

Errata and addenda for “Geometric Control of Mechanical Systems”

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This document records errors in the book “Geometric Control of Mechanical Systems” by Bullo and Lewis [2004]. These will be listed in order of page number and will be dated according to the order in which we discover them. Also included here will be style changes made to the text that are not necessarily errors.

Should you wish to make an error report, please contact the authors by visiting the webpage for the book:

<http://motion.mee.ucsb.edu/book-gcms/>

Errata and addenda

1. Cover (31/08/2004): Order of authors is incorrect.
2. Page 7, u13 (21/02/2004 from Alejandro Roa): There is an extra “also”.
3. Page 11, d13 (22/03/2005 from Akihiro Sato): Change “is the that of [Crouch 1984]” to “is that of Crouch [1984]”.
4. Page 21, d5 (28/03/2005 from John Mardlin): Change “ $x = x$ ” to the slightly more interesting “ $x = y$ ”.
5. Page 21, d12 (27/01/2006): Change “ $x \in S$ ” to $y \in S$ ”.
6. Page 24, u14 (27/01/2006): Change “ $v_1, v_1 \in V$ ” to “ $v_1, v_2 \in V$ ”.
7. Page 26, u7 (02/03/2007 from Pantelis Isaiah): Replace “This use of matrices is” with “This use of matrices in”.
8. Page 41, u18 (27/01/2006): Change

$$L(\underbrace{V^* \times \dots \times V^*}_{r \text{ copies}} \times \underbrace{V \times \dots \times V}_{s \text{ copies}}; \mathbb{R})$$

to

$$L(\underbrace{V^*, \dots, V^*}_{r \text{ copies}}, \underbrace{V, \dots, V}_{s \text{ copies}}; \mathbb{R}).$$

9. Page 46, u10 (28/02/2008): Replace “quadratic function” with “smooth quadratic function”.
10. Page 53, d8 (05/02/2013 from Andrew Staal): Replace “for every $a \in A$, there exists $\tilde{a} \in \tilde{A}$ ” with “for every $\tilde{a} \in \tilde{A}$, there exists $a \in A$ ”.
11. Page 57, d13 (03/07/2006): Make the replacement

$$\lim_{\mathbf{x} \rightarrow \mathbf{0}} \frac{o(\|\mathbf{x}\|_{\mathbb{R}^n}^k)}{\|\mathbf{x}\|_{\mathbb{R}^n}^k} = 0 \quad \implies \quad \lim_{\mathbf{x} \rightarrow \mathbf{0}} \frac{\|o(\|\mathbf{x}\|_{\mathbb{R}^n}^k)\|_{\mathbb{R}^m}}{\|\mathbf{x}\|_{\mathbb{R}^n}^k} = 0.$$

12. Page 59, d14 (22/11/2004): “ $o(\|\mathbf{v}\|_{\mathbb{R}^n}^{r+1})$ ” should be “ $O(\|\mathbf{v}\|_{\mathbb{R}^n}^{r+1})$ ”.
13. Page 74, d10 (01/31/2005 from David Tyner): Should be “ (y^1, \dots, y^m) ” rather than “ (y^1, \dots, y^n) ”.
14. Page 74, u9 (27/09/2005 from Bahman Ghahesifard): Missing parenthesis.
15. Page 78, u10 (03/10/2005 from Cesar Aguilar): “ $\mathcal{A} = \{\mathcal{U}_a, \phi_a\}_{a \in A}$ ” should be “ $\mathcal{A} = \{(\mathcal{U}_a, \phi_a)\}_{a \in A}$ ”.
16. Page 95, d18 (28/05/2008 from Jorrit Kirsten): Should be “ $A_X(x_0)$ the **linearization**”.
17. Page 104, u19 (21/05/2008 from Jorrit Kirsten): “pointwise assignments” should be “pointwise assignment”.
18. Page 109, d2 (21/05/2008 from Jorrit Kirsten): “If \mathcal{D} is a distribution” should be “If \mathcal{D} is a C^r -distribution”.
19. Page 120, d17 (18/08/2008 from Pantelis Isaiiah): “vector space” rather than “vector bundle”.
20. Page 127, u17 (04/11/2004): “it is” should be “is it”.
21. Page 127, u6 (04/11/2004): “it is” should be “is it”.
22. Page 132, u12 (18/08/2008): “it is” should be “is it”.
23. Page 133, d21 (22/12/2005): The proper formula for the components of the curvature tensor is

