A guide to examples and exercises from “Geometric Control of Mechanical Systems”

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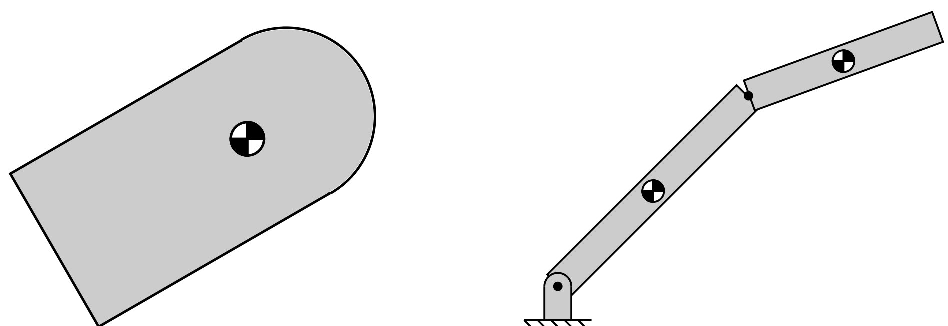
We list the physical systems that are considered in the book of Bullo and Lewis [2004], either as examples or as exercises. There are many other (not physical) examples in the text, and many other exercises. The intent is to provide a sampling of the sorts of systems one may treat using the methods detailed in the book, and to provide a guide as to where one can find discussions of the modeling, analysis, and control design issues for these systems. The order is that in which they appear in the book.

1. *Planar rigid body:* This system we saw in Section 1.1 and is depicted in Figure 1(a). This example appears in the following places, in the stated



(a) The planar rigid body   (b) The two-link planar manipulator

Figure 1:

contexts.

   (a) In Exercises E1.1 and E1.2 the reader is asked to derive the equations of motion for this system using elementary methods, and do some simulations to test their intuition.
   (b) The planar body is one of a few examples that recur throughout Chapter 4 to illustrate the mathematical modeling of mechanical systems. The reader is referred to Examples 4.3, 4.7, 4.31, 4.45, and 4.53, where all aspects of the system are put into our geometric framework, from the configuration space through to the control forces for the system.

(c) A modified version of this example, when its motion is in an ideal fluid, is also discussed in the text; see the discussion preceding Assumption 4.110. The reader is asked to do some basic modeling for this system in Exercises E4.25 and E5.24. In Example 6.60 the stability of relative equilibria for the system is investigated.

(d) The planar body system is considered as a system with a Lie group as a configuration space in Example 5.47.

(e) The controllability properties of the system are considered in Section 7.4.2.

(f) The controllability of the system as one whose configuration space is a Lie group is assigned as Exercise E7.21.

(g) A version of the system with different forces is considered in Exercise E7.14 and Example 8.11. The point of this modified system is to show that the low-order controllability results of Section 8.2 really are improvements over those of Section 7.3.

(h) Notions of kinematic reductions for the planar body are considered in Examples 8.22 and 8.26, and in Exercise E8.4.

(i) The kinematic reductions for the system are considered in Exercise E8.9, using the fact that the system has a Lie group as configuration space.

(j) In Exercise E9.9 the reader can perform simulations to test the series expansions presented in the book.

(k) A version of the planar body with gravitational effects considered serves as a model for a planar vertical takeoff and landing aircraft in Example 12.28.

(l) The motion planning strategies of Chapter 13 are demonstrated on the planar body in Section 13.3.1.

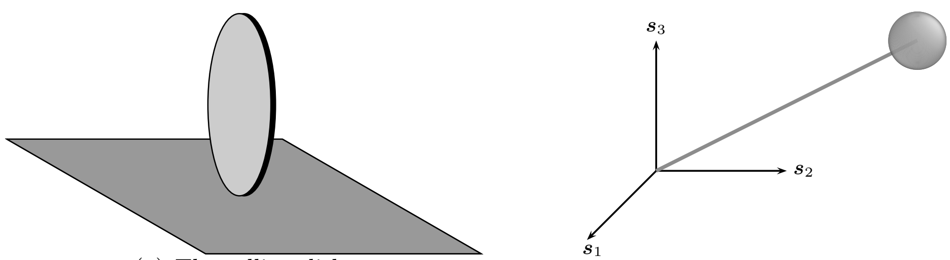
2. *Two-link planar manipulator:* This system was introduced in Section 1.2, and we reproduce the system in Figure 1(b). This example is treated in the following places in the text.

