Optimal Demand Response

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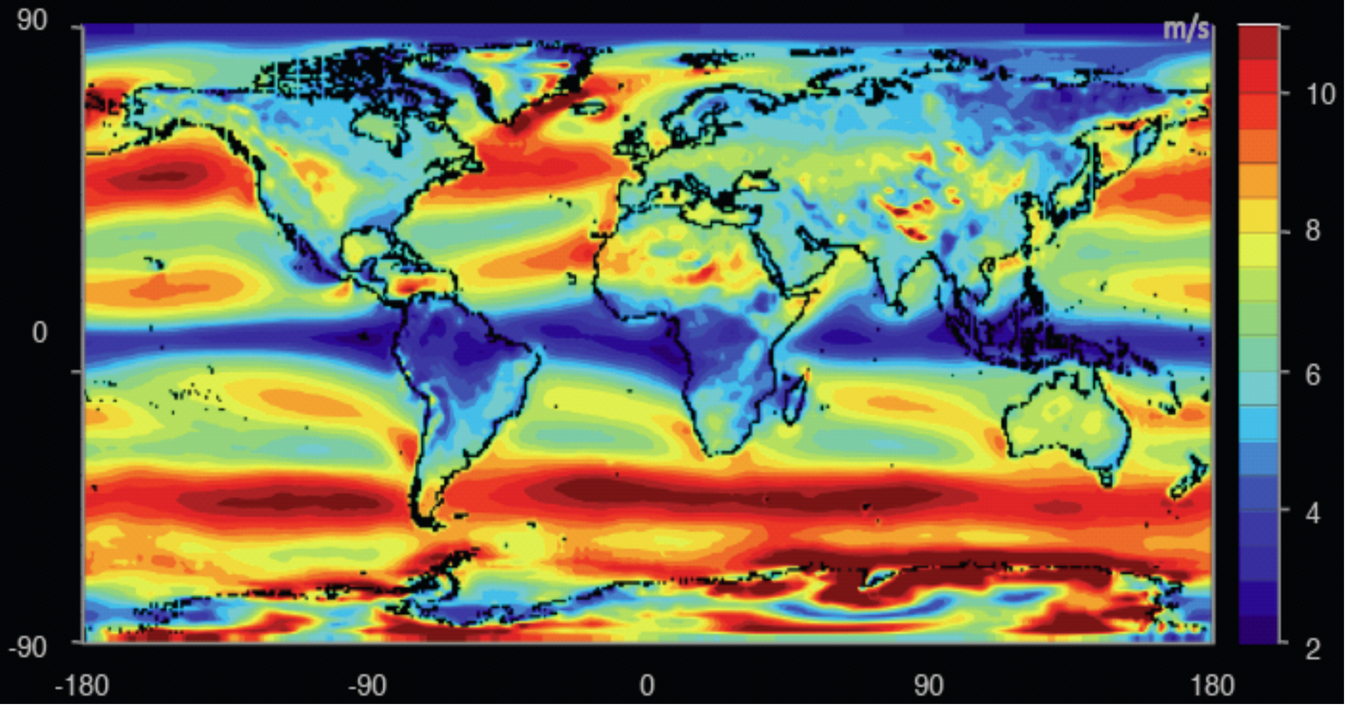
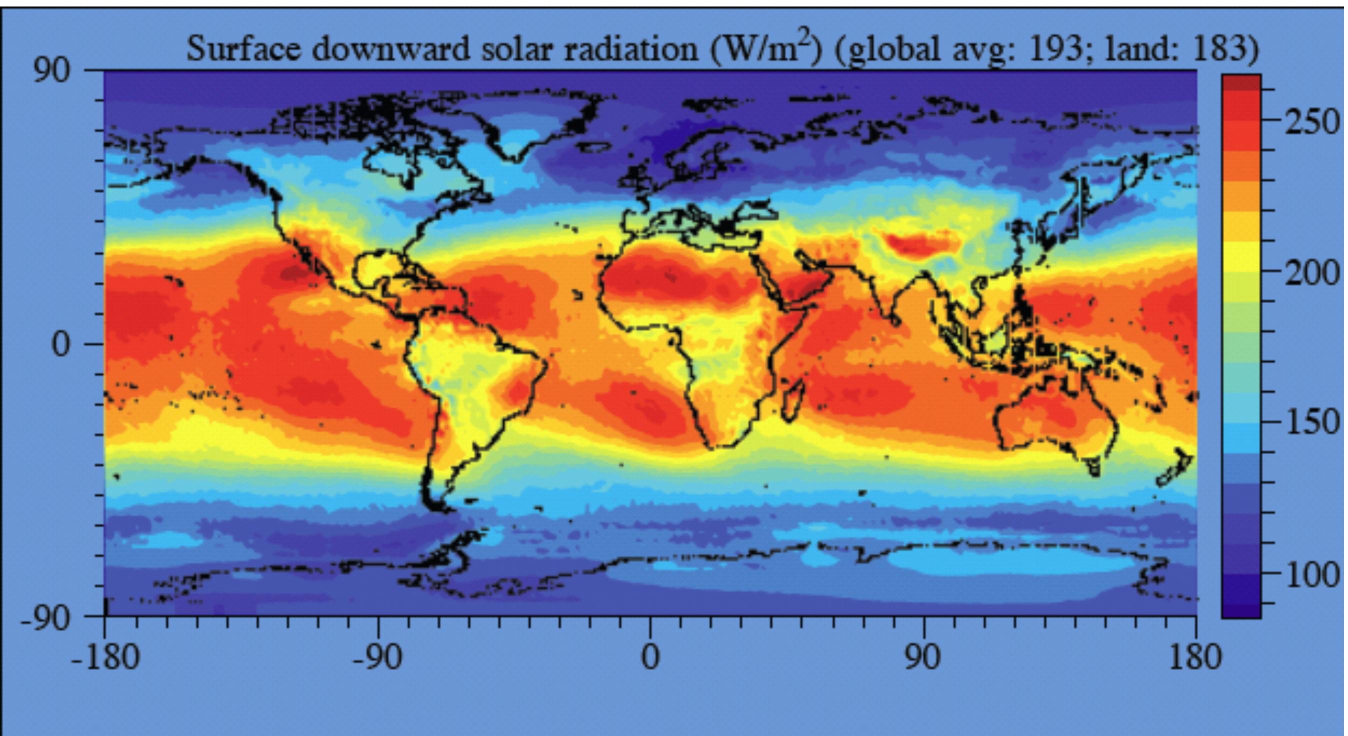
Outline

