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# The 2011 Santa Barbara Control Workshop



## Resource Allocation in Contention-Based WiFi Networks



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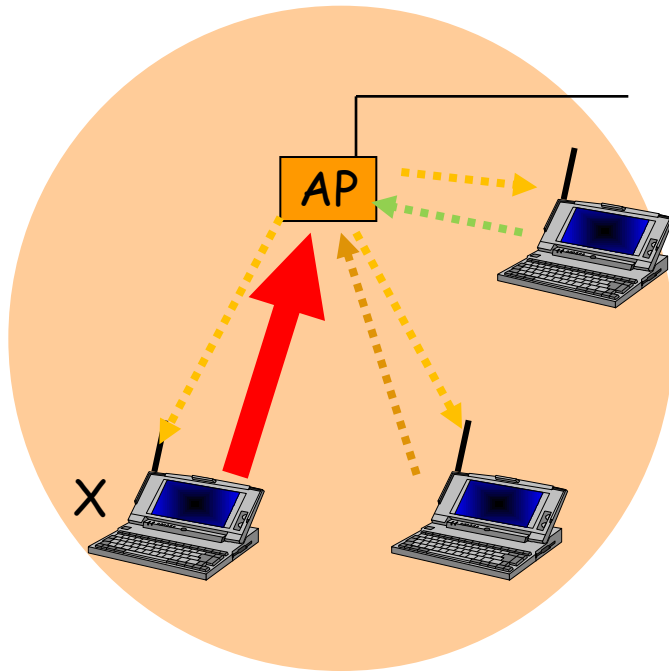
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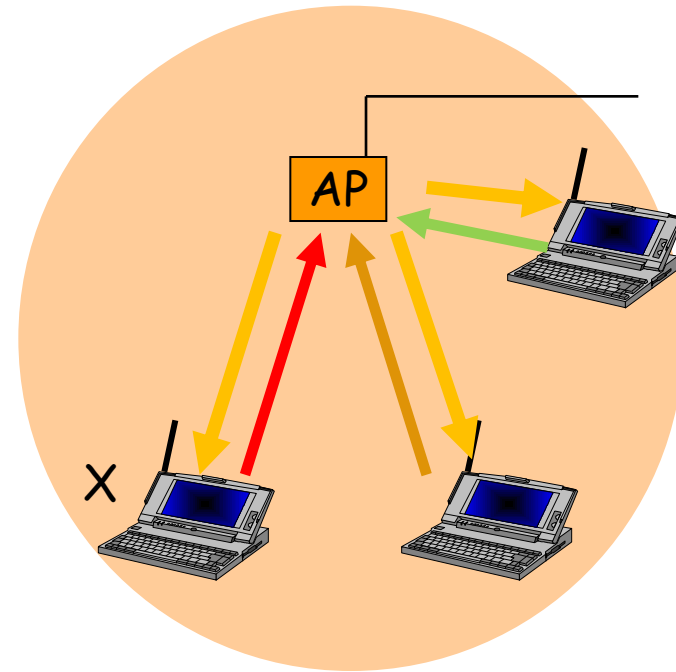
# Resource allocation in Contention-based networks

- Analysis with rational nodes:
  - Infrastructured IEEE 802.11 Networks
    - Achieving distributed fair bandwidth among nodes in non homogeneous bidirectional traffic to optimize throughput
    - Game theoretical analysis and design
- Schemes for multi-hop topologies
  - Wireless Ad-Hoc Networks
    - Grouping contending nodes (TDMA approach) in combination with Carrier Sense to access the channel (CSMA/CA)
    - Graph coloring solution to assign slots

# Infrastructured Networks



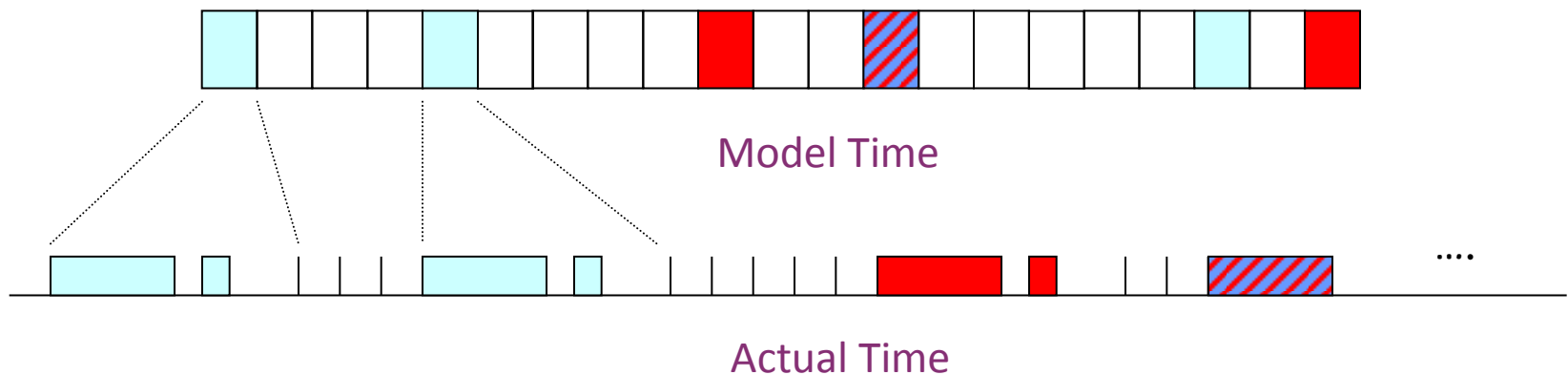
If station X tries to get all wireless resources  $\Rightarrow$  no space for the other stations, including the AP!



If station X leaves spaces to the AP  $\Rightarrow$  also the other stations able to transmit.  $k_i$  - desired up/down ratio for each station

# IEEE 802.11 DCF as a Slotted Access protocol

- **Distributed Coordination Function (DCF)** regulates access to the shared medium:
  - dynamic adaptation of the contention windows (short term unfairness)
  - use of **homogeneous** contention parameters among the contending nodes
- Protocol operations summarized in terms of **average access probability** in a slotted channel (with uneven or even slot size)
- In each system slot, each station accesses with probability  $\tau$  (and does not access with probability  $1-\tau$ ).
- Most protocols make  $\tau$  depending on the collision probability  $p$ ,  $\tau=f(p)$ , as a tradeoff between channel wastes due to collisions and idle slots.



# Game Theoretic Approach

- Thanks to open source drivers and programmable cards, we propose a **dynamic tuning** of the contention parameters used by the nodes via a **game-theoretic** approach.
- **AIM:** To guarantee a fair resource sharing among the nodes, while **optimizing** the per-node uploading and downloading bandwidth.
- **SOLUTION:**
  - Some DCF protocol EXTENSIONS able to cope with current resource sharing problems.
  - A non cooperative game where the contending stations act as the players
  - The stations works in saturated conditions and DCF can be modeled as a slotted access protocol while the station behavior is summarized in terms of per slot access probability

