Real-time estimation of distributed parameters systems: application to large scale infrastructure systems

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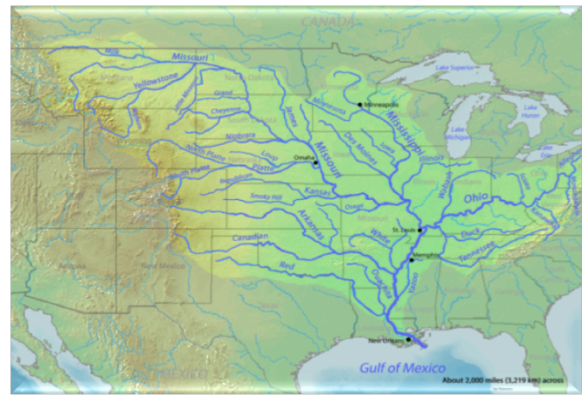
http://traffic.berkeley.edu **http://float.berkeley.edu**

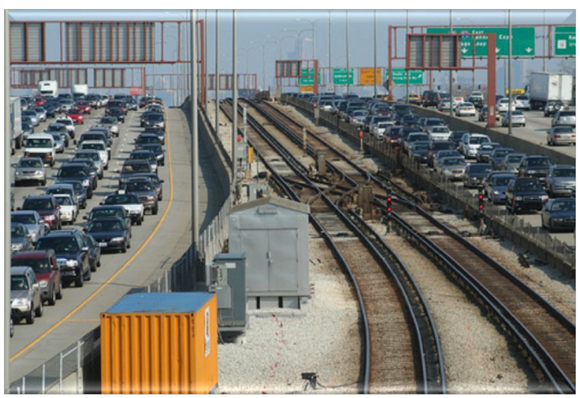
Distributed parameter systems

Distributed parameter systems integrate dynamical processes in which spatial variations play an integral role in their evolution

- **Examples include: structural systems, propagation of pollutants in air, water distribution networks, transportation networks, the power grid, smart buildings.**







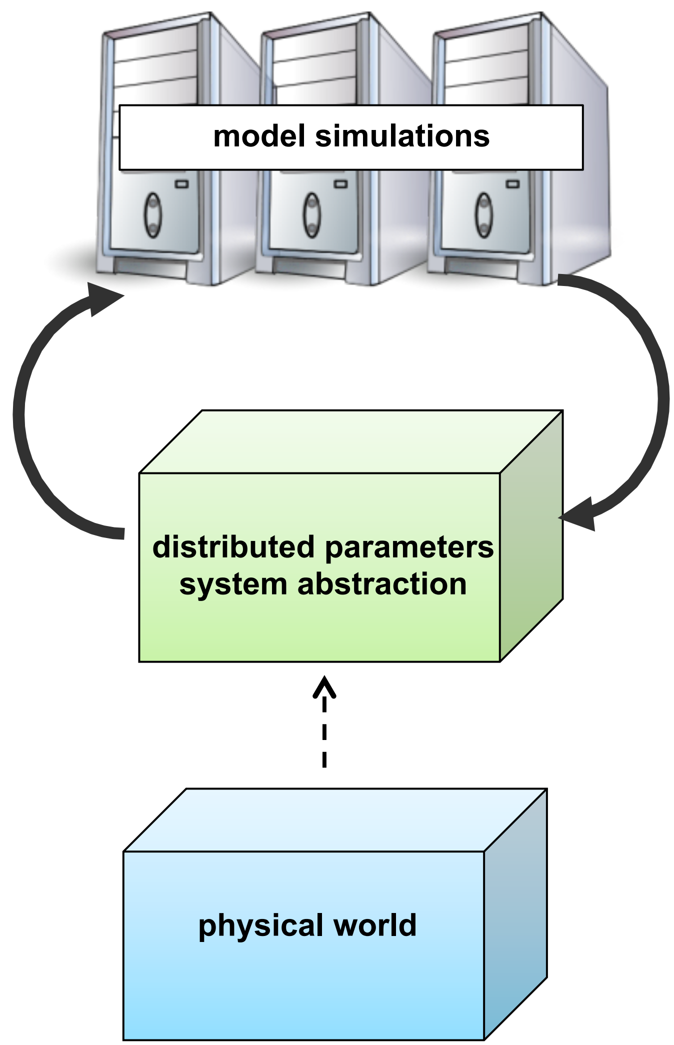
The direct problem (forward simulation)

Forward simulation requires a mathematical model, which is an abstraction of the system, for example:

- Partial differential equation (PDE)
- Ordinary differential equation (ODE)
- Finite element model (FEM)
- Finite difference model
- Computational code

Simulation numerically represents the evolution of the state of the system, and requires the usually unknown:

- Numerical parameters of the model
- Initial conditions (initialization)
- Boundary conditions



The inverse modeling problem

Inverse modelling characterizes the process of determining the numerical parameters of the model.