- Motivation
- Demand response model
- Some results



Wind power over land (outside Antartica): 70 – 170 TW

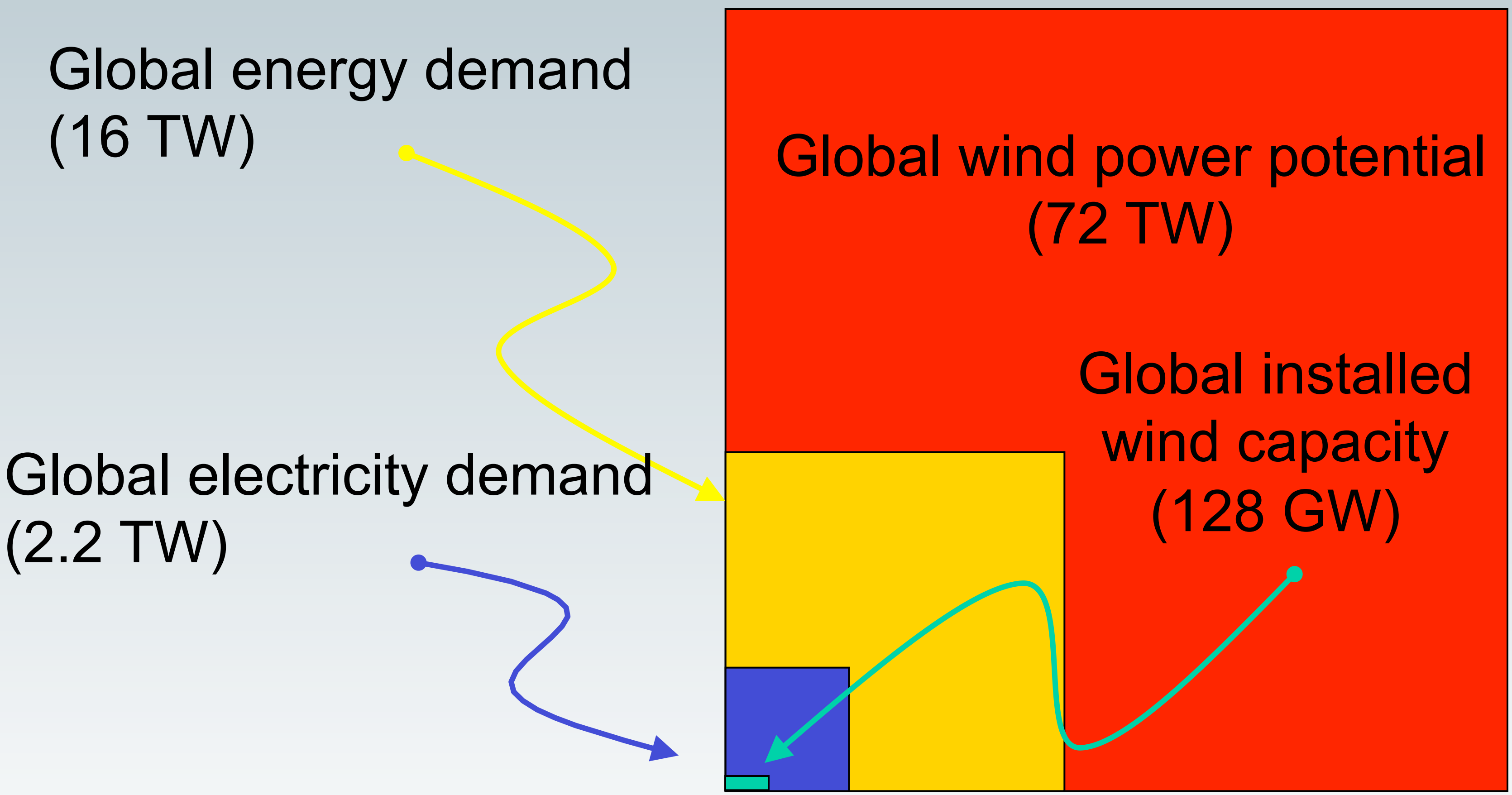