$$R_{jkl}^i = \frac{\partial \Gamma_{lj}^i}{\partial x^k} - \frac{\partial \Gamma_{kj}^i}{\partial x^l} + \Gamma_{km}^i \Gamma_{lj}^m - \Gamma_{lm}^i \Gamma_{kj}^m, \quad i, j, k, l \in \{1, \dots, n\}.$$

This uses the rule $R(\frac{\partial}{\partial x^k}, \frac{\partial}{\partial x^l}) \frac{\partial}{\partial x^j} = R_{jkl}^i \frac{\partial}{\partial x^i}$.

24. Page 135, u6 (10/02/2004 from David Tyner): Change “these operations make $T_x \mathbf{M}$ is \mathbb{R} -vector space” to “these operations make $T_x \mathbf{M}$ a \mathbb{R} -vector space”.
25. Page 137, d8 (13/02/2006): Replace “ $\Gamma^\infty(\mathbf{M})$ ” with “ $\Gamma^\infty(\mathbf{TM})$ ”.
26. Page 137, u13 (01/02/2006): Replace “this outlined” with “this is outlined”.
27. Page 154, d11 (06/02/2006 from Thomas Norman): A “ \times ” is missing between \mathbb{R}^2 and $]-\pi, \pi[$.
28. Page 156, d6 (04/11/2004): Replace “for the body (\mathcal{B}_a, μ_a) ” with “for the a th body”.
29. Page 158, d6 (08/03/2005 from Cesar Aguilar): Should read “ $(\mathbf{R}_0^T \mathbf{A}_{\mathbf{R}_0})^T = -\mathbf{R}_0^T \mathbf{A}_{\mathbf{R}_0}$ ”.
30. Page 158, u14 (09/03/2005 from Cesar Aguilar): The statement “ $\mathbf{SO}(3)$ is trivializable” should be “ $\mathbf{SO}(3)$ is parallelizable”.
31. Page 159, d5 and d8 (12/04/2005 from John Mardlin): Swap “ λ ” and “ ρ ” in the two displayed equations.
32. Page 169, u2 (01/12/2004): It is more consistent to say “ \mathbf{b}_3 ” rather than “ \mathbf{s}_3 ”. Also, the explicit expression for the inertia tensor can be given as

$$[\mathbb{I}_c] = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & J \end{bmatrix},$$

where an “ $*$ ” means a term whose value is immaterial.

33. Page 170, d19 (01/12/2004): It is more consistent to say “ \mathbf{b}_3 ” rather than “ \mathbf{s}_3 ”. Also, the explicit expressions for the inertia tensors can be given as

$$[\mathbb{I}_{1,c}] = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & J_1 \end{bmatrix}, \quad [\mathbb{I}_{2,c}] = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & J_2 \end{bmatrix},$$

where an “*” means a term whose value is immaterial.