# Contention-based access as a non-cooperative game

-Contending stations = players

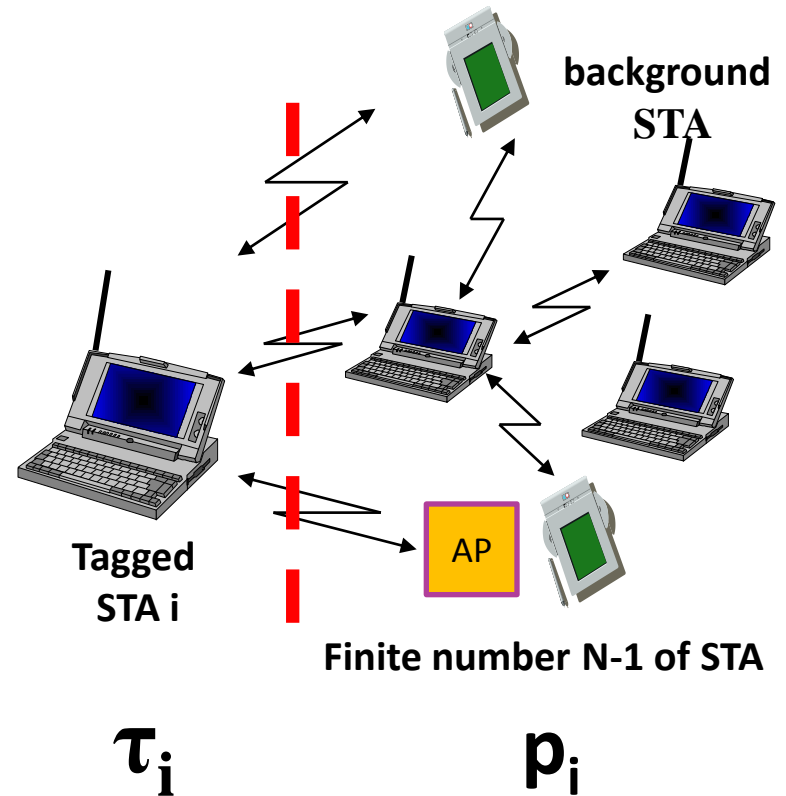
-Channel access probability  $\tau$  = player strategy

## Game definition:

N players,  $[0,1]^N$  set of strategies, node payoff  $(J_1, J_2, \dots, J_N)$

-Payoff perceived by each station depends on the whole set of probability  $(\tau_1, \tau_2, \dots, \tau_n)$  chosen by all the stations

$$(\tau_1, \tau_2, \dots, \tau_n) \rightarrow (\tau_i, p_i) \text{ with } p_i = 1 - \prod_{j \neq i} (1 - \tau_j)$$



# Node Payoff with Bidirectional Traffic

- Assumption: AP is a legacy station  $\tau_{AP}=f(p_{AP})$  equally sharing the downlink throughput among the stations.

- For the  $i$ -station:

- Uplink throughput: 
$$S^i_u(\tau_i, p_i) = \frac{\tau_i(1-p_i)(1-\tau_{AP})P}{E[slot]}$$
  $\uparrow \tau_i$

- Downlink throughput: 
$$S^i_d(\tau_i, p_i) = x_i \frac{\tau_{AP}(1-p_{AP})P}{E[slot]}$$
  $\downarrow \tau_i$

- The utility function with  $k_i$  in  $(0, \infty)$  
$$J^i(\tau_i, p_i) = \min(S^i_u, k_i S^i_d)$$

# Main Results

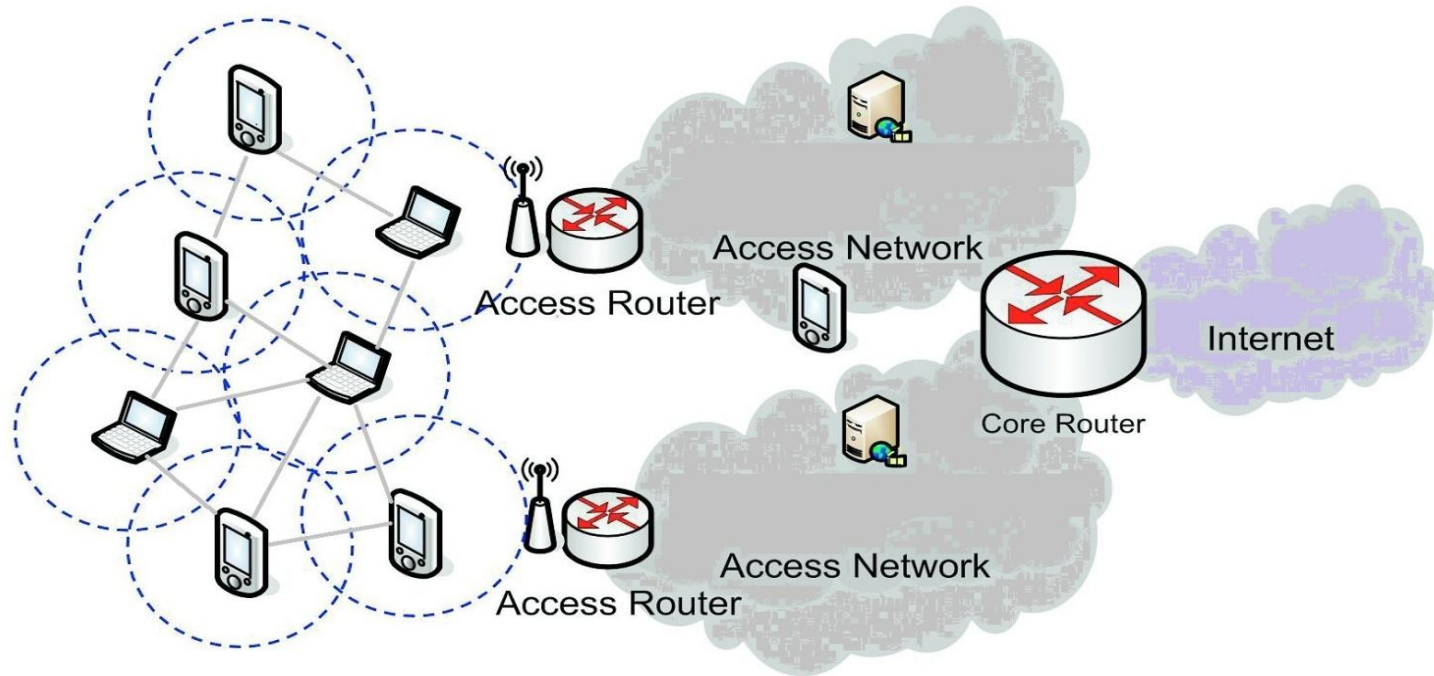
- Determination of Nash Equilibria and Pareto Optimality
- Mechanism design -> using of the AP to force desired equilibria
- Implementation of new DCF operations with best response strategy
- Implementation of Channel Monitoring functionalities (estimation of number of nodes and load conditions)
- Analysis of NE convergence and stability



# References

- L. Giarré, G. Neglia, I. Tinnirello. “Medium Access in WiFi Networks: Strategies of Selfish Nodes”, **IEEE Signal Processing Magazine (SPM)**, Vol. 26, Issue 5, 2009, pp.124-128
- I.Tinnirello, L. Giarré, G. Neglia. “ MAC design for WiFi infrastructure networks: a game theoretic approach. **IEEE Transaction on Wireless Communication**, in print, 2011.
- I. Tinnirello, L. Giarré, G. Neglia, “The role of the Access Point in Wi-Fi networks with selfish nodes”, **IEEE GAMENETS**, 2009.
- L. Giarré, G. Neglia, I. Tinnirello, “Performance Analysis of Selfish Access Strategies on WiFi Infrastructure Networks”, **IEEE Globecom** 2009
- L. Giarré, G. Neglia, I. Tinnirello, “Resource Sharing Optimality in WiFi infrastructure networks”, **IEEE CDC**, 2009
- I. Tinnirello, L. Giarré, G. Neglia. A Game Theoretic Approach to MAC Design for Infrastructure Networks, **IEEE CDC**, 2010.