World power demand: 16 TW



Solar power over land: 340 TW

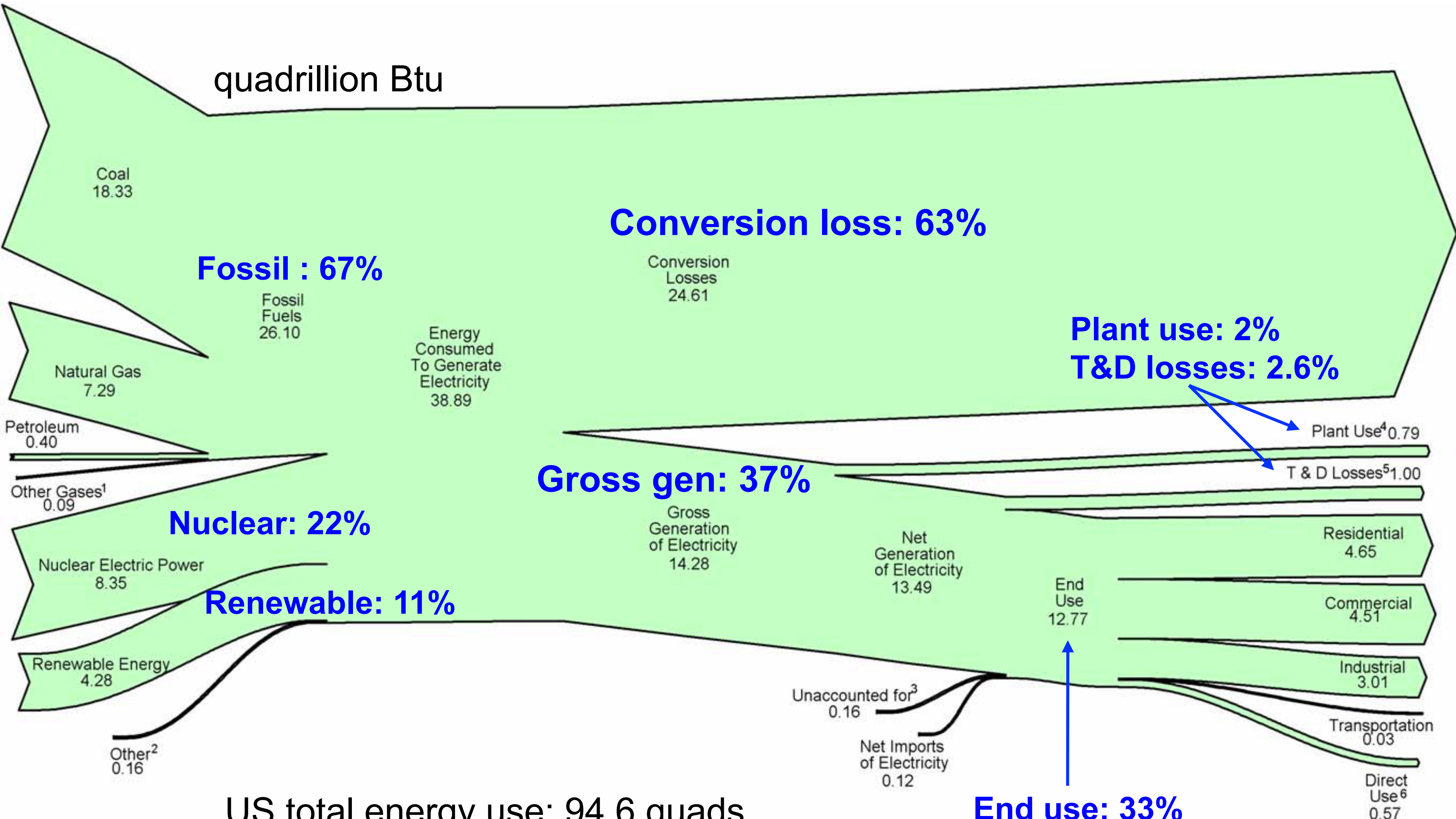
Source: M. Jacobson, 2011

Why renewable integration?



