Optimal Demand Response

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Outline

- Motivation
- Demand response model
- Some results
Wind power over land (outside Antarctica): 70 – 170 TW

World power demand: 16 TW

Solar power over land: 340 TW

Source: M. Jacobson, 2011
US electricity flow 2009

Conversion loss: 63%

Gross gen: 37%

Plant use: 2%
T&D losses: 2.6%

End use: 33%

US total energy use: 94.6 quads
For electricity gen: 41%

Source: EIA Annual Energy Review 2009
Renewables are exploding

Renewables in 2009
- 26% of global electricity capacity
- 18% of global electricity generation
- Developing countries have >50% of world’s renewable capacity
- In both US & Europe, more than 50% of added capacity is renewable

- Grid-connected PV has been doubling/yr for the past decade, 100x since 2000
- Wind capacity has grown by 27%/yr from 2004 – 09

Source: Renewable Energy Global Status Report, Sept 2010
Key challenges: uncertainty

High Levels of Wind and Solar PV Will Present an Operating Challenge!

Source: Rosa Yang
Today’s demand response

- Load side management has been practiced for a long time
  - E.g., direct load control

- They are simple and small
  - Centralized and static
  - Invoked rarely
    - Hottest days in summer
  - Small number of participants
    - Usually industrial, commercial

- Because
  - Simple system is sufficient
  - Lack of sensing, control, & 2-way communication infrastructure
Tomorrow's demand response

Benefits
- Adapt elastic demand to uncertain supply
- Reduce peak, shift load

Much more scalable
- Distributed
- Real-time dynamic
- Larger user participation

It'd be cheaper to use photons than electrons to deal with power shortages!
Automated Demand Response Saves Capacity and Energy

Electric load profile for PG&E participants on 8/30/2007

Source: Steven Chu, GridWeek 2009
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Features to capture

Wholesale markets
- Day ahead, real-time balancing

Renewable generation
- Non-dispatchable

Demand response
- Real-time control (through pricing)
Model: user

Each user has 1 appliance (wlog)
- Operates appliance with probability $\pi_i(t)$
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

\[ \underline{x}_i(t) \leq x_i(t) \leq \overline{x}_i(t) \quad \sum_t x_i(t) \geq \overline{X}_i \]

Demand at $t$:

\[ D(t) := \sum_i \delta_i x_i(t) \quad \delta_i = \begin{cases} 1 \text{ wp } \pi_i(t) \\ 0 \text{ wp } 1 - \pi_i(t) \end{cases} \]
Model: LSE (load serving entity)

Power procurement

- **Renewable power**: $P_r(t), \ c_r(P_r(t)) = 0$
  - Random variable, realized in real-time

- **Day-ahead power**: $P_d(t), \ c_d(P_d(t)), c_o(\Delta x(t))$
  - Control, decided a day ahead

- **Real-time balancing power**: $P_b(t), \ c_b(P_b(t))$
  - $P_b(t) = D(t) - P_r(t) - P_d(t)$

- Use as much renewable as possible
- Optimally provision day-ahead power
- Buy sufficient real-time power to balance demand
Key assumption

Simplifying assumption
- No network constraints
Questions

Day-ahead decision

- How much power $P_d$ should LSE buy from day-ahead market?

Real-time decision (at $t-$)

- How much $x_i(t)$ should users consume, given realization of wind power $P_r(t)$ and $\delta_i$?

How to compute these decisions distributively?