# Ad-hoc networks

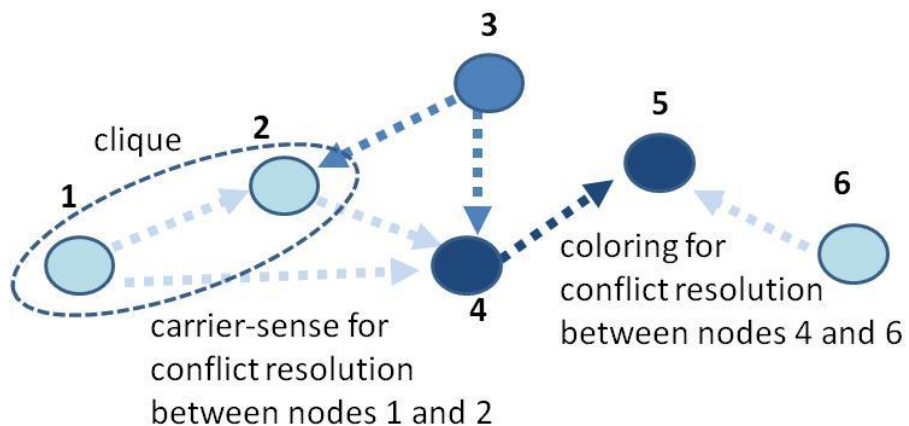


- Suitable for a large number of applications:
  - from **low-range** sensor networks targeted to distributed monitoring
  - to **high-range** mesh networks targeted to build infrastructure-less transport networks.

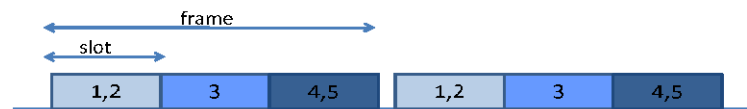
# Ad-hoc Networks

- Most ad-hoc networks rely on contention-based medium access protocols,
- regardless to the specific physical layer technology ( IEEE 802.15.4 PHY or 802.11a/b/g/n PHY, defining available bandwidth, transmission power, modulation coding scheme..)
- The use of carrier sense and random backoff mechanisms is a simple and well-established solution to manage multiple access over a shared channel bandwidth.
- CSMA/CA protocols exhibit very poor performance for **multi-hop** transmissions (inter-link interference due to imperfect carrier sensing).

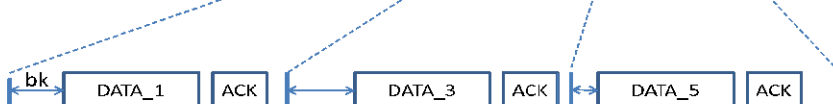
# Ad hoc Networks



## A) Pre-allocations



## B) Final scheduling



# Ad-Hoc networks

- **Aim:** Distributed resource allocation problems for multi-hop wireless networks.
- **Main idea:** Combining the TDMA approach for grouping the contending nodes in non-interfering sets) with the CSMA/CA approach (for managing the final access to the shared channel).
- **Solution:** determining the best number of slots in a frame and the best assignment of slots to different in terms of a map coloring problem, by trying to identify the most effective trade-offs between complexity, signaling overheads and performance gain.

# Main Results

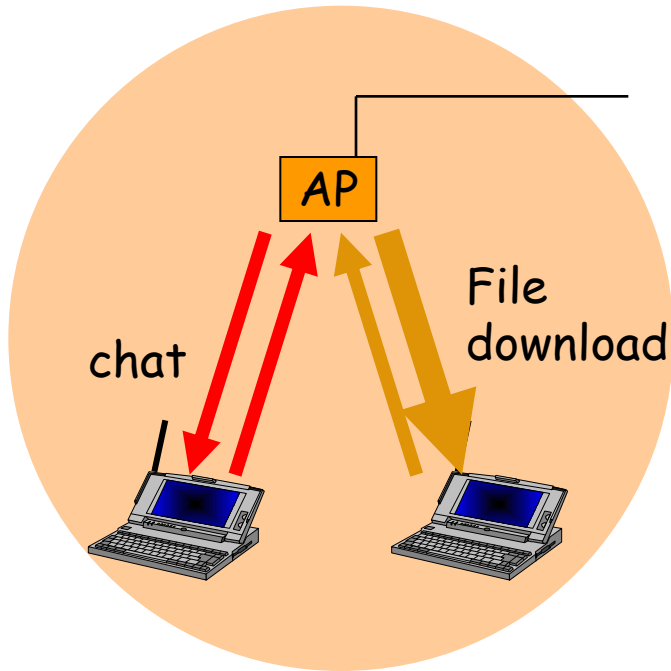
- **Problem:** Determine a distributed protocol setting the number  $x$  of slots in a frame and the slots allocations, in order to maximize **the per-node throughput** in saturation conditions
- Network transport capacity is critically affected by the number of slots  $x$ !
- **Incompatibility constraints:**
  - all neighbors and hidden nodes on different colors
  - only hidden nodes on different colors

# References

## (Ad hoc networks)

- I. Tinnirello, L.Giarrè and R. Pesenti. Decentralized Synchronization for Zigbee Sensor Networks in Multi-hop topology. **Necsys 2010**.
- L.Giarrè, F.G. La Rosa, R. Pesenti and I. Tinnirello. Coloring-based Resource Allocations in Ad-hoc Wireless Networks. **Medhoc 2011**.
- L.Giarrè, F.G. La Rosa, R. Pesenti and I. Tinnirello. Distributed Resource Allocation in Ad-hoc Wireless Networks. **CDC 2011**.

# Infrastructure Networks with heterogeneous applications



It might happen that  $k_1 \neq k_2$

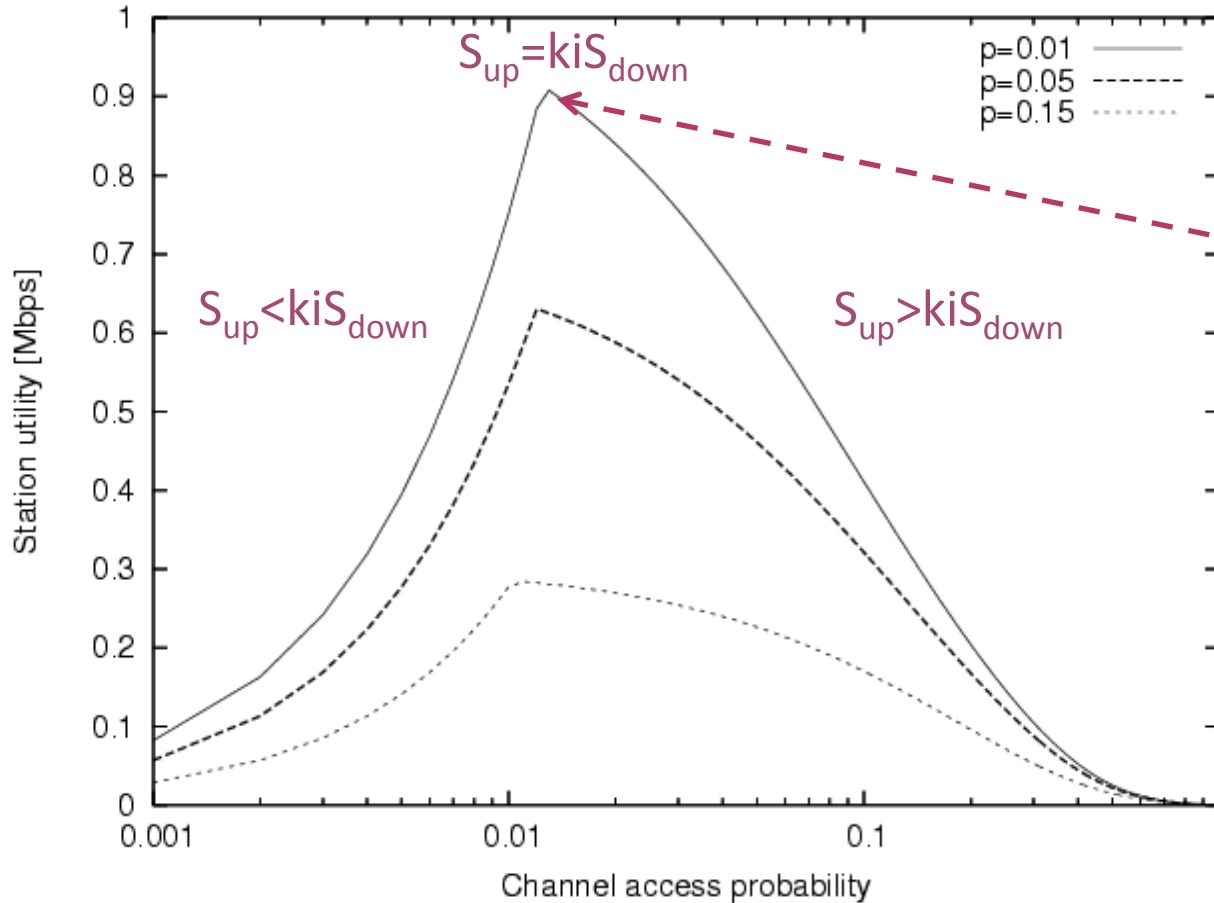
$$J_i(\tau_i, p) = \min(S_u^i, k_i S_d^i)$$