Source: Cristina Archer, 2010

US electricity flow 2009



US total energy use: 94.6 quads
For electricity gen: 41%

Source: EIA Annual Energy Review 2009

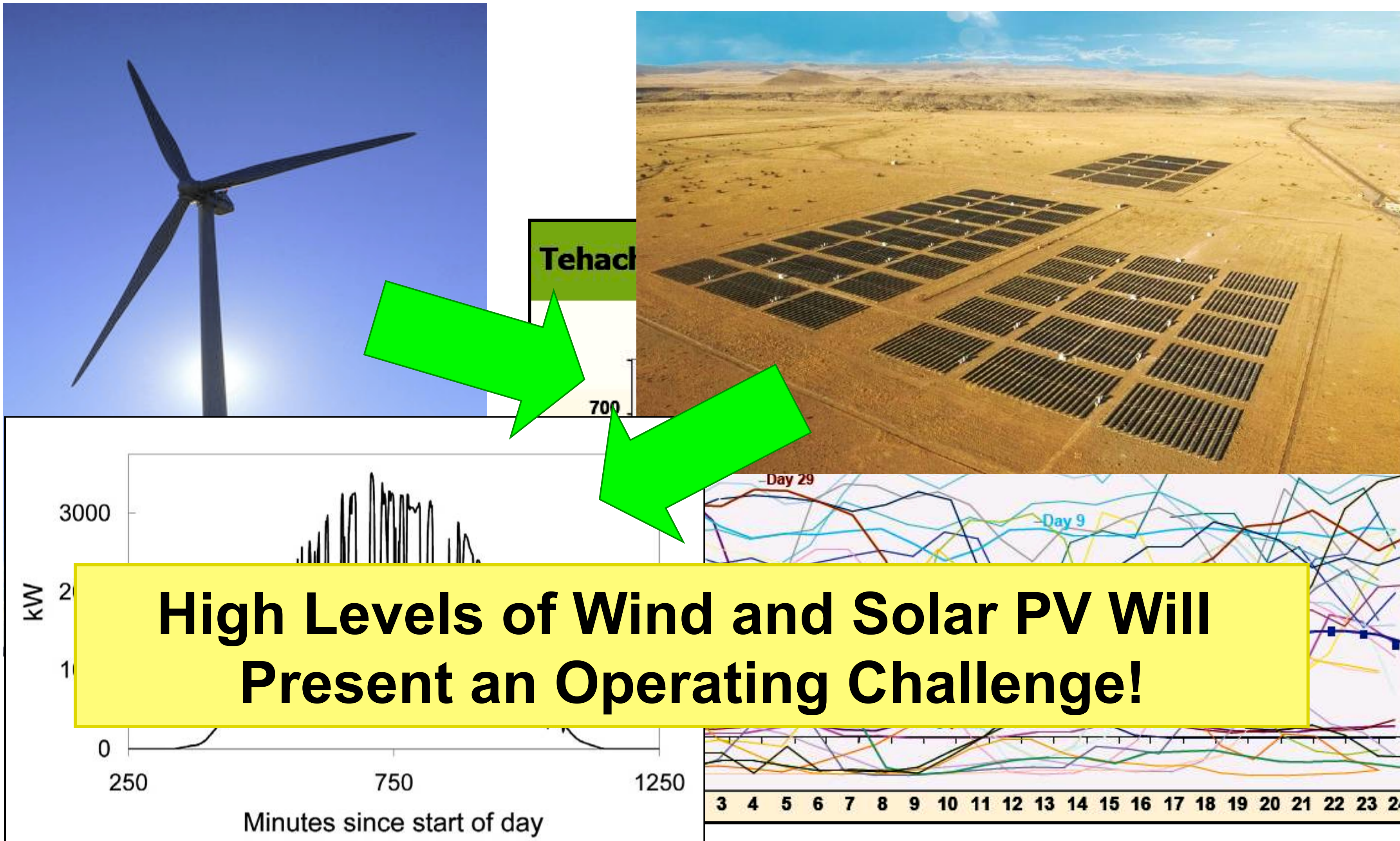
Renewables are exploding

- Renewables in 2009
  - 26% of global electricity capacity
  - 18% of global electricity generation
  - Developing countries have >50% of world's renewable capacity
  - In both US & Europe, more than 50% of added capacity is renewable

- Grid-connected PV has been doubling/yr for the past decade, 100x since 2000
- Wind capacity has grown by 27%/yr from 2004 – 09

Source: Renewable Energy Global Status Report, Sept 2010

Key challenges: uncertainty



Source: Rosa Yang

Today’s demand response

- Load side management has been practiced for a long time
  - E.g., direct load control
- They are simple and small
  - Centralized and static
  - Invoked rarely
    - Hottest days in summer
  - Small number of participants
    - Usually industrial, commercial
- Because
  - Simple system is sufficient
  - Lack of sensing, control, & 2-way communication infrastructure