(a) In Exercises E1.3 and E1.4 the reader can implement basic symbolic computations, derive the equations of motion for the system, and is asked to informally consider some controllability-type problems for the system.

(b) The two-link manipulator is another example that is used in Chapter 4 to demonstrate modeling techniques. The system is worked out in detail in Examples 4.4, 4.8, 4.32, 4.46, and 4.54.

   (c) The stability of the two-link manipulator in its various equilibrium configurations is worked out in Example 6.50.
   (d) In the absence of gravitational effects, the relative equilibria for the system, and their stability, are considered in Example 6.59.
   (e) Stabilization of the two-link manipulator using linear PD control is considered in Section 10.3.4.
   (f) The application of vibrational stabilization techniques to the system is considered in Example 12.24.

3. *Rolling disk:* This is the system introduced in Section 1.3. We show a pared down version of the system in Figure 2(a). This example appears



(a) The rolling disk

(b) The spherical pendulum

Figure 2:

in the following places.

   (a) The reader is asked to derive the equations of motion for the rolling disk in Exercise E1.5, using elementary methods.
   (b) The rolling disk is the last of the three examples used in the course of Chapter 4 to demonstrate the modeling of mechanical systems. This system illustrates how constraints are modeled in our setup. We refer to Examples 4.5, 4.9, 4.33, 4.55, 4.73, 4.81, 4.83, and 4.96 for the development of the mathematical model for the rolling disk.
   (c) The controllability properties of the rolling disk are considered in Section 7.4.3.

4. *Spherical pendulum:* The system consists of a pendulum that is fixed at a base point, but is not constrained to move in a plane. We show the system in Figure 2(b). This example appears in the following places, in the stated contexts.

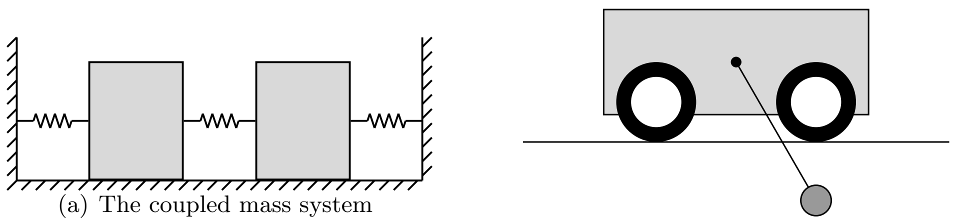
   (a) The reader is asked to do some basic modeling for the system in Exercise E1.6.

(b) The system is considered as a mechanical system with holonomic constraints in Example 4.99. In the absence of gravitational effects, the resulting equations of motion are the equations for geodesics on the two-sphere.

(c) The symmetries of the system are considered in Example 5.72, and the reader is asked to consider this further in Exercise E5.34.

(d) The spherical pendulum is treated as a fully actuated system for the problem of stabilization in Sections 11.1.3 and 11.3.2.

5. *Coupled masses:* This is a simple system, comprised of a pair of identical masses, coupled by three identical springs; see Figure 3(a). This system



(a) The coupled mass system

(b) The pendulum/cart system

Figure 3:

is considered as an exercise in the following places in the text.

(a) The reader is asked to do the modeling of the system, using the methods of Chapter 4, in Exercises E4.3, E4.17, and E4.29.

(b) This system is an example of a linear mechanical system, and its stability is determined as such a system in Exercise E6.5.

(c) The controllability of the system, as a linear mechanical control system, is the subject of Exercise E7.6.

