Regulation without calibration

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CDC2024 Contraction Workshop Milan, December 2024





Questions for a workshop on contraction theory

Is regulation possible without calibration?

How do animals regulate?

(Or, can we connect the internal model principle of control theory to the internal model principle of neuroscience?)

Regulation without calibration, R. Sepulchre, A. Cecconi, M. Bin, L. Marconi, submitted to IEEE Control Systems, 2024

The "Internal Model Principle" Between Mathematical and Conceptual Perspective. L. Marconi, plenary lecture CDC 2021.

Internal Models in Control, Bioengineering, and Neuroscience. M Bin, J. Huang, A. Isidori, L. Marconi, M. Mischiati, E. Sontag Annual Review of Control, Robotics, & Autonomous Systems, Vol. 5, pp. 55-79, 2022

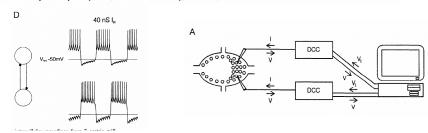
Principle Internal Models, R. Sepulchre. Editorial IEEE Control Systems, April 2023.

Motivation : A neurophysiological imitation game

JOURNAL OF NEUROPHYSIOLOGY Vol. 76, No. 2, August 1996. Printed in U.S.A.

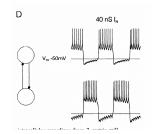
Mechanisms of Oscillation in Dynamic Clamp Constructed Two-Cell Half-Center Circuits

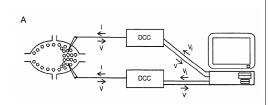
ANDREW A. SHARP, FRANCES K. SKINNER, AND EVE MARDER Volen Center for Complex Systems, Brandeis University, Waltham, Massachusetts 02254-9110



Question: how to emulate the biological oscillator D with an artificial circuit replacing one of the two neurons (A) ?

The question: how to regulate without calibration?





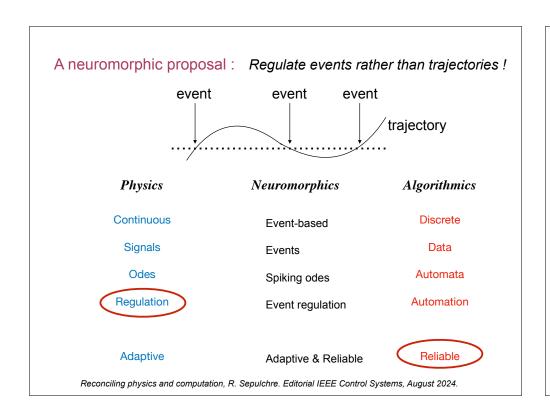
The *calibration* solution:

Tune the parameters of the artificial neuron to minimise the mismatch between the natural and artificial behaviors.

Claim: the internal model principle requires calibration

Fact: this cannot work in practice because the biological neuron is highly variable.

Th. Burghi, R. Sepulchre. Adaptive observers for biophysical neuronal circuits, IEEE TAC, vol.69, n 8, 2024.



- Regulation theory and the internal model principle
- Reliable internal models
- Reliable contraction
- Event regulation

Regulation theory

The Internal Model Principle of Control Theory*

B. A. FRANCIS† and W. M. WONHAM‡

The two conditions of regulation:

1. The error system is *contractive*

The purpose of the compensator is twofold. First, it is to provide closed loop stability.

2. e=0 is solution of the closed-loop behavior

Second, it is to regulate a variable z which is a given function of the plant output c and the reference signal r; typically z may be the tracking error r-c. A plant-compensator combination with these two properties is termed a synthesis, and a synthesis is called structurally stable if these two properties are preserved when certain system parameters are perturbed.

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Regulation requires calibration

Example:

 $\dot{y} = u + d$

Goal: regulate y=0 (disturbance rejection)

Solution:

$$u = -ky - \hat{d}$$

Regulation "only if" $d \equiv \hat{d}$

Regulation requires calibration

Example: $\dot{y} = u + d$

Goal: regulate y=0 (disturbance rejection)

Solution: $u = -ky - \hat{d}$

Regulation "only if" $d \equiv \hat{d}$

Internal model control: assume d is solution of an exosystem; include a copy of the exosystem in the controller in order to generate \hat{d}

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Constant disturbance: $\dot{d} = 0$ No calibration!

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Harmonic disturbance: $\ddot{d} + \omega_0^2 d = 0$ One parameter calibration!

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Regulation requires calibration

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Internal model control: assume d is solution of an exosystem; include a copy of the exosystem in the controller in order to generate \hat{d}

Constant disturbance: $\dot{d} = 0$ No calibration!

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"General" disturbance: $\dot{x} = f(x), d = h(x)$???

The internal model principle

No regulation without calibration, i.e, an *exact* copy of the exosystem

This principle holds for any *synchronization* problem:

Regulation, observer design, coordination, synchronization, ...

That is, ..., any control problem handled by contraction theory ...

B.A. Francis, W.M. Wonham, The internal model principle of control theory, Automatica, 1976.

P Wieland, R Sepulchre, F Allgöwer An internal model principle is necessary and sufficient for linear output synchronization Automatica, 2011.