34. Page 175, u13 (21/05/2008 from Jorrit Kirsten): “as can be see in the proof” should be “as can be seen in the proof”.
35. Page 175, u15 (01/10/2004): The sentence should read, “This is referred to as *Hamilton’s Principle*, with the original contributions appearing in [Hamilton 1834, 1835].” The papers of Hamilton should also be added to the bibliography.
36. Page 175, u11 (22/02/2006): The expression in the displayed equation should be set equal to zero.
37. Page 181, u1 (26/04/2007 from Elliot Johnson): Replace “ m_2^1 ” with “ m_2^2 ”.
38. Page 189, d15 (18/04/2005 from David Tyner): Replace “ ω_a ” with “ $\hat{\omega}_a$ ”.
39. Page 195, u11 (21/05/2008 from Jorrit Kirsten): Set this equation to be equal to zero.
40. Page 196, d18 (21/05/2008 from Jorrit Kirsten): Set this equation to be equal to zero.
41. Page 196, u5 (21/05/2008 from Jorrit Kirsten): Replace “call at a” with “call it a”.
42. Page 197, d10 (21/05/2008 from Jorrit Kirsten): Add the factor a_g to the last term in this expression.
43. Page 201, d8 (04/11/2004): Replace both occurrences of “regular” with “of class C^r ”.
44. Page 202, u2 (08/01/2014 from Jorrit Kirsten): Replace “measure” with “measured”.
45. Page 210, u13 (01/08/2014 from Jorrit Kirsten): Add condition that $X \in \Gamma^\infty(\mathcal{D})$.
46. Page 212, u1 (25/08/2008 from Pantelis Isaiah): Replace “takes” with “taken”
47. Page 217, d17 (18/08/2008 from Pantelis Isaiah): “is that curve” should be “is that a curve”.
48. Page 226, u4 (01/08/2014 from Jorrit Kirsten): Replace “these equations” with “the equations of the Poincaré representation”.
49. Page 230: u20 (04/11/2004): Replace “make” with “made”.
50. Page 232– (09/04/2006 from Thomas Norman): In part (e) of exercises E4.3, E4.4, E4.5, E4.6, E4.7, E4.8, and E4.9, replace “ $\iota_{\text{body}}|\mathbf{Q}$ ” and “ $\iota_{\text{spatial}}|\mathbf{Q}$ ” with “ $\iota_{\text{body}}|\mathbf{TQ}$ ” and “ $\iota_{\text{spatial}}|\mathbf{TQ}$ ”, respectively.
51. Page 239, d16 (28/05/2008 from Jorrit Kirsten): Should be “the work done by the gyroscopic force”
52. Page 239, d17 (17/01/2005 from David Tyner): “ $E(v_q) = \frac{1}{2}\mathbb{G}(v_q, v_q) - V(q)$ ” should be “ $E(v_q) = \frac{1}{2}\mathbb{G}(v_q, v_q) + V(q)$ ”.
53. Page 244, d17 (04/04/2010): The vector field X_1 is not correct as stated. It should be

$$X_1 = \ell \cos \phi \cos \theta \frac{\partial}{\partial x} + \ell \cos \phi \sin \theta \frac{\partial}{\partial y} + \sin \phi \frac{\partial}{\partial \theta},$$

and the constant ρ is not needed.