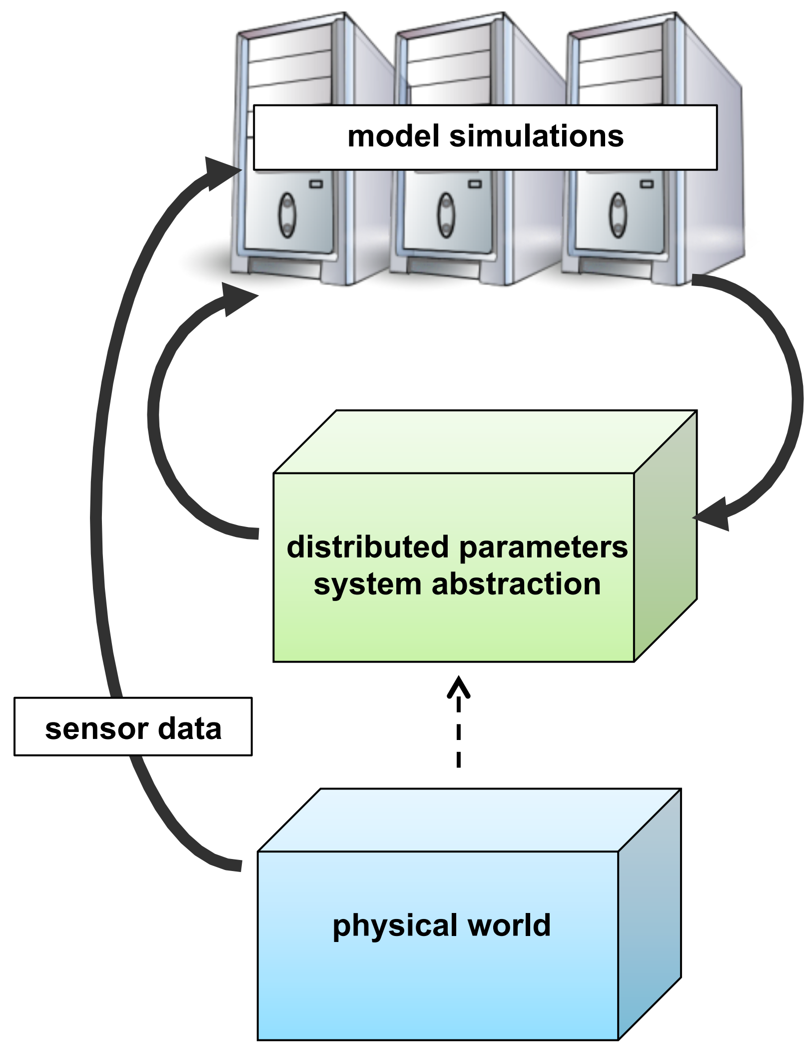
- System identification (in control theory)
- Learning (in machine learning)

In general it requires:

- To have a predefined mathematical model (abstraction)
- Experimental data

Challenges include:

- Modelling errors
- Measurement characteristics (noisy, sparse, etc.)



The data assimilation problem

Data fusion is sometimes used to characterize the process of integrating sensor data into the mathematical model to find the evolution of the state of the system over time. It is sometimes called:

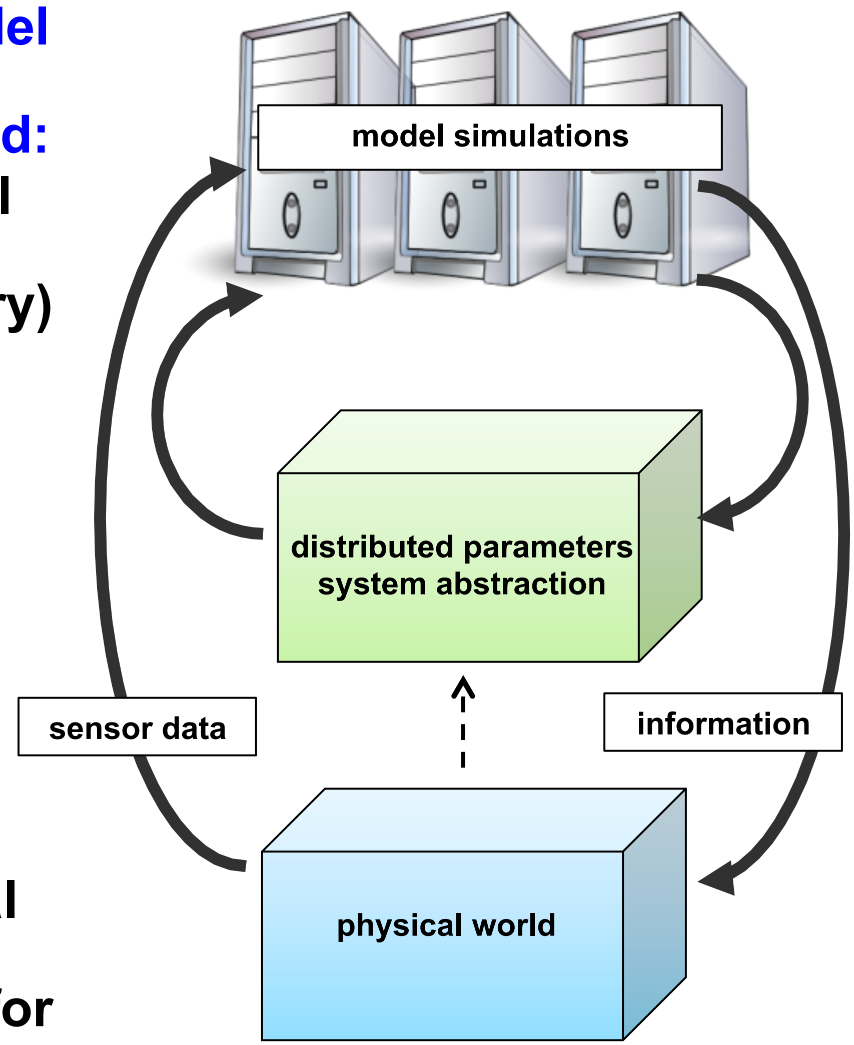
- **Data assimilation (in the physical sciences)**
- **State estimation (in control theory)**
- **Inference (in machine learning)**