J. Trumpf, H. Trentleman, J.C- Willems. An internal model principle for observers Automatica, IEEE CDC 2011.

M Bin, J. Huang, A. Isidori, L. Marconi, M. Mischiati, E. Sontag. Internal Models in Control, Bioengineering, and Neuroscience. Annual Review of Control, Robotics, & Autonomous Systems, Vol. 5, pp. 55-79, 2022

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The reliability experiment

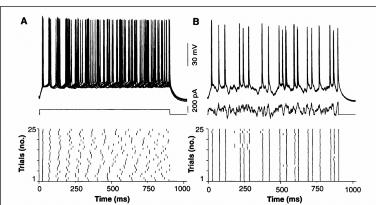
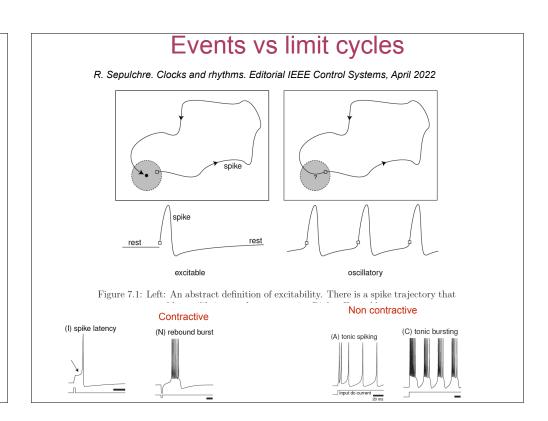


Fig. 1. Reliability of firing patterns of cortical neurons evoked by constant and fluctuating current. (**A**) In this example, a superthreshold dc current pulse (150 pA, 900 ms; middle) evoked trains of action potentials (approximately 14 Hz) in a regular-firing layer-5 neuron. Responses are shown superimposed (first 10 trials, top) and as a raster plot of spike times over spike times (25 consecutive trials, bottom). (**B**) The same cell as in (A) was again stimulated repeatedly, but this time with a fluctuating stimulus [Gaussian white noise, $\mu_s = 150$ pA, $\sigma_s = 100$ pA, $\tau_s = 3$ ms; see (14)].

Mainen & Sejnowski, Science, 1995

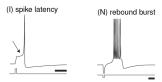
Events are reliable, limit cycle oscillations are not!



Contractive internal models

Event generators are contractive.

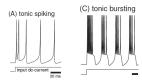
Contractive



Contractive exosystems *must* be *open*.

Limit cycles generators are non-contractive.

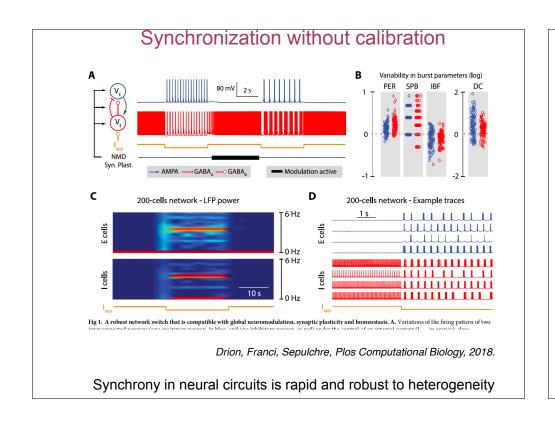
Non contractive



Regulation theory assumes *closed* exosystems.

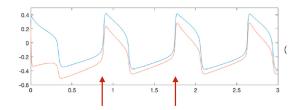
An anomaly ?

- Regulation theory and the internal model principle
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Reliable regulation

Synaptic rather than diffusive coupling enforces event-based error regulation.



Strong diffusive coupling where and when needed; No coupling where the dynamics is contractive

Event regulation results from the combination of *event* generators and *synaptic error* feedback

Synchrony without calibration

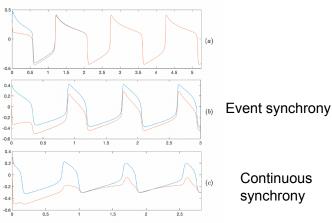


Fig. 1. Rapid synchronization of two identical (a) and non-identical (b) excitable systems under weak excitatory synaptic coupling. (c) Poor synchronization of the same non-identical excitable systems under strong diffusive coupling.

Rapid synchronization through fast threshold modulation, D. Somers, N. Kopell. Biological Cybernetics, 1992.

Rapid and robust synchronization via weak synaptic coupling, J.-G. Lee, RS, Automatica, 2024

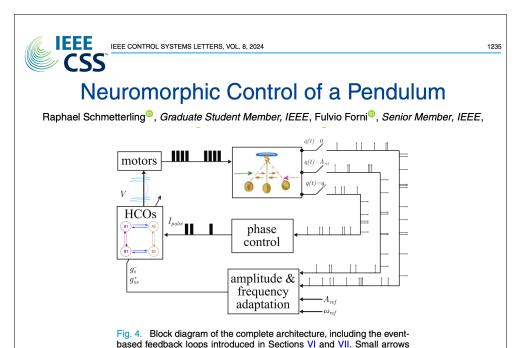
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Ingredients of neuromorphic regulation

The internal model is an event generator, made from neuromorphic circuits.

The feedback design comes from adaptive regulation control, but is designed to enforce strong coupling around the events and no coupling away from the events.

The modelling framework is *classical*: smooth odes, output feedback incremental passivity, ...



over signal transmission lines indicate event-based communication as described in Section III. The HCO block architecture is described in

Sections III and IV.

Conclusions for a workshop on contraction theory

Is regulation possible without calibration?

The internal model principle of continuous regulation theory is a *calibration* principle. Continuous trajectories *cannot* be regulated reliably in changing and uncertain environments.

How do animals regulate?

Regulation theory relies on *closed* and *non* contractive internal models. Lessons from neurophysiology: reliable regulation requires *open* and *excitable* internal models.

Neuromorphic proposal:

event regulation; excitable internal models; synaptic feedback

An event-based internal model principle

Only an internal model of reality - this working model in our minds- enables us to predict **events** which have not yet occured in the physical world, a process which saves time, expense, and even life. In other words the nervous system is viewed as a calculating machine capable of modelling or paralleling external **events**, and this process of paralleling is the basic feature of thought and of explanation

Kenneth Craik's, The Nature of Explanation (1943)