1. Does a best response policy lead to a NE?

2. How should the AP share the downlink throughput?  
(choice of  $\mathbf{x}_i$ )



# Node Best Response



$k_i = k = 1, x_i = 1/n$   
 $(\tau, p)$  outcome

$$\tau_{br} = \frac{k_i x_i \tau_{AP}}{1 - (1 - k_i x_i) \tau_{AP}}$$

where:

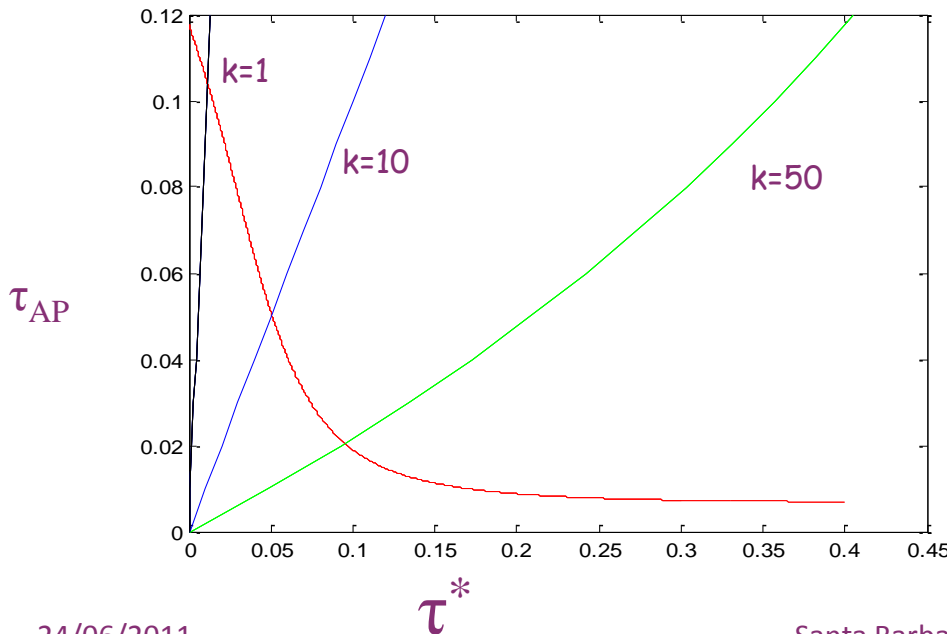
$\tau_{AP} = f(p_{AP})$  is function of the strategy set  $(\tau, p)$

# Nash Equilibrium ( $k_i=k=cost, x_i=1/n$ )

**Proposition:** The homogeneous strategy vector  $(\tau^*, \tau^*, \dots, \tau^*)$  such that

$$\tau^* = \frac{kf(1 - (1 - \tau^*)^n)}{n - (n - k)f(1 - (1 - \tau^*)^n)}$$

is the only Nash equilibrium in  $[0,1]^n$  of the game with non-null utility.



Proof sketch:

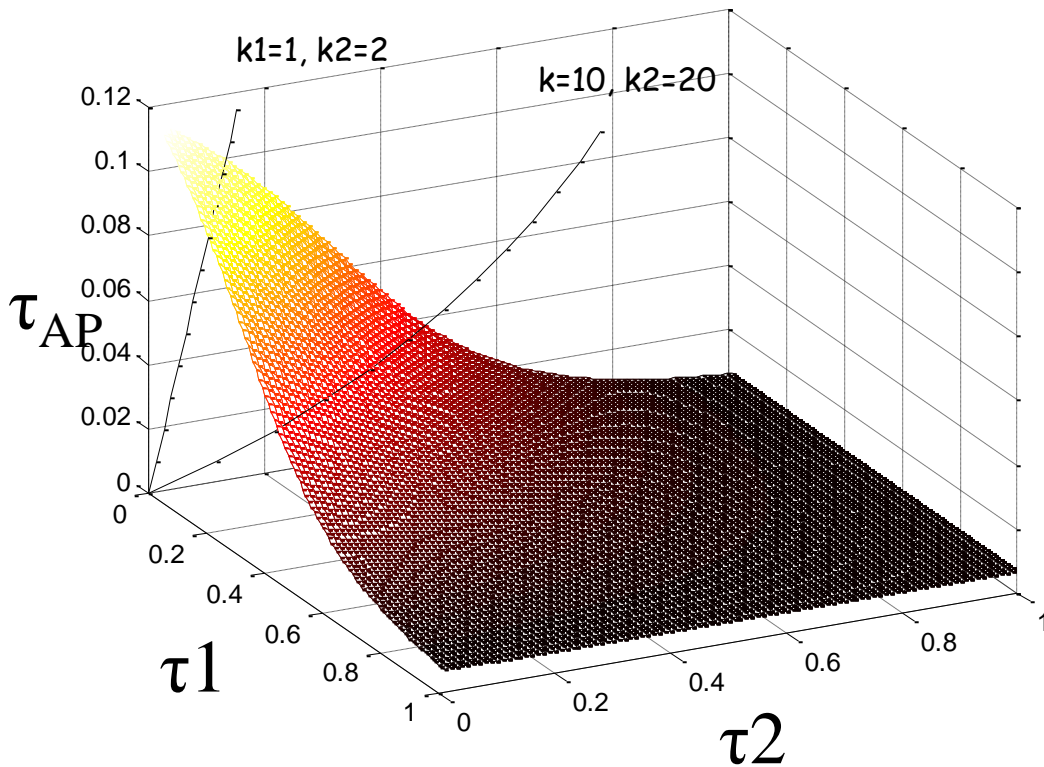
At the NE point, two conditions simultaneously hold:

$$\begin{cases} \tau^* = \frac{k\tau_{AP}}{n - (n - k)\tau_{AP}} = g(\tau_{AP}) \\ \tau_{AP} = f(1 - (1 - \tau^*)^n) \end{cases}$$

being  $f()$  decreasing in  $\tau^*$  starting from 0, and  $g()$  increasing in  $\tau_{AP}$ , a single intersection exists

# Nash Equilibrium ( $k_i, x_i=1/n$ )

**Proposition:** For a given vector  $\mathbf{k}=(k_1, k_2, ..k_n)$  of application requirements, by equally sharing the downlink throughput, it exists a unique NE with non-null utility.



