On modeling and locomotion of hybrid mechanical systems with impacts

Francesco Bullo
Control and Dynamical Systems
Mail Stop 107-81
California Institute of Technology
Pasadena, CA 91125
bullo@cds.caltech.edu

Miloš Žefran
Mechanical Engineering
Mail Stop 104-44
California Institute of Technology
Pasadena, CA 91125
milos@robby.caltech.edu

Abstract

Walking machines are mechanical systems that undergo impacts and changes in dynamic equations and can be viewed as a subclass of hybrid systems. In this work we focus on a class of planar mechanisms that can locomote through plastic impacts and clamping. This setting retains enough structure to investigate discrete phenomena in locomotion. We use a geometric framework to describe smooth phenomena such as inertial and constraint forces and discrete events such as impacts. In this setting hybrid mechanical control systems are described in terms of affine connections and linear jump transition maps. For this class of systems we perform a local controllability analysis. In particular we employ the notion of configuration controllability to identify systems that are able to locomote by clamping. Additionally, we discuss how to adapt smooth motion planning algorithms to this hybrid setting and present some instructive set of gaits.

Keywords: mechanical control systems, hybrid systems, impact models, nonlinear controllability

1 Introduction

An instructive subclass of hybrid systems are mechanical systems that interact discontinuously with their environment in order to move. Key motivating examples are walking multi-legged devices. Advances in control of smooth Lagrangian systems have recently led to a theoretical framework that encompasses numerous locomotion mechanisms. However, problems like motion planning and dynamic stabilization of a walking robot remain open.

The purpose of this paper is twofold. On one side we investigate geometric models for impact mechanics and relate them to this recently developed framework. We present a geometric model of mechanical systems subject to switching constraints. In particular we give an intrinsic definition of hybrid mechanical control system in terms of affine connections and linear jump transition maps. As an application, we consider some low dimensional examples of sliding and clamping mechanisms and design locomotion gaits for them. Secondly, we present a local nonlinear controllability analysis for these systems. This is an attempt to apply tools from nonlinear controllability theory to hybrid systems with controlled switches and jumps. Even though our results are specific to Lagrangian systems, we hope to gain insight into more general problems from the study of such structured examples.

Modeling and controllability results for smooth mechanical systems are discussed in various works by Lewis and Murray, see [10, 11, 12]. Some of these results are related to the general local controllability problem studied by Sussmann [14]. Also of relevance are contributions on the motion planning problem. These include results on kinematic systems by Leonard & Krishnaprasad [9] and Lafferriere and Sussmann [8]. Bullo et al. [5] developed a motion planning method for mechanical systems that evolve on a Lie group. Goodwine & Burdick [6] developed a controllability test and a planning method for a class of hybrid kinematic systems called stratified systems. A comprehensive treatment on impacts is given by Brogliato in [3].

The focus of this paper is on mechanical systems with changing dynamics. They represent a subclass of more general hybrid systems. A number of approaches to modeling and control of hybrid systems, as well as several applications are described in [1, 7]. However, these works consider a general setting. Our aim is instead to explore the inherent structure of the mechanical systems to derive our results. An approach in this direction is also [15].

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2 Smooth and hybrid mechanical systems

In this section we review some tools and results in modeling of smooth mechanical systems. We assume the reader to be familiar with the geometric machinery usually employed in nonlinear control theory [13].

Given a smooth manifold Q, an *affine connection* on Q is a smooth map that assigns to each pair of smooth vector fields X, Y a smooth vector field $\nabla_X Y$ such that for all functions f on Q

(i) $\nabla_{fX} Y = f \nabla_X Y$, and

(ii) $\nabla_X fY = f \nabla_X Y + (\mathcal{L}_X f) Y$

where $\mathcal{L}_X f$ denotes the Lie derivative of the function f with respect to X. Affine connections are a formal way of defining differentiation of vector fields and can be naturally extended to derivations on the set of tensor fields on Q, see [10] for further details.