(d) The stabilization of the system using linear PD control is the subject of Exercise E10.13.

6. *Pendulum/cart:* The system here is one of the standard examples, not just of a mechanical control system, but of a control system; the example appears in many control texts that deal with state space control. The system is a pendulum attached to a cart as depicted in Figure 3(b). This system is considered in the following exercises.

(a) The reader is asked to do the modeling on the pendulum/cart system, using the methods of Chapter 4, in Exercises E4.5, E4.19, and E4.31.

(b) The symmetries of the system are considered in Exercise E5.35.

(c) The stability of equilibria is considered in Exercise E6.14 and the stability of relative equilibria is considered in Exercise E6.18.

(d) The controllability properties of the pendulum/cart system are considered in Exercise E7.16 for various input types.

(e) Stabilization of the system using PD control is considered in Exercise E10.14.

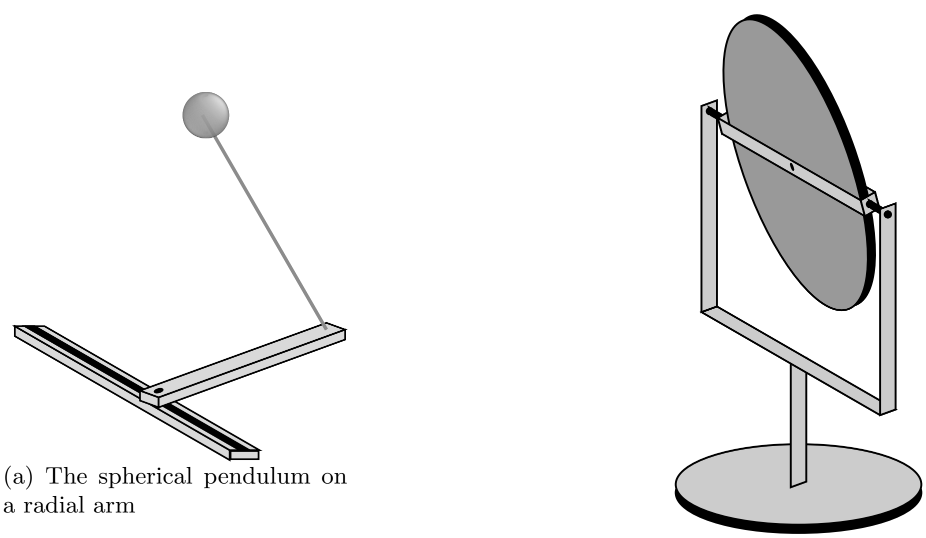
7. *Spherical pendulum on radial arm:* This is a spherical pendulum with the additional feature that the base point can be moved about in the plane. The system is depicted in Figure 4(a). This system is considered in the following exercises.

   (a) The reader is asked to do the modeling for the system in Exercises E4.6, E4.20, and E4.32.

   (b) The stability of equilibria for this spherical pendulum system is the subject of Exercise E6.15.

   (c) The controllability of the system from equilibrium configurations is considered in Exercise E7.17.

8. *Two-axis gyroscope:* The system here is depicted in Figure 4(b). The



(a) The spherical pendulum on a radial arm

(b) The two-axis gyroscope

Figure 4:

system is dealt with in the following exercises in the book.

(a) The reader can work out the model for the system in terms of the concepts of Chapter 4 in Exercises E4.7, E4.21, and E4.33.

(b) The stability of equilibria is considered in Exercise E6.16.

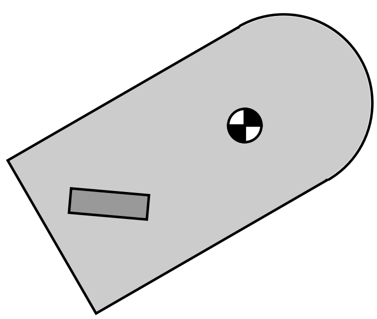
(c) Controllability from equilibrium configurations is considered in Exercise E7.18.

