Pairwise-optimal Discrete Coverage Control for Gossiping Robots

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Joint work with Ruggero Carli and Francesco Bullo
**The Team:** Robots with limited communication capabilities

**The Mission:** Spatially distributed load balancing
Challenges in Coverage Control:

- Reduce communication requirements
- Ensure solutions scales well for large teams
- Avoid local minima solutions

This work:

- Gossip communication: asynchronous, pairwise
- Computation scales with local partition size
- Better solution set than existing methods
Discretized Coverage

Domain is a weighted graph \( G = (Q, E, w) \)

**Required properties**

- \( G \) must be connected
- All edge-weights \( w \) must be positive

Partition vertices \( Q \) into regions for each robot: \( \{P_1, \ldots, P_n\} \)

such that each \( P_i \) induces connected subgraph
Discretized Coverage Cost Function

**Per agent cost**

Cost to cover $P_i$ from vertex $h \in P_i$:

$$H_i(h, P_i) = \sum_{k \in P_i} \text{dist}_{P_i}(h, k) \phi(k)$$

**Centroid:** $c_i \in P_i$ is vertex which minimizes $H_i(h, P_i)$

**Total coverage cost**

$$\mathcal{H}_{\text{multi-center}}(c, P) = \sum_{i=1}^{N} H_i(c_i, P_i)$$
Prior Work: Lloyd Algorithm

Theorem (Lloyd ’57 “least-square quantization”)

1. at fixed partition, optimal positions are centroids
2. at fixed positions, optimal partition is Voronoi

Lloyd’s Algorithm

For each update round:
1. move agents towards centroids of their regions
2. take Voronoi partition generated by agent positions

Result: convergence to set of centroidal Voronoi partitions
(i.e., positions are centroids of Voronoi partition they generate)
**Gossip Lloyd optimization**

Whenever two robots communicate:

1. Compute union of regions
2. Perform Voronoi partition of union using prior centroids
3. Update centroid of each agent for new region

Result: convergence to a centroidal Voronoi partition

(Durham, Frasca, Carli, Bullo ’09)
For pairwise updates: no need to use Lloyd separation!

⇒ Finding **optimal two partition of graph** is not computationally hard
**New Algorithm**

**Pairwise-optimal Coverage**

Whenever two robots communicate:

1. Compute union of regions
2. for sample vertex pair \((a, b)\) do
3. Perform Voronoi partition of union using \((a, b)\)
4. if Cost is lower then
5. Update centroids to \((a, b)\)
6. Update regions
7. end if
8. end for
Pairwise-optimal Coverage

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Let \(k = |P_i \cup P_j|:\)

\[\rightarrow\] From 1 to \(\mathcal{O}(k^2)\) samples
\[\rightarrow\] Finding vertex distances is either \(\mathcal{O}(k)\) or \(\mathcal{O}(k \log(k))\)

⇒ Algorithm is incremental, can be truncated if need be
Main Convergence Result

Theorem (Convergence under persistent gossip)

*If there exists a positive probability for any pairwise exchange in a finite time window, then the evolutions of $c$ and $P$ converge almost surely to a pairwise-optimal partition in finite time.*

**Proof sketch**

1) Regions remain connected
2) Total cost decreases with every pairwise update
3) Pairwise-optimal partitions are equilibrium set
Definition: A partition is a pairwise-optimal partition if, for every pair of neighboring robots \((i, j)\):

\[
H_i(c_i; P_i) + H_j(c_j; P_j) = \min_{a, b \in p_i \cup p_j} \left\{ \sum_{k \in P_i \cup P_j} \min \left\{ d_{P_i \cup P_j}(a, k), d_{P_i \cup P_j}(b, k) \right\} \right\}
\]

i.e., \((i, j)\) has reached lowest possible coverage cost of \(P_i \cup P_j\)

\[\Rightarrow\] Every pairwise-optimal partition is also centroidal Voronoi
Convergence results

Subset of Centroidal Voronoi

Cost: 12 hops

Cost: 10 hops

- Both are centroidal Voronoi
- Only lower cost is pairwise-optimal

⇒ Avoid all pairwise local minima
Convergence results

Statistical Results I

Initial cost: 1,032 \( m \)

100 sequences of pairwise exchanges

Gray - Pairwise-optimal Coverage
Black - Gossip Lloyd Coverage
Red - Centralized Lloyd Coverage
Convergence results

Statistical Results II

- 8 random initial conditions
- 20 sequences of pairwise exchanges

Gray - Pairwise-optimal Coverage
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Summary

Chief contributions

- Pairwise-optimal gossip coverage control algorithm
- Flexible computational requirement
- Improved performance over Lloyd-type methods

Future directions

- Dynamic teams and environments
- Pairwise optimization approach seems useful for wide class of problems