2.1 A geometric description of mechanical control systems

We start with systems that have Lagrangian equal to the kinetic energy and later generalize this model to include constraints [10] and impacts. A *mechanical control system* is defined by the following objects:

(i) an n-dimensional configuration manifold Q, with local coordinates $q = \{q^1, \dots, q^n\}$,

(ii) a metric $M_q : TQ \times TQ \to \mathbb{R}$ on Q (the kinetic energy), alternatively denoted by $\langle\!\langle \cdot, \cdot \rangle\!\rangle$,

(iii) an m-dimensional codistribution $\mathcal{F} = \mathrm{span}\{F^1, \dots, F^m\}$ defining the input forces.

Let $q(t) \in Q$ be the configuration of the system and $\dot{q}(t) \in T_q Q$ its velocity. Let the input vector fields be $Y_k = M_q^{-1} F^k$ and let $\mathcal{Y} = \mathrm{span}\{Y_1, \dots, Y_m\}$ denote the input distribution. The Euler-Lagrange equations for the system can be written in a coordinate independent form relying on the Riemannian connection ∇ of the metric M_q:

$$\nabla_{\dot{q}} \dot{q} = Y_k u^k, \tag{1}$$

where $(Y_k u^k)^i$ denotes the ith component of $(Y_k u^k)$, and where the input functions $\{u^k(t), t \in \mathbb{R}^+\}$ are assumed piecewise constant.

2.2 Holonomic and nonholonomic constraints

From the point of control, constraints on a mechanical system limit the set of directions in which the system can move. An intrinsic description of a constraint is therefore through a distribution on TQ, describing at each point the set of feasible velocities. If a constraint is nonholonomic, it is by definition described by such a distribution. If a constraint is holonomic, it is described by a smooth map $\varphi : Q \to \mathbb{R}^{n-p}$. If C is a regular value of φ, then the equation $\varphi(q) = C$ defines a submanifold R of Q. The set of feasible velocities is then $\mathcal{D}(q) = T_q R$, which formally corresponds to the null space of $\{\boldsymbol{d}\varphi_1(q), \dots, \boldsymbol{d}\varphi_{n-p}(q)\}$. Assuming that the holonomic constraint can be applied at any point (as is the case with clamping), and assuming that the set of regular values of φ contains an open non-empty neighborhood of $\varphi(q_0) \in \mathbb{R}^{n-p}$, the constraint distribution $\mathcal{D}(q)$ as the null space of $\{\boldsymbol{d}\varphi_1(q), \dots, \boldsymbol{d}\varphi_{n-p}(q)\}$ is well defined for each q in a neighborhood $W \subset Q$ of q_0.

It is important to note that both holonomic and nonholonomic constraints can be written in the form $\dot{q} \in \mathcal{D}(q)$, for an appropriate distribution $\mathcal{D}(q)$. In general, a mechanical control system together with a constrained distribution will be called a *constrained mechanical control system.* In what follows we denote one such system with

$$\Sigma = \{Q, M_q, \mathcal{F}, \mathcal{D}, U\}, \tag{2}$$

where $U = \{u^1(t), \dots, u^m(t), t \in \mathbb{R}^+\}$ is the set of piecewise constant inputs.

Let $\mathcal{D}^\perp$ denote the orthogonal complement to $\mathcal{D}$ with respect to the metric M_q. Accordingly, let $P : TQ \to TQ$ and $P^\perp : TQ \to TQ$ denote the orthogonal projections onto $\mathcal{D}$ and its complement, $P^\perp = I - P$. The equations of motion for the constrained system (2) are written in concise form as

$$\widetilde{\nabla}_{\dot{q}} \dot{q} = P(Y_k) u^k, \tag{3}$$

where the affine connection $\widetilde{\nabla}$ is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \left(\nabla_X P^\perp\right)(Y), \qquad \forall X, Y.$$