9. *Planar body with fan dynamics:* This is a modification of the example begun in Section 1.1, but now with a degree-of-freedom associated with the thrust fan as shown in Figure 5(a). This system comes up in the following places in the text.

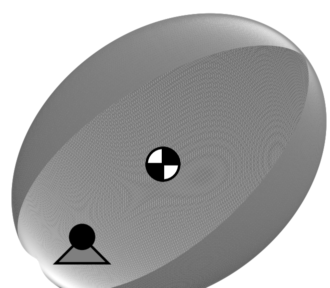
    (a) The modeling for this system is considered in Exercises E4.8, E4.22, and E4.34.

    (b) The properties of the system with respect to kinematic reductions are the topic of Exercise E8.8.

10. *Rigid body fixed at a point:* This is another standard example, and is depicted in Figure 5(b). This system is dealt with in the following places



(a) The planar rigid body with fan dynamics



(b) The rigid body fixed at a point

Figure 5:

in the text.

(a) The system is considered as one with a Lie group as configuration space in Example 5.48.

(b) Symmetries and conservation laws for the system are discussed in Example 5.74.

(c) The reader is asked to derive the equations of motion for a satellite with rotors in Exercise E5.25. This model amounts to a rigid body system with external forces.

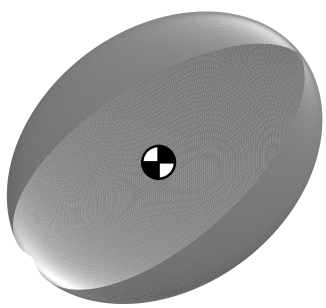
(d) The stability of rotations about the principal axes of the body is considered as stability of relative equilibria in Example 6.62.

(e) The controllability of the system as one whose configuration space is a Lie group is assigned as Exercise E7.22.

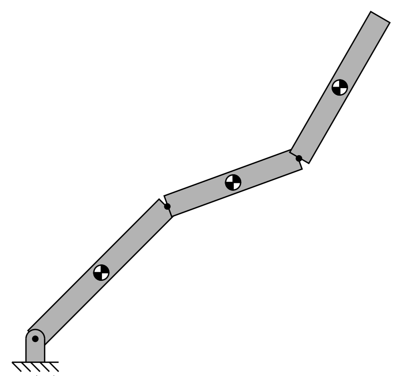
(f) In Exercise E8.10 the matter of kinematic reductions for the system is dealt with, using the Lie group structure of the configuration space.

(g) The problem of orienting a satellite in a desired direction is discussed in Section 11.4.3.

11. *Rigid body in an ideal fluid:* This system consists of a body moving freely, but its dynamics result from its being immersed in an ideal fluid, see Assumption 4.110. The system is depicted in Figure 6(a). The



(a) The rigid body in an ideal fluid



(b) The three-link manipulator

Figure 6:

system is considered in the following places.

(a) The modeling of the fluid-body interaction is briefly discussed in the text preamble to Exercise E4.25.

(b) The system is considered as one with a Lie group as configuration space in Example 5.49.

(c) A related example, a *planar* rigid body in an ideal fluid is considered in in Exercises E4.25 and E5.24 and in Example 6.60.

12. *Three-link manipulator:* This system is a three-link version of the two-link planar manipulator considered above, and is shown in Figure 6(b). This system appears in the following places.