54. Page 251, u4-u8 (01/08/2014 from Jorris Kirsten): Replace several occurrences of “ $\mathbf{0}$ ” with “ \mathbf{I}_n ”.
55. Page 253, d17 (01/08/2014 from Jorris Kirsten): Replace “ $-\mathbf{R}^T(t)\dot{\mathbf{r}}(t)$ ” with “ $\mathbf{R}^T(t)\dot{\mathbf{r}}(t)$ ”.
56. Page 253, u1 (01/08/2014 from Jorris Kirsten): Replace the first “ $\xi_s(t)$ ” with “ $\xi_b(t)$ ”.
57. Page 258, d1 (10/03/2005 from Cesar Aguilar): The domain and range of Eul^{-1} should be switched.
58. Page 259, d13 (08/09/2004): Extra parenthesis removed from the end of the line.
59. Page 261, d1 (01/08/2014 from Jorris Kirsten): Use the set of invertible diagonal matrices.
60. Page 262, u8-9 (22/11/2004): Change “ $o(t^2)$ ” to “ $O(t^2)$ ” and “ $o(t^3)$ ” to “ $O(t^3)$ ”. Also, remove the bit where the definition of “ o ” is recalled.
61. Page 269, d9 (01/08/2014 from Jorris Kirsten): Replace “ \simeq ” with “ \subset ”.
62. Page 277, u9 (01/08/2014 from Jorris Kirsten): Replace “to Lie algebra” with “to the Lie algebra”.
63. Page 283, u8, u11 (01/08/2014 from Jorris Kirsten): Add “and $g_1, g_2 \in \mathbf{G}$ ” to the conditionals.
64. Page 283, u8 (01/08/2014 from Jorris Kirsten): For the definition of a C^r -action, the group should be a Lie group.
65. Page 285, u7 (01/08/2014 from Jorris Kirsten): Add a factor “ a ” to each summand on the right.
66. Page 285, u4 (01/08/2014 from Jorris Kirsten): Change the sign of X_k .
67. Page 286, d6 (01/08/2014 from Jorris Kirsten): Should be “ $\xi_L + \xi_R$ ” with “ $\xi_R - \xi_L$ ”.
68. Page 287, u11 (07/09/2004): “fiber bundle map on TM ” should be replaced with “fiber bundle map from TM to T^*M over id_M ”.
69. Page 290, proof of Proposition 6.54 (16/02/2005): Replace all occurrences of “ $\langle\langle \cdot, \cdot \rangle\rangle$ ” with “ $\mathbb{G}(\cdot, \cdot)$ ”.
70. Page 291, Definition 5.68 (16/02/2005): Replace all occurrences of “ $\langle\langle \cdot, \cdot \rangle\rangle$ ” with “ $\mathbb{G}(\cdot, \cdot)$ ”.
71. Page 292, proof of Theorem 5.69 (16/02/2005): Replace all occurrences of “ $\langle\langle \cdot, \cdot \rangle\rangle$ ” with “ $\mathbb{G}(\cdot, \cdot)$ ”.
72. Page 294, d12 (16/08/2007 from Pantelis Isaiah): “ (r, θ_1, θ_2) ” should read “ (r, θ, ψ) ”.
73. Page 294, u3 (01/08/2014 from Jorris Kirsten): Replace “ \mathbb{R}^2 ” with “ \mathbb{R}^3 ”.
74. Page 295, u1 (01/08/2014 from Jorris Kirsten): Multiply right-hand side by m .
75. Page 296, d13 (16/02/2005): Replace “ $\langle\langle \xi_R(g), v_g \rangle\rangle$ ” with “ $\mathbb{G}_{\mathbb{I}}(\xi_R(g), v_g)$ ”.
76. Page 297, u15 (07/09/2004): Replace “where” with “for which”.
77. Page 302, u4 (16/12/2006): Replace “ $\overset{\mathbb{G}}{\nabla}_{\text{hlft}(X)} Y_k$ ” with “ $\overset{\mathbb{G}}{\nabla}_X \text{hlft}(Y_k)$ ”.
78. Page 305, u6 (11/05/2005 from Cesar Aguilar): Should read “ $\exp(\mathbf{A})\mathbf{B} = \mathbf{B}\exp(\mathbf{A})$ ”.
79. Page 307, u12 (01/08/2014 from Jorris Kirsten): Replace “ $T_e R_{\gamma(t)}$ ” with “ $T_e R_{\gamma(t)^{-1}}$ ”.
80. Page 320, d3 (23/03/2007): Replace “intersection with $\mathbb{R} \setminus \bar{B}_r(x_0)$ ” with “intersection with $\mathbb{R}^n \setminus \bar{B}_r(x_0)$ ”.
81. Page 321, d16 (18/03/2009): Add a “:” after “chain of implications”.