## Proof sketch:

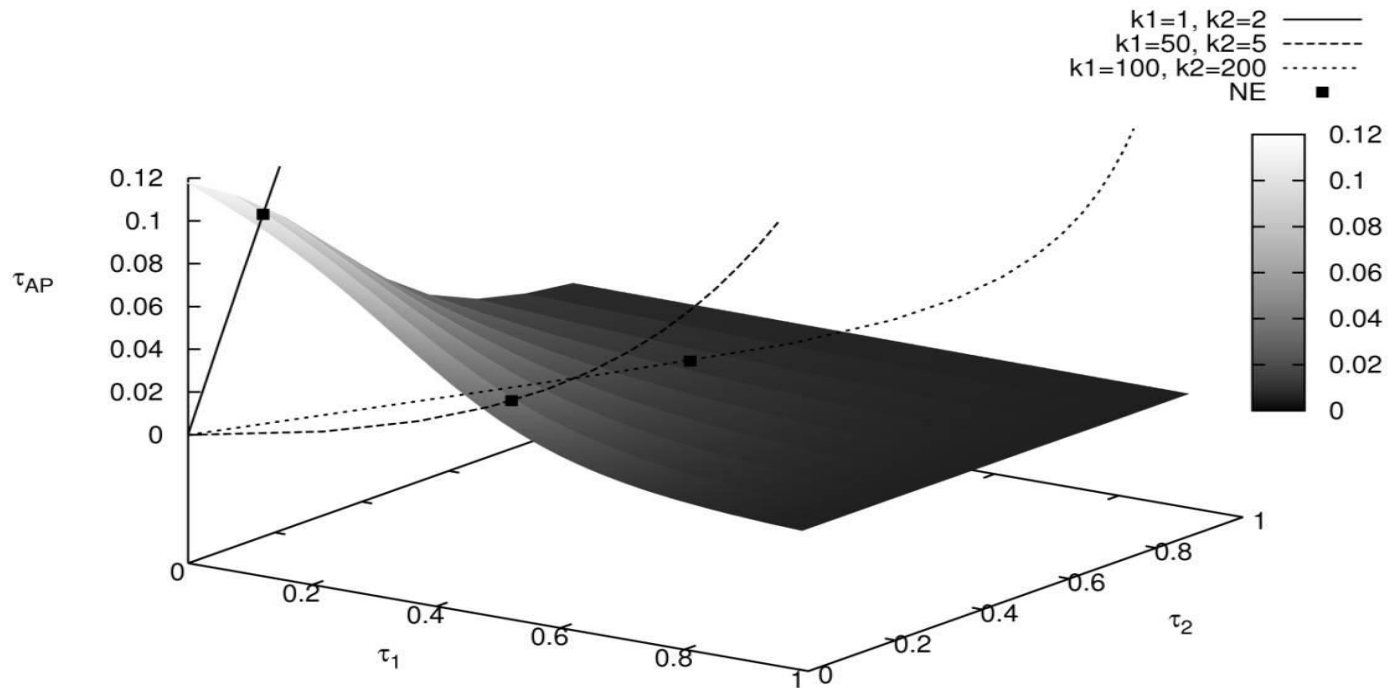
At the NE point,  $N+1$  conditions simultaneously hold:

$$\left\{ \begin{array}{l} \tau_1 = \frac{k_1 \tau_{AP}}{n - (n - k_1) \tau_{AP}} \\ \tau_2 = \frac{k_2 \tau_{AP}}{n - (n - k_2) \tau_{AP}} \\ \dots \\ \tau_n = \frac{k_n \tau_{AP}}{n - (n - k_n) \tau_{AP}} \\ \tau_{AP} = f(1 - \prod(1 - \tau_i)) \end{array} \right.$$

The first  $N$  conditions represent a 1-dim curve in a  $N+1$  space; the last one a surface..

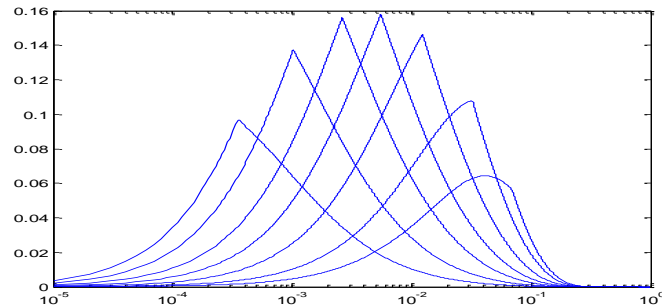
# Nash Equilibrium ( $k_i, x_i$ )

**Proposition:** For a given vector  $\mathbf{k}=(k_1, k_2, ..k_n)$  of application requirements, and a given vector of downlink throughput coefficients  $(x_1, x_2, .., x_n)$ , it exists a unique NE with non-null utility.



# Mechanism design

- Can the AP play the role of arbitrator in order to improve the performance of its access network?
  1. Using  $\tau_{AP}$  as a configuration parameter (rather than  $f(p_{AP})$ )