We refer to [10] for a complete treatment. Observe that the input distribution for the constrained system is a projection of the input distribution of the unconstrained system. Since equation (3) is formally identical to equation (1), it is possible to provide a unified treatment of both constrained and unconstrained mechanical control systems, see [10]. In particular, given a system (2) (set $\mathcal{D} = TQ$ for an unconstrained system), the forced equations of motion can be expressed in terms of the pair $(\widetilde{\nabla}, \mathcal{Y})$, where we define the set of input vector fields as

$$\mathcal{Y} = \mathrm{span}\{P(Y_k) \mid Y_k = M_q^{-1} F^k, k = 1, \dots, m\}.$$

2.3 Plastic and elastic impacts

Loosely speaking, an impact causes a switch in the equations of motions and a jump in the system's velocity. Let $(Q, M_q, \mathcal{F})$ be a mechanical control system, let $\mathcal{D}^-$ and $\mathcal{D}^+$ be two constraint distributions, and let $(\nabla^-, \mathcal{Y}^-)$ and $(\nabla^+, \mathcal{Y}^+)$ be the corresponding affine connections and input distributions. We say that the mechanical systems undergoes an *impact* at time t if the following events occur

(i) the dynamic equations switch from $(\nabla^-, \mathcal{Y}^-)$ to $(\nabla^+, \mathcal{Y}^+)$,

(ii) the state $(q, \dot{q})$ undergoes a discontinuous change in velocity described by a tensor field $J_q : T_qQ \to T_qQ$. In other words[1]:

$$q(t^+) = q(t^-)$$
$$\dot{q}(t^+) = J_q\left(\dot{q}(t^-)\right).$$

This definition recovers the classic notions of purely plastic and elastic impacts as special cases. For example, if a particle hits a surface with nonzero velocity, then the linear operator J_q annihilates the normal component of the velocity in the plastic impact case and reverses it in the elastic impact case (module a coefficient of restitution e). Formally, we define:

Plastic impact: The two constraint distributions $\mathcal{D}^-$ and $\mathcal{D}^+$ are distinct (for example $\mathcal{D}^- = TQ$ and $\mathcal{D}^+ = TR$ is the tangent space of a submanifold $R \subset Q$). The operator $J_q = P_{\mathcal{D}^+}$ is the orthogonal projection from T_qQ onto $\mathcal{D}^+$.

Elastic impact: The equations of motion do not change, as connection and input distributions do not change. There exist a submanifold R such that

$$J_q = P_{TR} + (-e)P_{TR}^{\perp},$$

where P_{TR} is the orthogonal projection onto the tangent space to R and where $0 < e < 1$ is the coefficient of restitution.

We note that the above definition of impact applies to both holonomic and nonholonomic impacts, that is to impacts that possibly involve either holonomic or nonholonomic or both type of constraint distributions. This is an important advantage of the geometric framework we advocate.

2.4 Hybrid mechanical control systems

In this section we introduce a special class of hybrid systems by merging the notion of "control systems on manifolds with an affine connection," see [12], and that of "controlled general hybrid dynamical system," see [2].

[1]The notations $q(t^-)$ and $q(t^+)$ refer to the limiting processes $\lim_{s\to t^-} q(s)$ and $\lim_{s\to t^+} q(s)$.

The fundamental discrete phenomena we model are controlled switches between distinct sets of constraints, resulting in impacts. The underlying structure is a mechanical control system $(Q, M_q, \mathcal{F})$ together with a given set of constraint distributions $\mathcal{D}_i$, where i belongs to an index set I. Each constraint $\mathcal{D}_i$ leads to a constrained mechanical control system $\Sigma_i = [Q, M_q, \mathcal{F}, \mathcal{D}_i, U]$, with associated affine connection ∇_i and input distribution $\mathcal{Y}_i$. Formally, we define the *hybrid mechanical control system* as

$$\text{HMCS} = [I, Q, \Sigma_Q, \mathbf{V}, \Delta] \tag{4}$$

where:

(i) I is the *index set* of constraints,

(ii) Q is the n-dimensional *configuration manifold*,

(iii) $\Sigma_Q = \{\Sigma_i = [Q, M_q, \mathcal{F}, \mathcal{D}_i, U]\}_{i\in I}$ is the collection of constrained mechanical control systems on Q,

(iv) $\mathbf{V} = \{V_{ij}\}_{i,j\in I}$ is the set of discrete controls. We require $V_{ij} \neq \emptyset$.