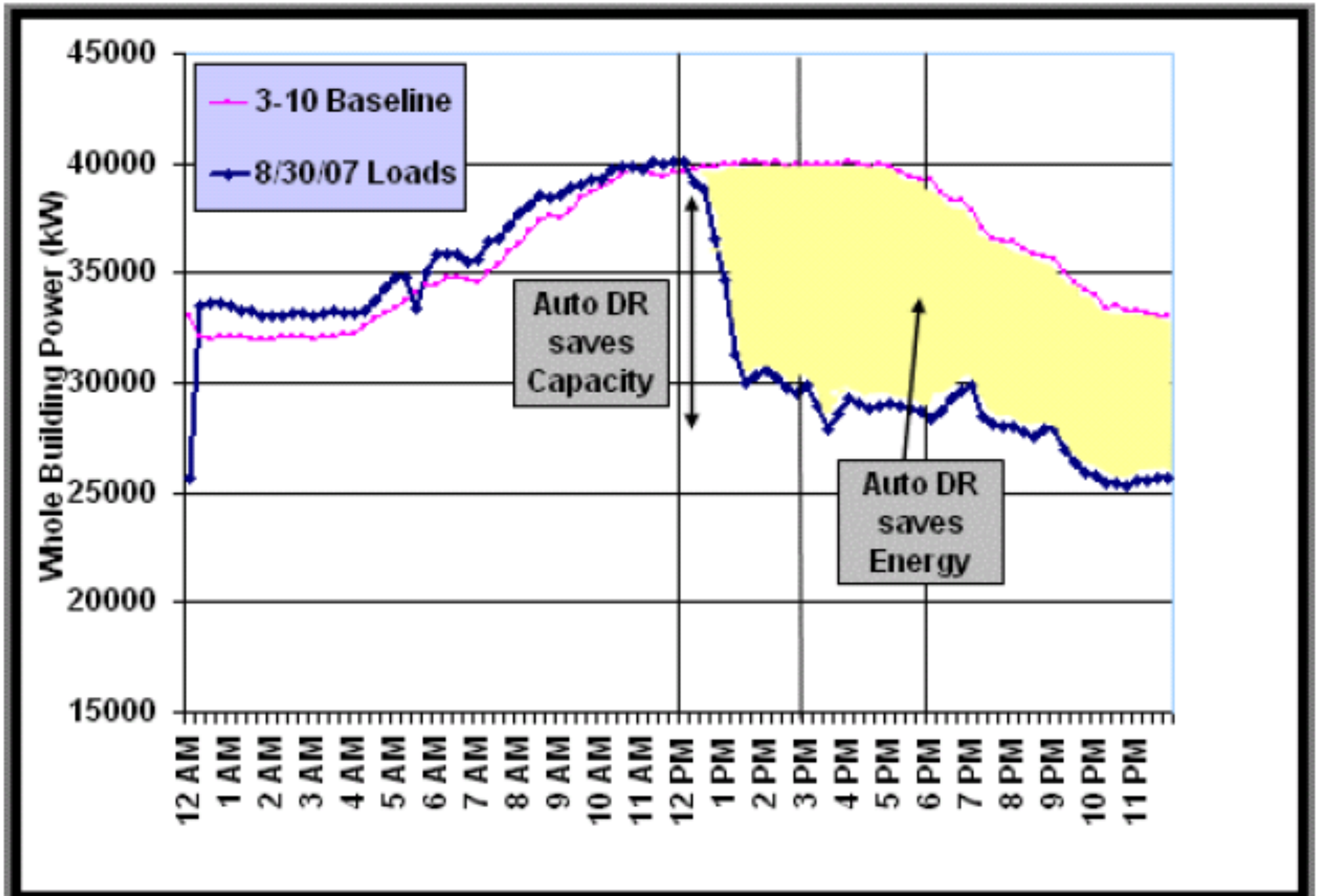
Tomorrow’s demand response

- Benefits
  - Adapt elastic demand to uncertain supply
  - Reduce peak, shift load
- Much more scalable
  - Distributed
  - Real-time dynamic
  - Larger user participation

It’d be cheaper to use photons than electrons
to deal with power shortages!

Automated Demand Response Saves Capacity and Energy

Electric load profile for PG&E participants on 8/30/2007



Source: Steven Chu, GridWeek 2009

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Features to capture

Wholesale markets

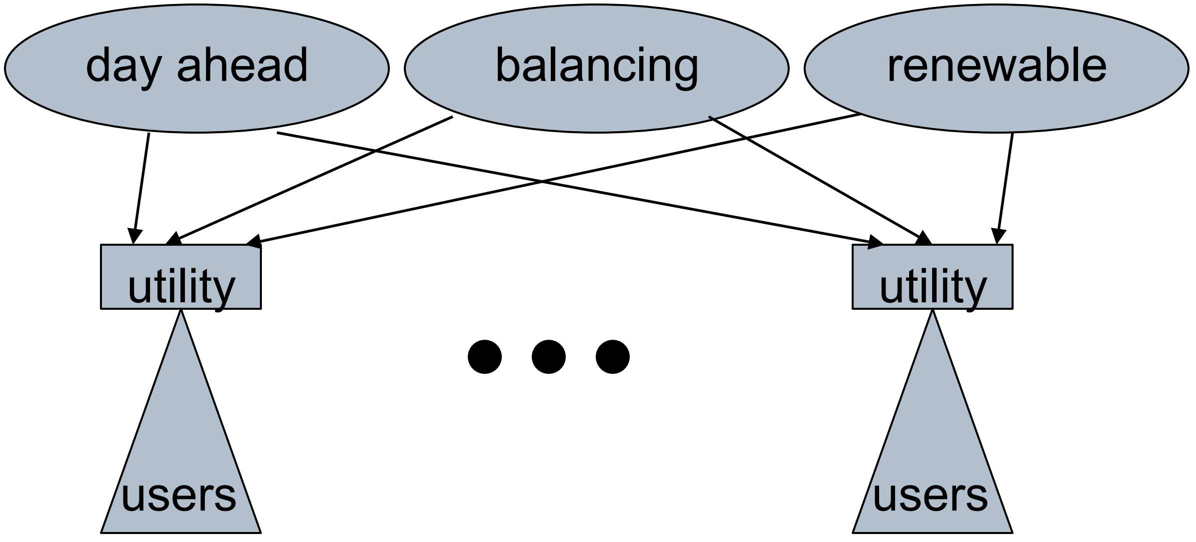
- Day ahead, real-time balancing

Renewable generation

- Non-dispatchable

Demand response

- Real-time control (through pricing)