82. Page 326, d16 (22/11/2004): Change “ $o(\|(x^1, \dots, x^n)\|_{\mathbb{R}^n}^3)$ ” to “ $O(\|(x^1, \dots, x^n)\|_{\mathbb{R}^n}^3)$ ”.
83. Page 333, d5 (13/01/2005 from David Tyner): “ $L(\mathbb{T}_{q_0}; \mathbb{T}_{q_0}^* \mathbb{Q})$ ” should be “ $L(\mathbb{T}_{q_0} \mathbb{Q}; \mathbb{T}_{q_0}^* \mathbb{Q})$ ”.
84. Page 334, d9 (23/03/2009): Replace this statement with: “ $\lambda \in \mathbb{C}$ is an eigenvalue of $A_{M,K}$ of algebraic multiplicity k if and only if there exists an eigenvalue $\ell \in \mathbb{R}$ of $M^\sharp \circ K^\flat$ of algebraic multiplicity $\frac{k}{2}$ such that $\lambda = \pm\sqrt{-\ell}$ ”
85. Page 349, d18 (22/01/2005): There is no reason why constraints cannot be included in the definitions for relative equilibria and their stability. This is done here by replacing “ $\Sigma = (\mathbb{Q}, \mathbb{G}, V, F)$ ” with “ $\Sigma = (\mathbb{Q}, \mathbb{G}, V, \mathcal{D}, F)$ ”. See item 90 for the other change that needs to be made.
86. Page 350, Figure 6.9 (16/12/2006): The figure has been altered; it is confusing as it is.
87. Page 350, d4 (16/02/2005): Should be “ $T\pi_B \circ \gamma'(t)$ ”.
88. Page 350, d8 (16/02/2005): Should be “ $T\pi_B \circ \gamma'(t)$ ”.
89. Page 350, d14 (16/01/2005): Add a comma after “fiber stable”.
90. Page 351, d18 (22/01/2005): There is no reason why constraints cannot be included in the definitions for relative equilibria and their stability. This is done here by making the first sentence in this section, “Next, we endeavor to provide existence and stability criteria for a relative equilibrium of a C^∞ -forced simple mechanical system $\Sigma = (\mathbb{Q}, \mathbb{G}, V, F)$ where F is time-independent.” See item 85 for the other change that needs to be made.
91. Page 351, u9 (05/09/2004): Added comma to the end of the line.
92. Page 351, u2 (16/02/2005): Change “ $\langle\langle v_q, X(q) \rangle\rangle$ ” to “ $\mathbb{G}(v_q, X(q))$ ”.
93. Page 353, d4 (16/02/2005): Replace “ $V_{X,v}^{\text{eff}}$ ” with “ $V_{X,v(t)}^{\text{eff}}$ ”.
94. Page 353, u8 (16/12/2004): The “on the dynamical system” is better as “for the dynamical system”.
95. Page 354, d16 (16/02/2005): Item (ii) in the statement of the lemma should begin “if q_0 is a critical point for ψ , then. . .”
96. Page 354, d19 (15/12/2004): The beginning of the proof does not need to say what is being proved.
97. Page 354, u15 (22/11/2004): Change “ $o(\|(q^1, \dots, q^n)\|_{\mathbb{R}^n}^3)$ ” to “ $O(\|(q^1, \dots, q^n)\|_{\mathbb{R}^n}^3)$ ”.
98. Page 360, text before Exercise 6.4 (18/08/2008 from Pantelis Isaiiah): All argument of h should match its domain, e.g., “ $h(x, 1)$ ” instead of “ $h(1, x)$ ”.
99. Page 362, u9 (20/04/2006): Change “converges to a critical point” to “converges to a connected component of the set of critical points”.
100. Page 373, u4 (09/04/2006 from Bahman Ghahesifard): Change “an family” to “a family.”
101. Page 378, d8 (01/08/2007 from Cesar Aguilar): The interior in this equation should be relative to
- $$\text{aff}\left\{f_0(x_0) + \sum_{a=1}^m u^a f_a(x_0) \mid u \in U\right\}.$$
102. Page 380, d7 (18/08/2008 from Pantelis Isaiiah): “other element” should be “other elements”.
103. Page 382, d1 (18/08/2008 from Pantelis Isaiiah): “The” instead of “Let”.