(v) $\Delta = \{\delta_{ij} | i, j \in I\}$ is the set of *jump transition maps*, where $\delta_{ij} : V_{ij} \times \cup_{q\in Q}\mathcal{D}_i(q) \to \cup_{q\in Q}\mathcal{D}_j(q)$, and $\delta_{ij}(v)(q, \dot{q}) = (q, J_{ij}(q, v) \cdot \dot{q})$. the operator $J_{ij}(q, v)$ is linear for each $q \in Q$ and each $v \in V_{ij}$.

The evolution of a hybrid mechanical control system can be described as follows. The system starts in a state $((q, \dot{q}), i) \in TQ \times I$ and it evolves according to the dynamics given by ∇_i and the chosen set of controls. At any point, we can choose to jump to any other discrete state through impact and in general we also have several different impacts to choose from (indexed by the set of discrete inputs V_{ij}).

2.5 Remarks

There are a number of differences between our definition and the definition in [2]. The controlled jump sets are in our case equal to Q and are thus omitted. Furthermore, we impose more structure on the controlled jump destination maps: at each point $q \in Q$ we can choose to change the discrete state from i to j by undergoing an impact. During the impact, the velocity is mapped from $\mathcal{D}_i(q)$ to $\mathcal{D}_j(q)$ through a linear operator. However, in general there is more than one way of changing the discrete state between i and j. The choices are indexed by the set of discrete inputs V_{ij}.

If we require that we can only perform a finite number of impacts in a finite time interval, we can guarantee the existence and uniqueness of the solution of the equations of motion. Since impacts are under our control, this is not a very restrictive assumption.

3 Nonlinear controllability and motion planning

In this section we investigate controllability and motion planning for HMCS. Our controllability analysis is strongly motivated by notions of configuration and equilibrium controllability [12]. We are thus naturally led to study the case when the velocity at the impact is zero (in the terminology of [2], the system undergoes controlled switches rather than controlled jumps).

3.1 Equilibrium controllability for a smooth mechanical control system

For any discrete regime i, we have the control system $\Sigma_i = [Q, M_q, \mathcal{F}, \mathcal{D}_i, U]$ with associated connection and input distribution $(\nabla_i, \mathcal{Y}_i)$. The equations of motion are

$$\nabla^i_{\dot q}\dot q = Y^i_k u^k(t), \tag{5}$$

where $\{Y^i_1, \ldots, Y^i_m\}$ is a base for $\mathcal{Y}_i$. Let q_0 be a point in Q and let W be a neighborhood of q_0. The reachable set of q_0 within W is

$$\mathcal{R}^W_Q(q_0, \leq T) = \cup_{t \leq T}\{x \in Q \mid \exists \text{ a solution to (5) s.t. } \dot q(0) = 0,\ q(t) \in W \text{ for } t \in [0,T], \text{ and } q(T) = x\}.$$

The system (5) is *locally configuration controllable* at q_0 if there exist a time T such that $\mathcal{R}^W_Q(q_0, \leq T)$ contains a non-empty open subset of Q, and *equilibrium controllable* on on $W \subset Q$, if, for $q_1, q_2 \in W$, there exist an input $\{u^k(t), t \in [0,T]\}$ and a solution $\{q(t), t \in [0,T]\}$ such that $q(0) = q_1$, $q(T) = q_2$, $q(t) \in W$ for all $t \in [0,T]$, and $\dot q(0) = 0$, $\dot q(T) = 0$. These controllability notions are quite useful since there exists algebraic tests for them. Before presenting these tests, we review a few definitions. Given a pair of smooth vector fields X, Y on Q, we use two operations on them, Lie bracket and symmetric product. The latter is defined as:

$$\langle X : Y\rangle = \nabla_X Y + \nabla_Y X.$$

Corresponding to these operations between pairs of vector fields, we introduce two operations on a family of vector fields $\mathcal{X} = \{X_1, \ldots, X_m\}$. We let $\mathrm{Lie}(\mathcal{X})$ be the closure of $\mathcal{X}$ under the Lie bracket operation (the involutive closure), and we let $\mathrm{Sym}(\mathcal{X})$ be the closure of $\mathcal{X}$ under the symmetric product operation. Within the set $\mathrm{Sym}(\mathcal{X})$, we define the *order* of a symmetric product to be the number of vector fields X_j present in it. We say that a symmetric product is *bad* if it contains even number of each X_i. Otherwise the product is said to be *good*. The controllability tests are then