Model: user

Each user has 1 appliance (wlog)

- Operates appliance with probability $\pi_i(t)$
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

$$\underline{x}_i(t) \le x_i(t) \le \overline{x}_i(t) \qquad \sum_t x_i(t) \ge \overline{X}_i$$

Demand at t:

$$D(t) := \sum_i \delta_i x_i(t) \qquad \delta_i = \begin{cases} 1 & \text{wp } \pi_i(t) \\ 0 & \text{wp } 1 - \pi_i(t) \end{cases}$$

Model: LSE (load serving entity)

Power procurement

capacity

energy

- Renewable power: $P_r(t),\ c_r\big(P_r(t)\big) = 0$
  - Random variable, realized in real-time
- Day-ahead power: $P_d(t),\ c_d\big(P_d(t)\big),\ c_o\big(\Delta x(t)\big)$
  - Control, decided a day ahead
- Real-time balancing power: $P_b(t),\ c_b\big(P_b(t)\big)$
  - $P_b(t) = D(t) - P_r(t) - P_d(t)$

- Use as much renewable as possible
- Optimally provision day-ahead power
- Buy sufficient real-time power to balance demand

Key assumption

Simplifying assumption

- No network constraints

Questions

Day-ahead decision

- How much power P_d should LSE buy from day-ahead market?

Real-time decision (at t-)

- How much $x_i(t)$ should users consume, given realization of wind power $P_r(t)$ and δ_i ?

How to compute these decisions distributively?

How does closed-loop system behave ?

available info: $u_i(\cdot), \pi_i, F_r(\cdot)$ $\quad$ $\delta_i, P_r(t), P_d^*$

decision: P_d^* $\quad$ $x_i^*(t)$

Our approach

Real-time (at *t*-)

- Given P_d and realizations of $P_r(t), \delta_i$ choose optimal $x_i^*(t) = x_i^*\left(P_d; P_r(t), \delta_i\right)$ to max social welfare, through DR

Day-ahead

- Choose optimal P_d^* that maximizes expected optimal social welfare

$t - 24hrs$ t-

available info: $u_i(\cdot), \pi_i, F_r(\cdot)$ $\delta_i, P_r(t), P_d^*$

decision: P_d^* $x_i^*(t)$

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  - T=1 case: distributed alg
  - General T: distributed alg
  - Impact of uncertainty

T=1 case

Each user has 1 appliance (wlog)

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Demand at t:

$$D(t) := \sum_i \delta_i x_i(t) \qquad \delta_i = \begin{cases} 1 & \text{wp} \ \ \pi_i(t) \\ 0 & \text{wp} \ \ 1 - \pi_i(t) \end{cases}$$

Welfare function

Supply cost

$$c\left(P_d, x\right) = c_d\left(P_d\right) + c_o\left(\Delta(x)\right)_0^{P_d} + c_b\left(\Delta(x) - P_d\right)_+$$

$$\Delta(x) := \sum_i \delta_i x_i - P_r$$

excess demand

Welfare function

Supply cost

$$c\left(P_d, x\right) = c_d\left(P_d\right) + c_o\left(\Delta(x)\right)_0^{P_d} + c_b\left(\Delta(x) - P_d\right)_+$$

