Possible courses that can be taught from “Geometric Control of Mechanical Systems”

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22/03/2006

Courses have been taught using this book at the University of Illinois, Urbana-Champaign (a first-year graduate course for engineering students) and Queen’s University (an advanced undergraduate/first-year graduate course for engineering and mathematics students). Let us roughly describe these courses.

Use as an undergraduate text. Clearly one cannot attempt to cover a lot of the material in the book at the undergraduate level. However, it is possible to gather bits and pieces from this book to make a coherent and interesting undergraduate course, provided that students have the analysis, linear algebra, and differential equations prerequisites described above. At Queen’s University the course is core for a group of students from the Mathematics & Engineering program. These are engineering students who have, by North American standards, a strong background in mathematics. There are occasionally Mathematics undergraduates in the class. The course is simultaneously given to graduate students, usually in Mathematics, who do extra work for their graduate credit. At the undergraduate level, there is a compromise that must be made in terms of the level of detail and rigor. The extent of this will depend on the instructor and the students.

The course at Queen’s roughly proceeds as follows.

1. The hardest part of the course is delivering the background in differential geometry. What has worked in practice is to present this background simultaneously with the modeling. Thus, selected material from Chapters 2 and 3, and Chapter 4 is covered at the same time. It is attempted to make the treatment of differential geometry as concrete as possible. One way to achieve this—one that has worked in the classroom—is to place the emphasis on coordinate representations and their transformation properties. This enables students to perform computations in examples, but at the same time become acquainted with the important idea of coordinate invariance. Nonholonomic constraints are not covered. At the end of this development, which normally takes up over half of a twelve-week course, students can understand the control systems described in Section 4.6 that do not involve constraints.

2. The next portion of the course is devoted to control theory. After a brief description of the typical control problems (controllability, stabilizability, motion planning, etc.), the classical approach of linearization is considered. Linear mechanical systems and their stability properties are discussed (Section 6.2.1). Then linearizations, and the relationships between linear and nonlinear stability are discussed (Section 6.2.2). If time permits, stability with dissipation is considered (Section 6.2.3). Then stabilization problems using linear methods are discussed (Section 10.3). Undergraduate students at Queen's have had two courses in linear control theory, so reference can be made for these students to Linear Quadratic Regulator theory. However, the graduate students often have no background in control, so not too much emphasis is placed on standard topics in linear systems theory.
3. The final topic in the course is motion planning for affine connection control systems. This uses material from Section 8.3 and Chapter 13, as well as a little material from Chapter 7. This is fairly sophisticated material for undergraduates, but it goes quite smoothly in the classroom, provided that the students have grasped the material from Chapters 3 and 4. The presentation here begins with a discussion of driftless systems, their controllability (using Lie brackets), and some words about motion planning for these systems. The problem of relating motion control problems for affine connection control systems to that for driftless systems is posed. This provides motivation for talking about kinematic reductions, particularly decoupling vector fields. A few words are said about the solution of motion planning problems in practice (this problem is generally insoluble), and students are shown simulations for a few examples.

Often the above material can be covered in about ten weeks. The material in the latter part of the course goes quite quickly provided that sufficient time is spent on the first part. Usually the time left over is used to cover special topics like dissipative forces and the LaSalle Invariance Principle, or systems with nonholonomic constraints.

Use as a graduate text. The structure of a graduate course using this book as a text will depend largely upon the background of the students and of the instructor. For instance, for engineering students, the undergraduate course described above, possibly sped up and with some additional material, could form a good course. Indeed, at the University of Illinois at Urbana-Champaign, this text has been used along these lines. First, as suggested above, the modeling and differential geometry background are presented simultaneously. Later, building on the prerequisite on linear control theory, more time is dedicated to nonlinear analysis and design methods. The course typically entails a final project, prepared by individual students or by small teams, on one of the more advanced chapters. We believe that, provided the instructor is comfortable with the more advanced material in the book (e.g., general discussions of controllability, perturbation methods, etc.), this material can be taught at the graduate level to engineering students.

To students with a stronger mathematical background than first-year engineering students, it is possible to be quite flexible with the sort of course one teaches. Some examples are the following.

1. For students with an undergraduate degree in mathematics, one could give students a rigorous course merely on modeling and stability of mechanical systems, using material from Chapters 2, 3, 4, 5, and 6. Such a course need not present any control theory at all.
2. For students with some background in geometric mechanics, a course could be given that provided an introduction to the principles of nonlinear control, and that illustrated how these concepts applied to mechanical systems. Such a course could be given from (probably a subset of) the material in Chapters 7, 9, 8, 10, 11, 12, and 13.
3. Students with a background in nonlinear control could receive a course on how to apply these methods to mechanical systems. This course would involve material from (probably a subset of) Section 3.8, and Chapters 4, 5, 7, 8, 9, 10, 11, 12, and 13, but now it would be possible to skip over, or only quickly review, the overview material on control theory in each of the last five chapters.

In any course in which a subset of the control material is to be covered, the instructor is advised that the material from the following sets of chapters go naturally together:

1. Chapters 7, 8, and 13;
2. Chapters 9 and 12;
3. Chapters 6, 10, and 11.

In any of the above courses, one fundamental decision affecting the course structure is whether the material on Lie groups and symmetry will be covered. If it is, then this will necessarily form a significant portion of the course material, since students will not generally have the background in Lie groups such as is required (and provided) in the book. For this reason, it is worth keeping in mind that all material on Lie groups and symmetry can be skipped, and what is left will be logically consistent. On the other hand, if one wishes to focus on systems on Lie groups or with symmetry, it is possible to do this as well.