How does closed-loop system behave?

\[
\begin{align*}
\text{available info:} & \quad u_i(\cdot), \pi_i, F_r(\cdot) \quad \text{\(t - 24\text{hrs}\)} \quad \delta_i, P_r(t), P_d^* \\
\text{decision:} & \quad P_d^*, \quad x_i^*(t) \quad \text{\(t-\)}
\end{align*}
\]
Our approach

Real-time (at $t-$)
- Given $P_d$ and realizations of $P_r(t), \delta_i$ choose optimal
  $x_i^*(t) = x_i^*(P_d; P_r(t), \delta_i)$ to max social
  welfare, through DR

Day-ahead
- Choose optimal $P_d^*$ that maximizes expected
  optimal social welfare

$t - 24$hrs  
$\delta_i, P_r(t), P_d^*$

available info: $u_i(\cdot), \pi_i, F_r(\cdot)$

decision: $P_d^*$, $x_i^*(t)$
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Some results
- T=1 case: distributed alg
- General T: distributed alg
- Impact of uncertainty
T=1 case

Each user has 1 appliance (wlog)

- Operates appliance with probability $\pi_i(t)$
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

$$\underline{x}_i(t) \leq x_i(t) \leq \overline{x}_i(t)$$

$$\sum_i x_i(t) \geq \bar{X}_i$$

Demand at $t$:

$$D(t) := \sum_i \delta_i x_i(t)$$

$$\delta_i = \begin{cases} 1 & \text{wp } \pi_i(t) \\ 0 & \text{wp } 1 - \pi_i(t) \end{cases}$$
Welfare function

Supply cost

\[ c(P_d, x) = c_d(P_d) + c_o(\Delta(x))^{P_d}_0 + c_b(\Delta(x) - P_d)_+ \]

\[ \Delta(x) := \sum_i \delta_i x_i - P_r \] excess demand
Welfare function

Supply cost

\[ c(P_d, x) = c_d(P_d) + c_o(\Delta(x))_0^{P_d} + c_b(\Delta(x) - P_d)_+ \]

\[ \Delta(x) := \sum_i \delta_i x_i - P_r \]

excess demand
Welfare function

Supply cost

\[ c(P_d, x) = c_d(P_d) + c_o(\Delta(x))P_d^0 + c_b(\Delta(x) - P_d) \]

\[ \Delta(x) := \sum_i \delta_i x_i - P_r \]

excess demand

Welfare function (random)

\[ W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x) \]

user utility  supply cost
Optimal operation

Welfare function (random)
\[ W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x) \]

Optimal real-time demand response
\[
\max_{x} W(P_d, x) \quad \text{given realization of } P_r, \delta_i
\]
Optimal operation

Welfare function (random)

\[ W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x) \]

Optimal real-time demand response

\[ x^*(P_d) := \arg \max_x W(P_d, x) \] given realization of \( P_r, \delta_i \)
Optimal operation

Welfare function (random)
\[ W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x) \]

Optimal real-time demand response
\[ x^*(P_d) := \arg \max_x W(P_d, x) \text{ given realization of } P_r, \delta_i \]

Optimal day-ahead procurement
\[ P_{d}^* := \arg \max_{P_d} \mathbb{E} W(P_d, x^*(P_d)) \]

Overall problem:
\[ \max_{P_d} \mathbb{E} \max_x W(P_d, x) \]
Algorithm 1 (real-time DR)

$$\max_{P_d} \mathbb{E} \max_{x} W(P_d, x)$$

Active user $i$ computes $x_i^*$
- Optimal consumption

LSE computes
- Real-time price $\mu_b^*$
- Optimal day-ahead power to use $y_o^*$
- Optimal real-time balancing power $y_b^*$
Algorithm 1 (real-time DR)

Active user $i$:
\[
x_{i}^{k+1} = \left( x_{i}^{k} + \gamma (u_i'(x_i^k) - \mu_b^k) \right)_{x_i^k} \]
inc if marginal utility $>\,$ real-time price

LSE:
\[
\mu_{b}^{k+1} = \left( \mu_{b}^{k} + \gamma \left( \Delta(x^k) - y_{o}^{k} - y_{b}^{k} \right) \right)_{+}
\]
inc if total demand $>\,$ total supply

- Decentralized
- Iterative computation at $t$-
Theorem: Algorithm 1

Socially optimal

- Converges to welfare-maximizing DR \( x^* = x^*(P_d) \)
- Real-time price aligns marginal cost of real-time power with individual marginal utility

\[
\mu_b^* = c_b(y_b^*) = u_i'(x_i^*)
\]

