilizability of hybrid systems," *Proceedings of the IEEE*, vol. 88, no. 7, pp. 1069–1082, 2000.