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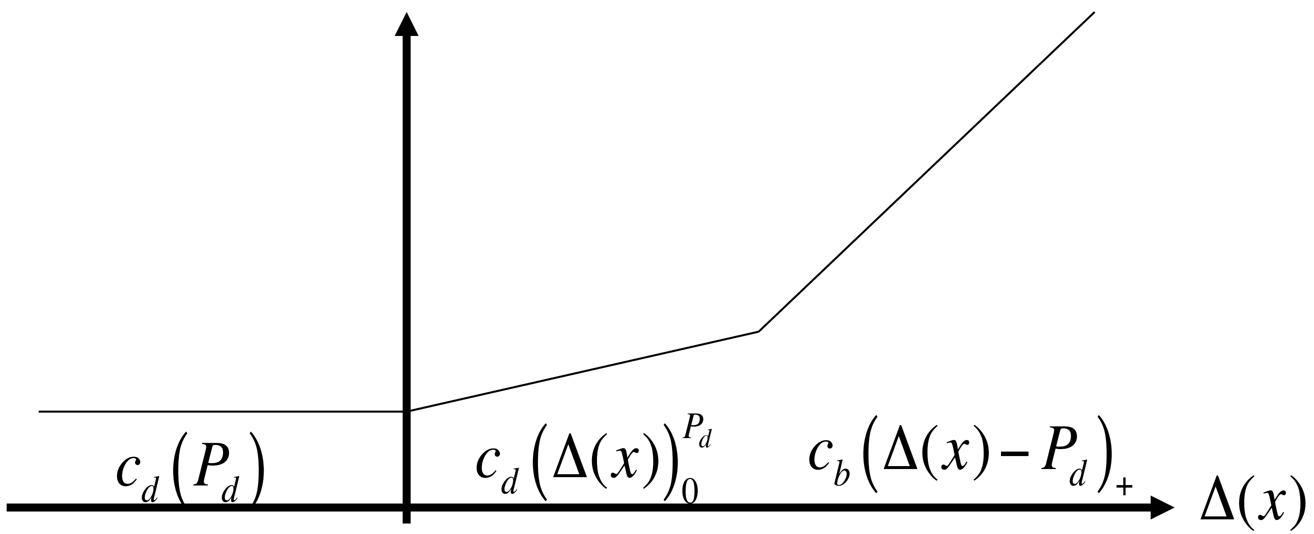
excess demand



Welfare function

Supply cost

$$c\left(P_d, x\right) = c_d\left(P_d\right) + c_o\left(\Delta(x)\right)_0^{P_d} + c_b\left(\Delta(x) - P_d\right)_+$$

$$\Delta(x) := \sum_i \delta_i x_i - P_r$$

excess demand

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

user utility supply cost

Optimal operation

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

Optimal real-time demand response

$$\max_x W\left(P_d, x\right)$$

given realization of P_r, δ_i

Optimal operation

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

Optimal real-time demand response

$$x^*\left(P_d\right) \quad := \quad \arg \quad \max_x W\left(P_d, x\right)$$

given realization of P_r, δ_i

Optimal operation

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

Optimal real-time demand response

$$x^*\left(P_d\right) \ := \ \arg \ \max_x W\left(P_d, x\right)$$

given realization of P_r, δ_i

Optimal day-ahead procurement

$$P_d^* \ := \ \arg \ \max_{P_d} \ \mathrm{E}W\left(P_d, x^*\left(P_d\right)\right)$$

Overall problem: $\max_{P_d} \ \mathrm{E} \ \max_x W\left(P_d, x\right)$

Algorithm 1 (real-time DR)

$$\max_{P_d} \ \mathrm{E} \ \underbrace{\max_{x} W\left(P_d, x\right)}_{\text{real-time DR}}$$

Active user i computes x_i^*

- Optimal consumption

LSE computes

- Real-time price μ_b^*
- Optimal day-ahead power to use y_o^*
- Optimal real-time balancing power y_b^*

Algorithm 1 (real-time DR)

Active user i :

$$x_i^{k+1} = \left(x_i^k + \gamma \left(u_i{}' \left(x_i^k \right) - \mu_b^k \right) \right)_{\underline{x}_i}^{\overline{x}_i}$$

inc if marginal utility > real-time price

LSE :

$$\mu_b^{k+1} = \left(\mu_b^k + \gamma \left(\Delta \left(x^k \right) - y_o^k - y_b^k \right) \right)_+$$

inc if total demand > total supply

- Decentralized
- Iterative computation at t-

Algorithm 1 (real-time DR)

Theorem: Algorithm 1

Socially optimal

- Converges to welfare-maximizing DR $x^* = x^*\left(P_d\right)$
- Real-time price aligns marginal cost of real-time power with individual marginal utility

$$\mu_b^* = c_b'\left(y_b^*\right) = u_i'\left(x_i^*\right)$$

Incentive compatible

- x_i^* max i's surplus given price μ_b^*

Algorithm 1 (real-time DR)

Theorem: Algorithm 1

Marginal costs, optimal day-ahead and balancing power consumed:

$$c_b'\left(y_b^*\right) = c_o'\left(y_o^*\right) + \mu_o^*$$

$$\mu_o^* = \frac{\partial W}{\partial P_d}\left(P_d^*\right)$$

Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} \ \mathrm{E}W\left(P_d, x^*\left(P_d\right)\right)$$

LSE: $$P_d^{m+1} = \left(P_d^m + \gamma^m\left(\mu_o^m - c_d{}'\left(P_d^m\right)\right)\right)_+$$

calculated from Monte Carlo
simulation of Alg 1
(stochastic approximation)

Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} \ \mathrm{E}W\left(P_d, x^*\left(P_d\right)\right)$$

LSE: $$P_d^{m+1} = \left(P_d^m + \gamma^m\left(\mu_o^m - c_d{}'\left(P_d^m\right)\right)\right)_+$$

Given δ^m, P_r^m : $$\mu_o^m = \frac{\partial W}{\partial P_d}\left(P_d^m\right)$$

$$\mu_b^m = \mu_o^m + c_o^{'}\left(y_o^m\right)$$

Algorithm 2 (day-ahead procurement)