Incentive compatible

- \( x_i^* \) maximizes \( i \)'s surplus given price \( \mu_b^* \)
Algorithm 1 (real-time DR)

**Theorem: Algorithm 1**

Marginal costs, optimal day-ahead and balancing power consumed:

\[ c_b'(y_b^*) = c_o'(y_o^*) + \mu_o^* \]

\[ \mu_o^* = \frac{\partial W}{\partial P_d}(P_d^*) \]
Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} EW\left(P_d, x^*(P_d)\right)$$

LSE: $$P_{d}^{m+1} = \left(P_{d}^{m} + \gamma^{m} \left(\mu_{o}^{m} - c_{d} '\left(P_{d}^{m}\right)\right)\right)_{+}$$

calculated from Monte Carlo simulation of Alg 1 (stochastic approximation)
Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} \text{EW}(P_d, x^*(P_d))$$

LSE:

$$P_{d}^{m+1} = \left(P_d^m + \gamma^m \left(\mu_o^m - c_d^d(P_d^m)\right)\right)_+$$

Given $$\delta^m, P_r^m$$:

$$\mu_o^m = \frac{\partial W}{\partial P_d}(P_d^m)$$

$$\mu_b^m = \mu_o^m + c_o^d(y_o^m)$$
Algorithm 2 (day-ahead procurement)

Theorem

Algorithm 2 converges a.s. to optimal $P_d^*$ for appropriate stepsize $\gamma^k$
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General T case

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Coupling across time
⇒ Need state

Demand at $t$:

$$D(t) := \sum_i \delta_i x_i(t) \quad \delta_i = \begin{cases} 1 & \text{wp } \pi_i(t) \\ 0 & \text{wp } 1 - \pi_i(t) \end{cases}$$
**Time correlation**

- **Example: EV charging**
  - Time-correlating constraint: \[ \sum_{t=1}^{T} x_i(t) \geq R_i, \forall i \]
- **Day-ahead decision and real-time decisions**

\[
\begin{align*}
\text{Day-ahead} & \quad t- \quad t = 1, 2, \ldots, T \\
\text{available info:} & \quad u_i(\cdot), F_r(\cdot) \quad P_r(t), P_d^*(t), R_i(t) \\
\text{decision:} & \quad P_d^*(t), \forall t \quad x_i^*(t), \forall i
\end{align*}
\]

- **(1+T)-period dynamic programming**

**Remaining demand**
Main idea
- Solve deterministic problem in each step using conditional expectation of $P_r$ (distributed)
- Apply decision at current step

One day ahead, decide $P_d^*$ by solving
\[
\max_{P_d, x} \sum_{\tau=t}^{T} W \left( P_d (\tau), x(\tau); \bar{P}_r (\tau) \right) \quad \text{s.t.} \quad \sum_{\tau=1}^{T} x_i(\tau) \geq R_i
\]

At time $t-$, decide $x^*(t)$ by solving
\[
\max_{x} \sum_{\tau=t}^{T} W \left( P_d^* (\tau), x(\tau); \bar{P}_r (\tau \mid t) \right) \quad \text{s.t.} \quad \sum_{\tau=t}^{T} x_i(\tau) \geq R_i(t)
\]
Algorithm 3 ($T>1$)

**Theorem: performance**

- Algorithm 3 is optimal in special cases

\[ J^{A3} - J^* \leq \frac{1}{2} T \bar{P}_r^2 \]
Effect of renewable on welfare

Renewable power:

\[ P_r(a,b) := a \cdot \mu_r + b \cdot V_r \]

Optimal welfare of (1+T)-period DP

\[ W(P_r(a,b)) \]
Effect of renewable on welfare

\[ P_r(a,b) := a \cdot \mu_r + b \cdot V_r \]

**Theorem**
- Cost increases in \( \text{var} \) of \( P_r \)
- \( W(P_r(a,b)) \) increases in \( a \), decreases in \( b \)
- \( W(P_r(s,s)) \) increases in \( s \)