(i) if the rank of $\mathrm{Lie}(\mathrm{Sym}(Y_1, \ldots, Y_m))$ is full and if every bad symmetric product at q is a linear combination of lower order good symmetric products, then the system (5) is locally configuration controllable at q,

(ii) if these conditions are verified at every $q \in W$, then the system is equilibrium controllable on W.

Intuitively, the symmetric closure of the input vector fields describes what velocities are reachable, while the involutive closure describes what configuration are reachable. See [12] for details.

3.2 Equilibrium controllability for the hybrid mechanical control system

The definition of equilibrium controllability only relies on the properties of the solutions to the motion equations. Since these solutions are well–defined for hybrid mechanical control systems, the definition above is also applicable in this setting. We now provide an algebraic test for equilibrium controllability of a hybrid mechanical control system in the case when the velocity at the impact is zero.

Proposition 3.1 *The hybrid mechanical control system* (4) *is equilibrium controllable on an open set* W *if the following two conditions hold:*

(i) in each discrete state i*, every bad symmetric product is a linear combination of lower order good symmetric products*

(ii) the rank of $\mathrm{Lie}(\sum_{i\in I}\mathrm{Sym}_i(\mathcal{Y}_i))(q)$ *is full for all* $q \in W$.

This statement is proven in [4]. The proof relies on two facts. First, by (i) the system in each regime is equilibrium controllable if restricted to the maximal integral manifold of the distribution $\mathrm{Lie}(\mathrm{Sym}_i(\mathcal{Y}_i))(q)$. In other words, we can reach any configuration on this submanifold and we can reach it at zero velocity. These last observation implies that the control problem is now kinematic as opposed to dynamic. Secondly, we can switch from each smooth regime i to any other j at any time and configuration (this is guaranteed by point (iv) in the definition of HMCS where set $V_{ij} \neq \emptyset$). We are thus dealing with a kinematic control system with input vector field in $\mathrm{Lie}(\mathrm{Sym}_i(\mathcal{Y}_i))(q)$ and $\mathrm{Lie}(\mathrm{Sym}_j(\mathcal{Y}_j))(q)$. By Chow's theorem the set of reachable points is

$$\mathrm{Lie}\left(\mathrm{Lie}(\mathrm{Sym}_i(\mathcal{Y}_i)) + \mathrm{Lie}(\mathrm{Sym}_j(\mathcal{Y}_j))\right) \qquad \forall i, j.$$

3.3 Motion Planning

Numerous results are available on the motion planning problem for smooth mechanical control systems. We

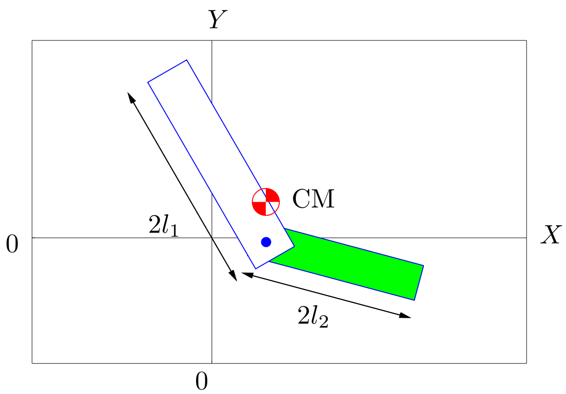

Figure 1: Sliding and clamping device with two legs parameterized by (θ_i, x_i, y_i). The second leg is allowed to fully clamp on the floor reducing the system's degrees of freedom from 4 to 1.