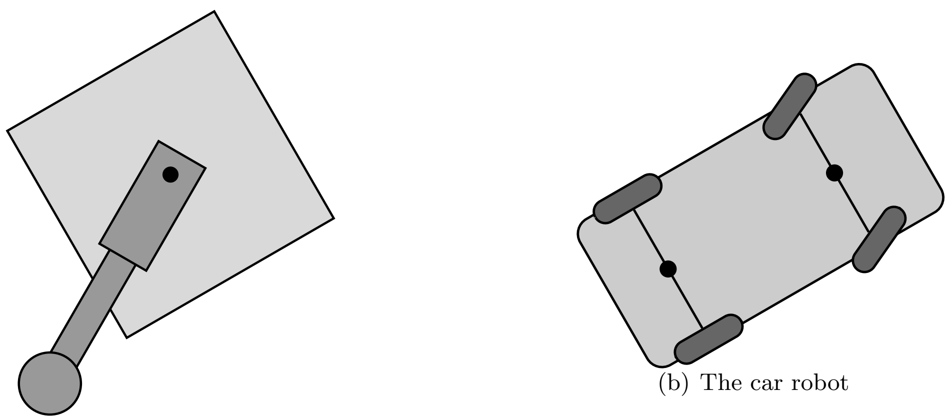
    (a) The reader is asked to use Mathematica® to do the modeling for the system in Exercise E4.37.

    (b) Kinematic reductions for the three-link manipulator are considered in Exercise E8.14.

    (c) Simulations for series expansions for the system are given in Exercise E9.11.

    (d) The kinematic reductions are used to consider motion planning problems in Exercise E13.8.

13. *Robotic leg:* The system here consists of a body moving in the plane and fixed at a point, and attached to it an extensible leg with a mass of the end, as shown in Figure 7(a). The system is treated in the text in



(a) The robotic leg

(b) The car robot

Figure 7:

the following places.

    (a) This system is introduced in Example 5.71 as a system with symmetry. Since this is the first appearance of the system, the model resulting from the techniques of Chapter 4 is also presented in this example.

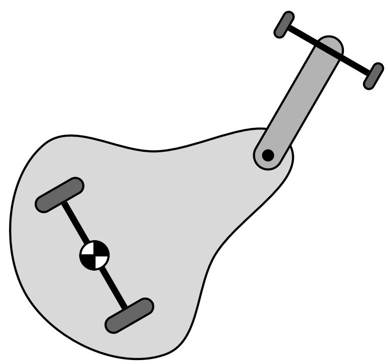
    (b) The controllability of the system is considered in Section 7.4.1. The reader is asked to explore the controllability of a single-input variant of the system in Exercise E7.13.

(c) Kinematic reductions for the robotic leg are considered in Example 8.29.

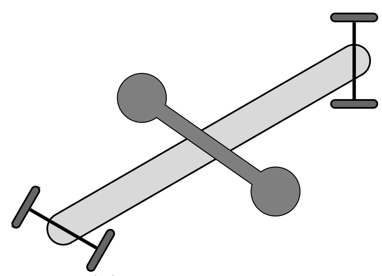
(d) The motion planning methods of Chapter 13 are applied to the system in Section 13.3.2.

14. *Car robot:* This system is a simplified model for a car as shown in Figure 7(b). This system is considered in the following places. The modelling of the system is considered in Exercises E4.9, E4.23, E4.35, and E4.41. The system is also given as an exercise on kinematic reductions in Exercise E8.7.

15. *Roller racer:* This system is a wheeled vehicle that uses articulation to generate motion. The model we use is shown in Figure 8(a). This system appears in the following places in the text.



(a) The roller racer



(b) The snakeboard

Figure 8:

(a) The modeling for the system is given as Exercise E4.42.

(b) In Exercise E9.10 the reader can do some simulations on the system to test some of the series expansions presented in the book.

16. *Snakeboard:* The final example used in the text is the snakeboard, which is a popular example in the literature on nonholonomic mechanics. A model for the system is depicted in Figure 8(b) and it consists of a bar with a set of actuated wheels on either end. Atop the bar sits an inertia wheel that is also actuated. The system is considered in Section 13.4 as a nontrivial example of implementing the motion planning strategies of Chapter 13.

References

Bullo, F. and Lewis, A. D. [2004] *Geometric Control of Mechanical Systems: Modeling, Analysis, and Design for Simple Mechanical Systems*, number 49 in Texts in Applied Mathematics, Springer-Verlag, New York–Heidelberg–Berlin, ISBN 0-387-22195-6.