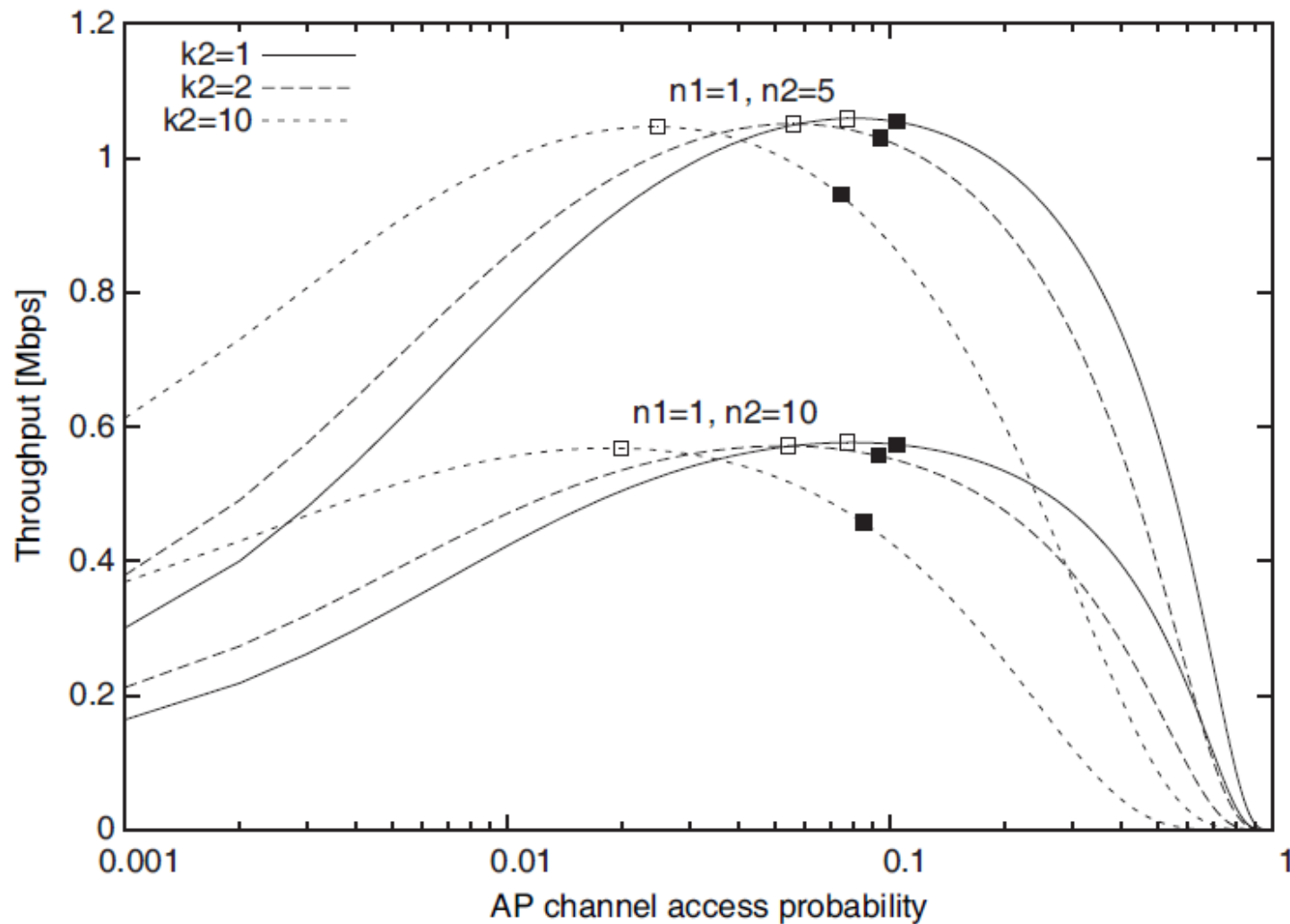


2. Employing a downlink scheduling according to the application requirement  $\mathbf{k}=(k_1,k_2,k_2,\dots,k_n)$

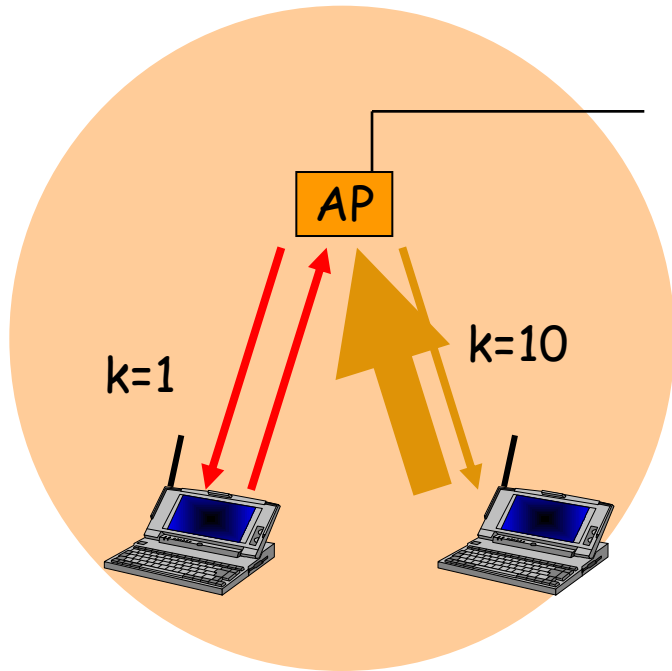
# Tuning the AP channel access probability

- The best response is 
$$\tau_i^+ = \frac{k_i x_i \cdot c}{1 - (1 - k_i x_i)c}$$
- The NE becomes the intersection between an hyperplane  $T_{AP}=c$  and the parametric curve identified by the best response equations

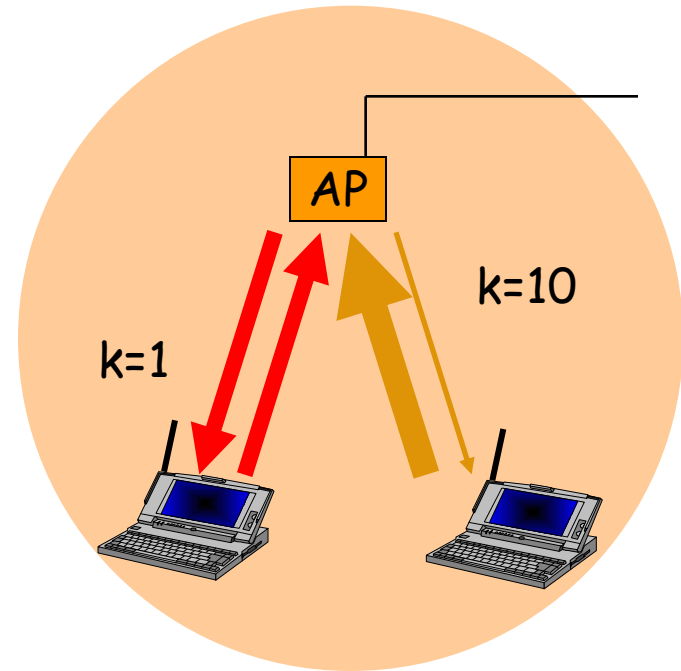
# Per-station total bandwidth



# Downlink Scheduling



By equally sharing the downlink, stations with higher  $k$  get an higher total up-down capacity



A fairer criterion could be an equal repartition of the per-station up+down capacity!

$S_d^i = x_i S_{AP}$  with 
$$x_i = \frac{1}{\sum_{j=1}^n \frac{1}{k_j+1}}$$

The unique NE still exists



# Scheduling policies

- AA: Application Agnostic

The AP is **not aware** of the per station application requirements ( $K_i$ ). AP equally shares the downlink throughput among the stations ( $S_d = S_{AP}/n$ ).

– At NE each station perceives a throughput of  $(1+k_i)S_d$

- AW: Application aWare

The AP is **aware** of  $K_i$ . AP can allocate and heterogeneous downlink throughput  $S_d^i = x_i S_{AP}$

with

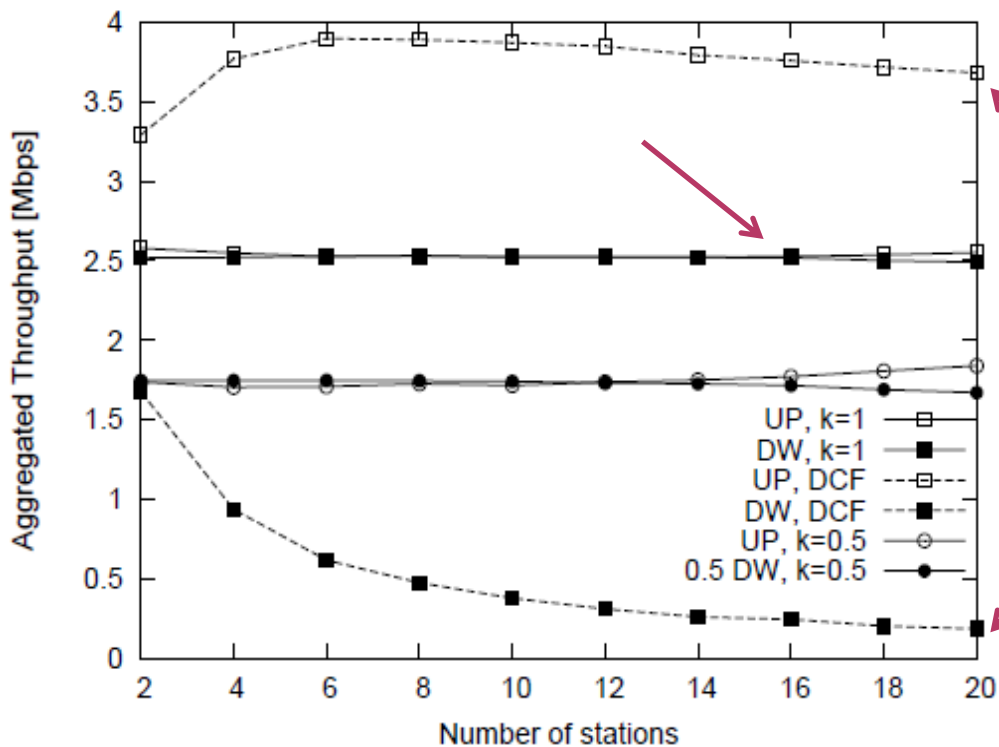
$$x_i = \frac{\frac{1}{k_i+1}}{\sum_{j=1}^n \frac{1}{k_j+1}}$$