propose to employ a scheme composed of local and global planning algorithms. The method in [5, 9] can be directly adapted for motion planning in the smooth regimes. For planning motions that span different discrete states, we combine this local scheme with the discontinuous motion planning schemes proposed in [8]. We should note that there are still open questions about motion planning for smooth systems and here we do not attempt to address these questions. For example, it is still not clear how to perform motion planning for generic under-actuated mechanical systems. However, the existing techniques already cover a broad class of systems (fully actuated mechanical systems, systems with symmetries and conserved quantities) and as new techniques are developed for smooth systems they can be directly employed in the hybrid setting.

3.4 Example: sliding and clamping devices

We present a simple example: two homogeneous bars of unit density and lengths (l_1, l_2), connected by a joint (Figure 1). In the figure, CM denotes the center of mass of the two body system. The coordinates of the center of mass of the jth joint are (θ_j, x_j, y_j), while $(x_{\mathrm{CM}}, y_{\mathrm{CM}})$ are the coordinates of CM. We assume that the joint is actuated and that we can instantaneously clamp the second bar to the ground, resulting in fixing the position and orientation $\varphi_1(q) = (\theta_2, x_2, y_2)$.

The configuration manifold of the two body system is $Q = \mathbb{T}^2 \times \mathbb{R}^2$, with a configuration $q = (\theta_1, \theta_2, x_{\mathrm{CM}}, y_{\mathrm{CM}})$. The inertia matrix M_q is

$$l^{-1} \begin{bmatrix} \frac{5}{6}(l_1^4 + l_1^2 l_2^2) + 2l_1^4 l_2^2 & 2l_1^3 l_2^3 \cos(\theta_1 - \theta_2) & 0 & 0 \\ 2l_1^3 l_2^3 \cos(\theta_1 - \theta_2) & \frac{5}{6}(l_1^2 l_2^2 + l_2^4) + 2l_1^2 l_2^4 & 0 & 0 \\ 0 & 0 & 2l & 0 \\ 0 & 0 & 0 & 2l \end{bmatrix},$$

where $l = 6\left(l_1^2 + l_2^2\right)^2$. The input codistribution is $\mathcal{F} = \mathrm{span}\{\boldsymbol{d}\theta_1 - \boldsymbol{d}\theta_2\}$.

When the second link is clamped, the system is confined to the submanifold $R_1(q_0) = \{q \in Q \mid \varphi_1(q) = \varphi_1(q_0) = (\theta_{20}, x_{20}, y_{20})\}$. This holonomic constraints induces the constraint distribution $\mathcal{D}_1(q)$

$$\mathrm{span}\left\{(l_1^2 + l_2^2)\frac{\partial}{\partial \theta_1} + l_1^3 \sin(\theta_1)\frac{\partial}{\partial x_{\mathrm{CM}}} - l_1^3 \cos(\theta_1)\frac{\partial}{\partial y_{\mathrm{CM}}}\right\}.$$

If we set $\mathcal{D}_0(q) = T_q Q$, we thus have two distinct constrained mechanical control systems $[Q, M_q, \mathcal{F}, \mathcal{D}_i]$, for $i \in \{0, 1\}$.

Next we present the input vector fields on the two regimes. The input in the unconstrained regime is

$$Y_0 = \eta l \left(\left(5l_2^2 + l_1^2\left(5 + 12l_2^2\right) + 12l_1^3 l_2 \cos(\theta_1 - \theta_2)\right) \frac{\partial}{\partial \theta_1} - \left(5l_2^2 + l_1^2\left(5 + 12l_2^2\right) + 12l_1 l_2^3 \cos(\theta_1 - \theta_2)\right) \frac{\partial}{\partial \theta_2} \right).$$

By projecting Y_0 onto the appropriate constraint distributions, we get

$$Y_1 = \zeta \left((l_1^2 + l_2^2)\frac{\partial}{\partial \theta_1} + l_1^3 \sin(\theta_1)\frac{\partial}{\partial x_{\mathrm{CM}}} - l_1^3 \cos(\theta_1)\frac{\partial}{\partial y_{\mathrm{CM}}} \right)$$

for an appropriate scalar function $\zeta = \zeta(\theta_1, \theta_2)$.