104. Page 382, u14 (01/07/2005 from Cesar Aguilar): In fact, $[\xi_1, [\xi_0, \xi_1]] + [\xi_2, [\xi_0, \xi_2]]$ is in $\mathcal{B}^w(\xi)$ when $w = (1, 1, 3)$.
105. Page 383, d1 (30/12/2008 from Pantelis Isaiah): The 2 at the beginning of the line should not be there.
106. Page 384, d18 (18/08/2008 from Pantelis Isaiah): “Theorem 7.40” should be “Theorem 7.20”.
107. Page 387, d16 (09/04/2006 from Bahman Gharesifard): Should be “ $\mathcal{D}_{\gamma(t_0)}$ ” rather than the garble that is there.
108. Page 390, u3: “has” instead of “have”.
109. Page 393, u1 (29/10/2009 from Maria Barbero): Delete the “2” in front of this equation.
110. Page 393, u3 (29/10/2009 from Maria Barbero): The argument on the left side of the equals sign should be “ 0_{q_0} ”.
111. Page 417, d1 (23/05/2012 from Alisina Azhang): “ $\{\text{sym}_{Y_1}, \dots, \text{sym}_{Y_m}\}$ ” should be “ $\{\text{sym}_{Y_{k+1}}, \dots, \text{sym}_{Y_m}\}$ ”.
112. Page 449, d5 (04/11/2004): Should be “using the variation of constants formula”.
113. Page 483, u5 (05/09/2004): Added some commas to arrive at “if, in part (vii), one can take”.
114. Page 488, d21 (15/01/2007 from Elsa Hansen): Replace “there are systems that do not satisfy the necessary conditions, but are nonetheless locally asymptotically stabilizable by C^0 -state feedback” with “there are systems that satisfy the necessary conditions, but are nonetheless not locally asymptotically stabilizable by C^0 -state feedback”.
115. Page 497, d12 (25/09/2004): Should be “ K_P^\sharp ” and “ K_D^\sharp ”.
116. Page 498, u3 (25/09/2004): Should be “ K_P^\sharp ” and “ K_D^\sharp ”.
117. Page 502, u12 (25/09/2004): Should be “ K_P^\sharp ” and “ K_D^\sharp ”.
118. Page 503, d13 (25/09/2004): Should be “ K_P^\sharp ”.
119. Page 503, d14 (22/01/2005): Add a comma after “Furthermore”.
120. Page 509, d14 (23/09/2004): The sentence should read: “That is to say, if K_D is positive-definite, then derivative control of a simple mechanical system is an example of dissipative control for the corresponding control-affine system.”
121. Page 512, u14 (23/09/2004): The sentence here reads better as, “We now state a sufficient condition for stabilizability by PD control.”
122. Page 513, d6 (27/09/2004): Should be “ $F_B^*(b)$ ”.
123. Page 536, u5 (07/06/2011 from Cesar Aguilar): Should be “ $\gamma_{\text{ref}}(t_0) = r$ ”.
124. Page 537, d2 (07/06/2011 from Cesar Aguilar): “ $\mathcal{T}(q, X_q) = X_q$ ” should be “ $\mathcal{T}(q, X_q) = (X_q, q)$ ”.
125. Page 563, d18 (06/09/2004): Replace “analog” with “analogous”.
126. Page 603, d10 (16/02/2005): Replace both occurrences of “ $\langle\langle \cdot, \cdot \rangle\rangle$ ” with “ $\mathbb{G}(\cdot, \cdot)$ ”.
127. Page 621, u9 (10/11/2004): This item should be replaced with, “A property P holds *almost everywhere* (a.e.) on I , or for *almost every* $t \in I$ (a.e. $t \in I$) if there exists a subset $N \subset I$ of zero measure such that P holds for all $t \in I \setminus N$.”

128. Page 667, u10 (21/05/2008 from Jorrit Kirsten): Replace “Grundgesetz” with “Grundgesetz”.
129. Page 671, d15 and d18 (21/05/2008 from Jorrit Kirsten): Replace “Dynamics” with “Dynamik”
130. Page 673, d1 (21/05/2008 from Jorrit Kirsten): Replace “Rotationskorpers” with “Rotationskörpers” and “Flussigkeit” with “Flüssigkeit”.
131. Page 676, d16 (25/08/2008 from Pantelis Isaiiah): Add “Configuration” to beginning of title.
132. Page 679, u5 (17/01/2006): Should be “Lie” and not “lie”.
133. Page 681, d6 (17/12/2004): The title should be appropriately capitalized, and it is “number”, not “volume”.

References

- Bullo, F. and Lewis, A. D. [2004] *Geometric Control of Mechanical Systems: Modeling, Analysis, and Design for Simple Mechanical Systems*, number 49 in Texts in Applied Mathematics, Springer-Verlag, New York/Heidelberg/Berlin, ISBN 0-387-22195-6.
- Hamilton, W. R. [1834] *On a general method in dynamics; by which the study of motions of all free systems of attracting or repelling points is reduced to the search and differentiation of one central relation, or characteristic function*, Philosophical Transactions of the Royal Society, pages 247–308.
- [1835] *Second essay on a general method in dynamics*, Philosophical Transactions of the Royal Society, pages 95–144.