<u>Theorem</u>

Algorithm 2 converges a.s. to optimal P_d^* for appropriate stepsize γ^k

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General T case

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Coupling across time
➔ Need state

Demand at t:

$$D(t) := \sum_i \delta_i x_i(t) \qquad \delta_i = \begin{cases} 1 & \text{wp } \pi_i(t) \\ 0 & \text{wp } 1 - \pi_i(t) \end{cases}$$

Time correlation

- Example: EV charging
  - Time-correlating constraint: $\sum_{t=1}^{T} x_i(t) \geq R_i, \forall i$
- Day-ahead decision and real-time decisions

	Day-ahead	*t-*	$t = 1,2,\ldots, T$
available info:	$u_i(\cdot), F_r(\cdot)$	$P_r(t), P_d^*(t), R_i(t)$	
decision:	$P_d^*(t), \forall t$	$x_i^*(t), \forall i$	

Remaining demand

- (1+T)-period dynamic programming

Algorithm 3 (T>1)

- Main idea
  - Solve deterministic problem in each step using conditional expectation of P_r (distributed)
  - Apply decision at current step
- One day ahead, decide P_d^* by solving

$$\max_{P_d, x} \sum_{\tau=t}^{T} W\left(P_d(\tau), x(\tau); \overline{P}_r(\tau)\right) \quad s.t. \sum_{\tau=1}^{T} x_i(\tau) \geq R_i$$

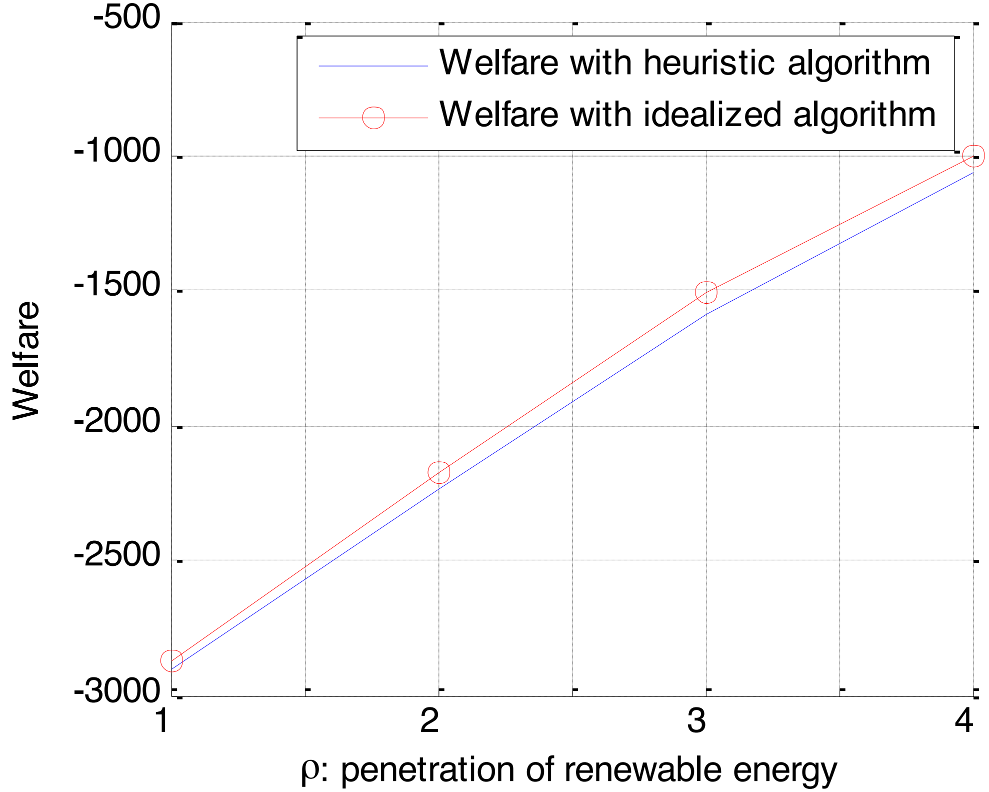
- At time *t*-, decide $x^*(t)$ by solving

$$\max_{x} \sum_{\tau=t}^{T} W\left(P_d^*(\tau), x(\tau); \overline{P}_r(\tau \mid t)\right) \quad s.t. \sum_{\tau=t}^{T} x_i(\tau) \geq R_i(t)$$

Algorithm 3 (T>1)

Theorem: performance

- Algorithm 3 is optimal in special cases
- $J^{A3} - J^{*} \le \frac{1}{2} T \overline{P}_r^2$



Effect of renewable on welfare

Renewable power:

$$P_r(a,b) := a \cdot \underbrace{\mu_r}_{\text{mean}} + b \cdot \underbrace{V_r}_{\text{zero-mean RV}}$$

Optimal welfare of (1+T)-period DP

$$W\left(P_r(a,b)\right)$$

Effect of renewable on welfare

$$P_r(a,b) := a \cdot \mu_r + b \cdot V_r$$

Theorem

- Cost increases in var of P_r
- $W\left(P_r(a,b)\right)$ increases in a, decreases in b
- $W\left(P_r(s,s)\right)$ increases in s