We now have all the necessary tools to check for equilibrium controllability. In particular we perform the operation of symmetric closure on the two regimes: $\mathrm{Sym}_i(Y_i)$. For both i, we have

$$\langle Y_i : Y_i \rangle_i \in \mathrm{span}\{Y_i\},$$

and therefore all (good and bad) symmetric products are linear combination of lower order symmetric products. Next we look at the Lie brackets computations on the manifold Q. It is straightforward to compute that

$$\mathrm{rank}\left\{Y_0, Y_1, [Y_0, Y_1], [Y_0, [Y_0, Y_1]]\right\}(q) = 4$$

in a neighborhood of the point $(\theta_1, \theta_2, x_{\mathrm{CM}}, y_{\mathrm{CM}}) = (0, 0, 0, 0)$. Therefore, the hybrid mechanical control system $\{\Sigma_0, \Sigma_1\}$ is equilibrium controllable.

We conclude this section exhibiting some simple locomotion gaits for this hybrid mechanical system. At the point $q_0 = (0, 0, 0, 0)$

$$[Y_0, Y_1](q_0) = \zeta \frac{\partial}{\partial x_{\mathrm{CM}}}$$

where ζ is a scalar function. Piecewise constant out of phase inputs can be employed to generate a displacement along these two Lie bracket directions. Figures 2 illustrates this idea. As discussed above, we refer to the work in [8] and plan to present an extension of that algorithm to our setting in a subsequent work.

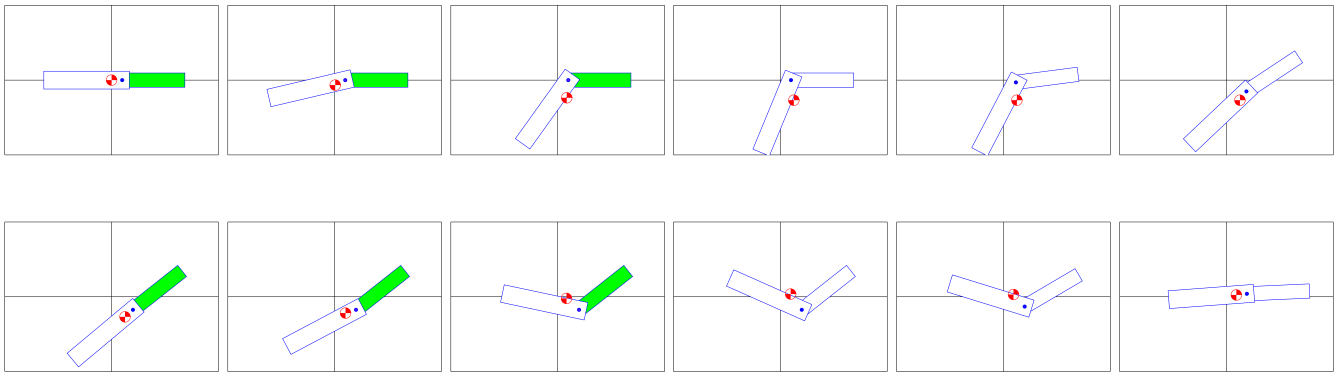

Figure 2: Translation gait for the $\{\Sigma_0, \Sigma_1\}$ hybrid mechanical system. When the second leg is clamped to the ground, it is colored in gray. Notice the final displacement of the center of mass.

4 Conclusions

We have presented a rigorous and comprehensive framework for the study of hybrid mechanical control system. This class of hybrid systems has interesting features such as a very structured and well understood smooth dynamics (a set of affine connections) and linear jump transition maps (the classic models of plastic and elastic impact). We have presented a controllability test that characterizes the reachable set via zero velocity impacts, discussed the motion planning problem and designed a set of simple gaits for a low dimensional examples. Our next goal is to investigate the motion planning problems with jumps at non-zero velocity.

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