# Game-based MAC Scheme implementation and evaluation

- Each station has an two estimators for probing uplink and downlink load conditions
- The station best response depends not only on the application requirements ( $K_i$ ) but also on the uplink load ( $n$ ) and downlink load ( $T_{ap}$ )
- Cases
  - 1) AP as a legacy
  - 2) AP implementing the adaptive tuning mechanism of the channel access probability
- Algorithms: AA (Application Agnostic scheduling) and AW (Application aWare scheduling)

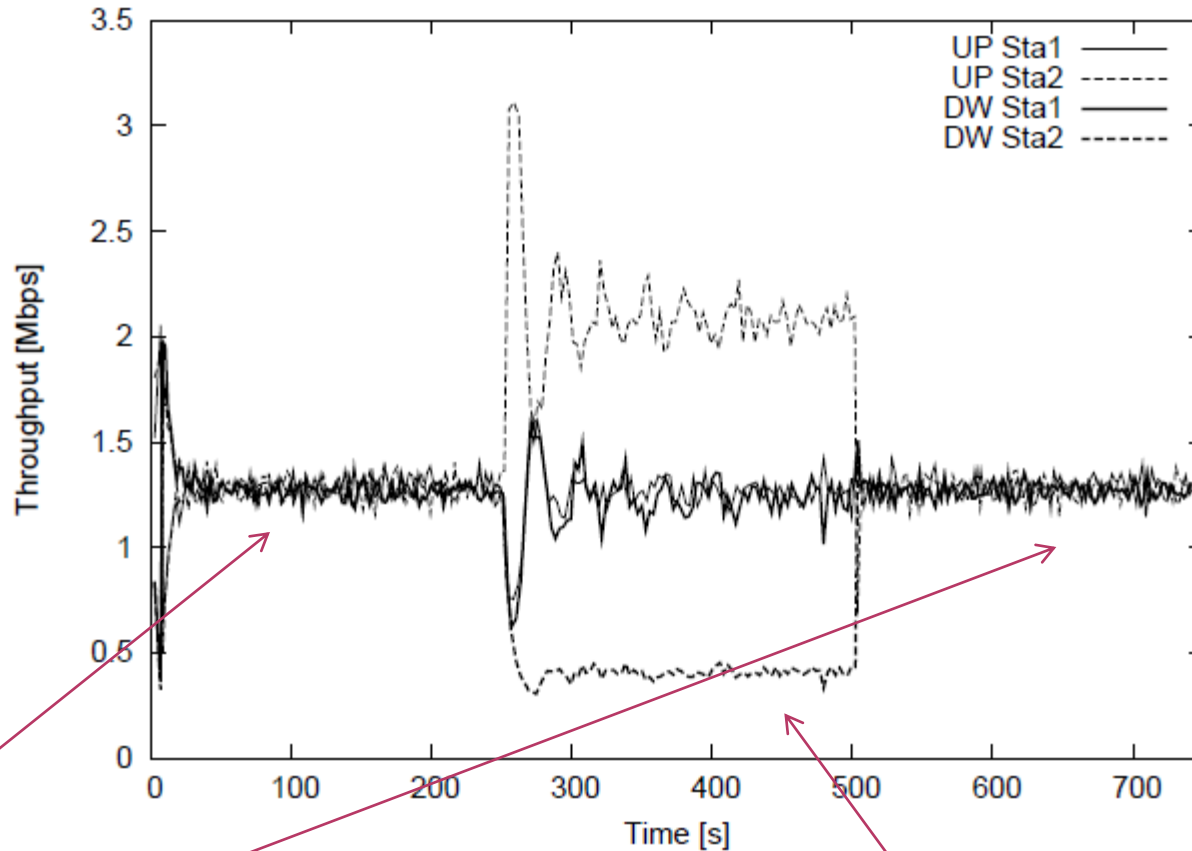
# Numerical Example: Resource Repartition

- Custom-made simulation platform;
- Interval update:0.5 seconds; 802.11b; P=1500 bytes



Standard  
DCF

# Effects of best response strategy (Time-varying Application requirements )



$k_1=k_2=1$

$k_2=5, k_1=1$

# Final Remarks on Infrastructure Networks

- **Contention-based access protocols can be defined in terms of non-cooperative games**
  - Standards are somehow limited with the proliferation of open-source drivers
- In infrastructure networks, the node strategies converge to Nash equilibria with non-zero payoff, by considering both uplink and downlink bandwidth requirements of user applications
- AP can be used for mechanism design, in order to force desired equilibrium conditions
  - by tuning its channel access probability
  - by employing scheduling policies for improving the network fairness

# Ad-hoc Networks

## Minimum Graph Coloring

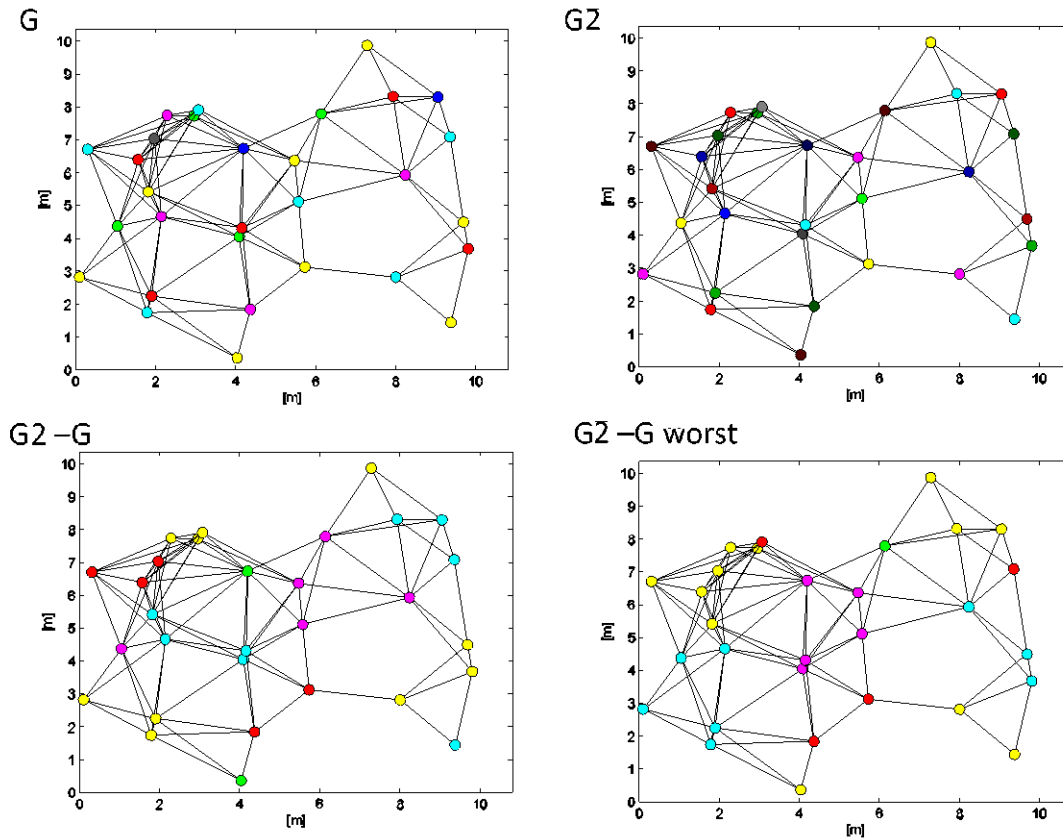
- **Minimum Graph Coloring (MGC)** problem on an incompatibility graph, built on the basis of network topology  $G = (V, E)$   $V$ : nodes  $i$  of the network,  $E$ : pairs of nodes
- $H_e$  is the **Incompatibility graph type I**  $(V, F_e)$ , where for each  $e \in 2^E$ :  $F_e = \{(j, k) : \exists i \in V \text{ s.t. } (j, i), (i, k) \in E\}$ 
  - $(j, k)$  frame may collide if transmitted simultaneously
  - $H_e = G^2$ : all nodes have non-interfering allocations and we can guarantee a collision-free throughput proportional to  $r/x$
- $H_0$  is the **Incompatibility graph type II**  $(V, F_0)$ , where for each  $e \in 2^E$ :  $F_0 = \{(j, k) : \exists i \in V \text{ s.t. } (j, i), (i, k) \in E, \text{ but } (j, k) \notin E\}$ 
  - $(j, k)$  collide and reciprocally hidden
  - $H_\emptyset = G^2 - G$ : visible nodes share the same allocations

# Coloring Algorithms

- **Select and Compare (SC):**
  1. **First coloring** Randomly pick a color from a list of available colors.
  2. **Conflict Resolution** If none of your (1-hop or 2-hop) neighboring nodes has chosen the same color, keep it as definitive color, otherwise remove it from the list and try again the next step.
  3. **List update** If the color list is empty, add new colors. The list is updated starting from  $\min(c+1, x_{\max})$  color, where  $c = \max(\text{neighboring node colors})$
- **Choose the First Available color (CFA) :**

Instead of randomly picking a color from the available ones, each node first updates the list of available colors and then selects the color with the lowest index

# Example of colored network



*A network topology colored with different CFA maps for the incompatibility graphs  $G, G^2, G^2-G$*



# Performance Evaluation

- Given a graph  $H_e$ , the maximum number of needed colors is upper bounded by  $\Delta_e+1$ , where  $\Delta_e$  is the maximum node degree of the graph.
- Let  $x_e$  the number of colors required in  $H_e$  and  $c_e$  the number of cliques. After coloring, the throughput sum perceived by all the nodes belonging to each clique is obviously  $r/x_e$ , thus resulting in a total throughput equal to:  $\rho_{tot}^e = \frac{r}{x_e} c_e$
- Average per-node throughput as  $\rho_{tot}/n = r / x_e E[d_e]$   
( $E[d_e] = n/c_e$  represents the average after coloring clique size).

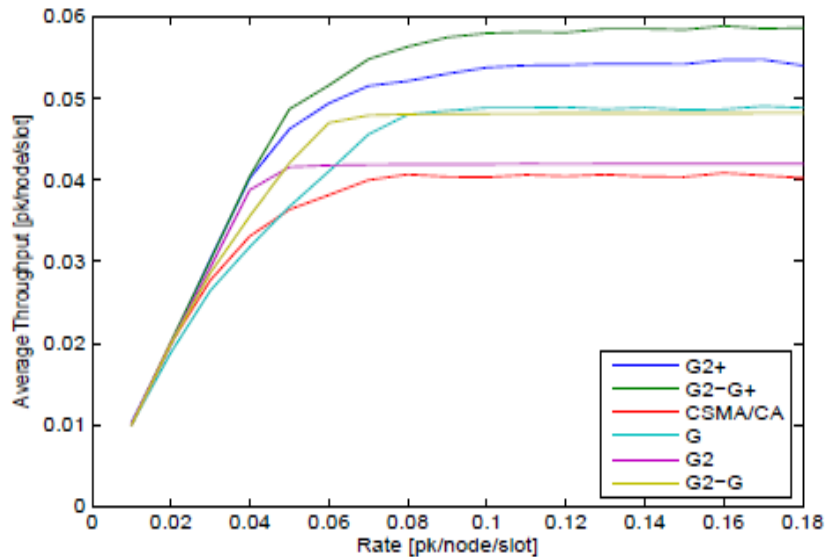
# Performance Evaluation

Topology	$H_e$	$x_e$	$c^e$	$\hat{\rho}^e/\tau$	$E[\rho^e]/\tau$
1	$G^2$	16	30	0.0624	0.0625
1	$G^2 - G$	5	12	0.0792	0.0800
2	$G^2$	13	30	0.0768	0.0769
2	$G^2 - G$	5	11	0.0730	0.0733
3	$G^2$	15	30	0.0666	0.0667
3	$G^2 - G$	5	13	0.0863	0.0867

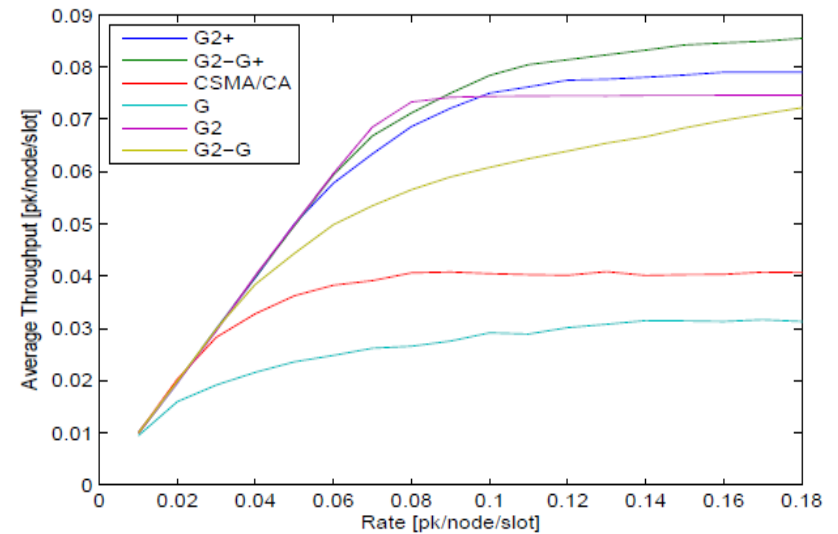
TABLE I

MEASUREMENTS AND ESTIMATES OF THROUGHPUT.

# Performance Evaluation



*Average throughput under the SC coloring scheme*



*Average throughput under the CFA coloring scheme*

# Some Observations

1. **Coloring G** can be useless, because the carrier sense functionality is already able to avoid interference among adjacent nodes. For the CFA case, the performance obtained under the G coloring are even worse than the ones obtained with the CSMA/CA protocol, because the slot allocations may synchronize hidden nodes for lower packet generation rates.
2. **Coloring G2** can be more efficient (CFA case) or less efficient (SC case) than coloring G2-G, according to the network topology and to the effectiveness of the coloring scheme in selecting a limited number of colors and/or leaving a limited number of bottlenecks.
3. **If we allow node  $i$**  to transmit during the slots associated to its color and to colors different from the ones of its adjacent nodes (schemes G2+ and G2-G+), we can further improve the network performance

# From PALERMO to SANTA BARBARA



THANKS!