For online systems implementation, it requires:

- **Streaming sensor data**
- **Real time computation**

Specific to cyberphysical systems:

- **Coupling between the physical processes and the computational processes**
- **Need to run faster than physics for nowcast (and forecast)**



Outline

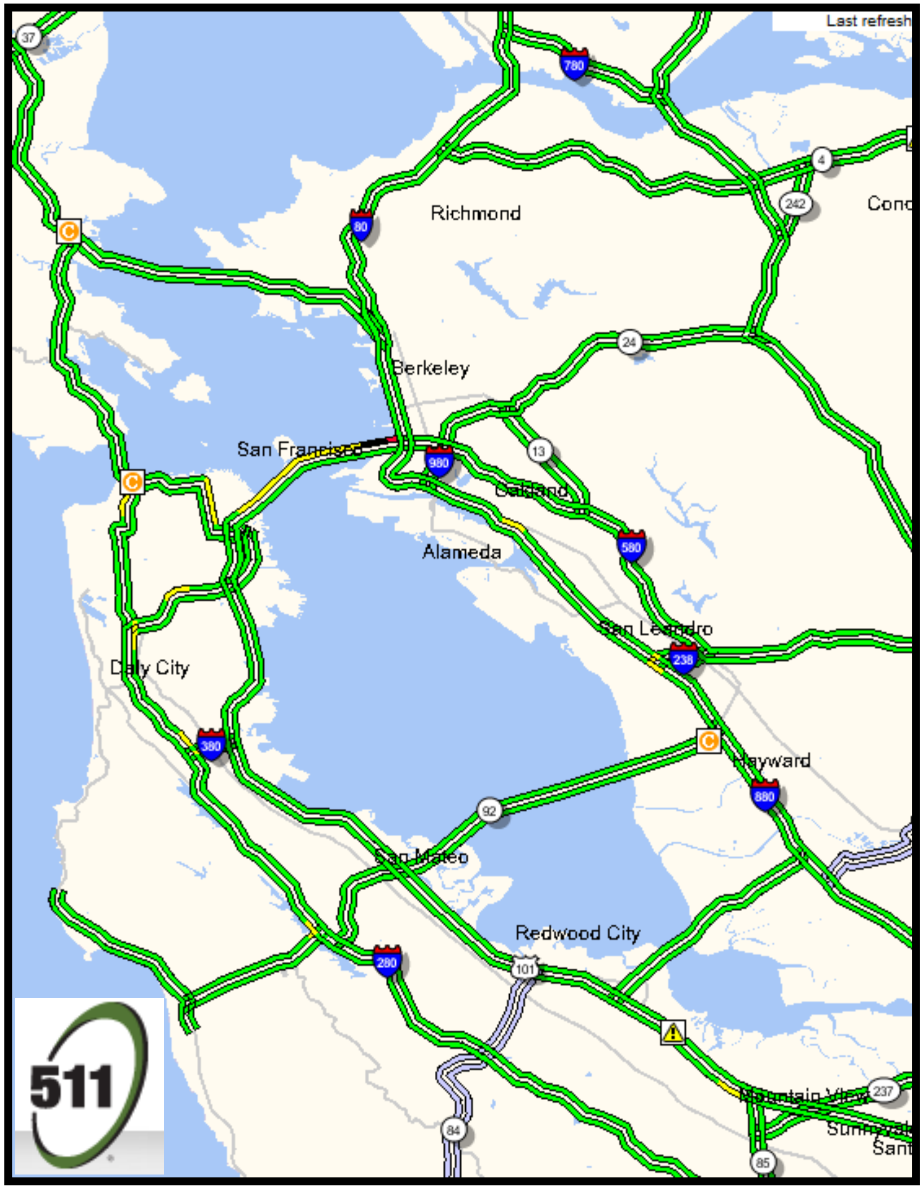
1. Traffic information systems at the age of web 2.0

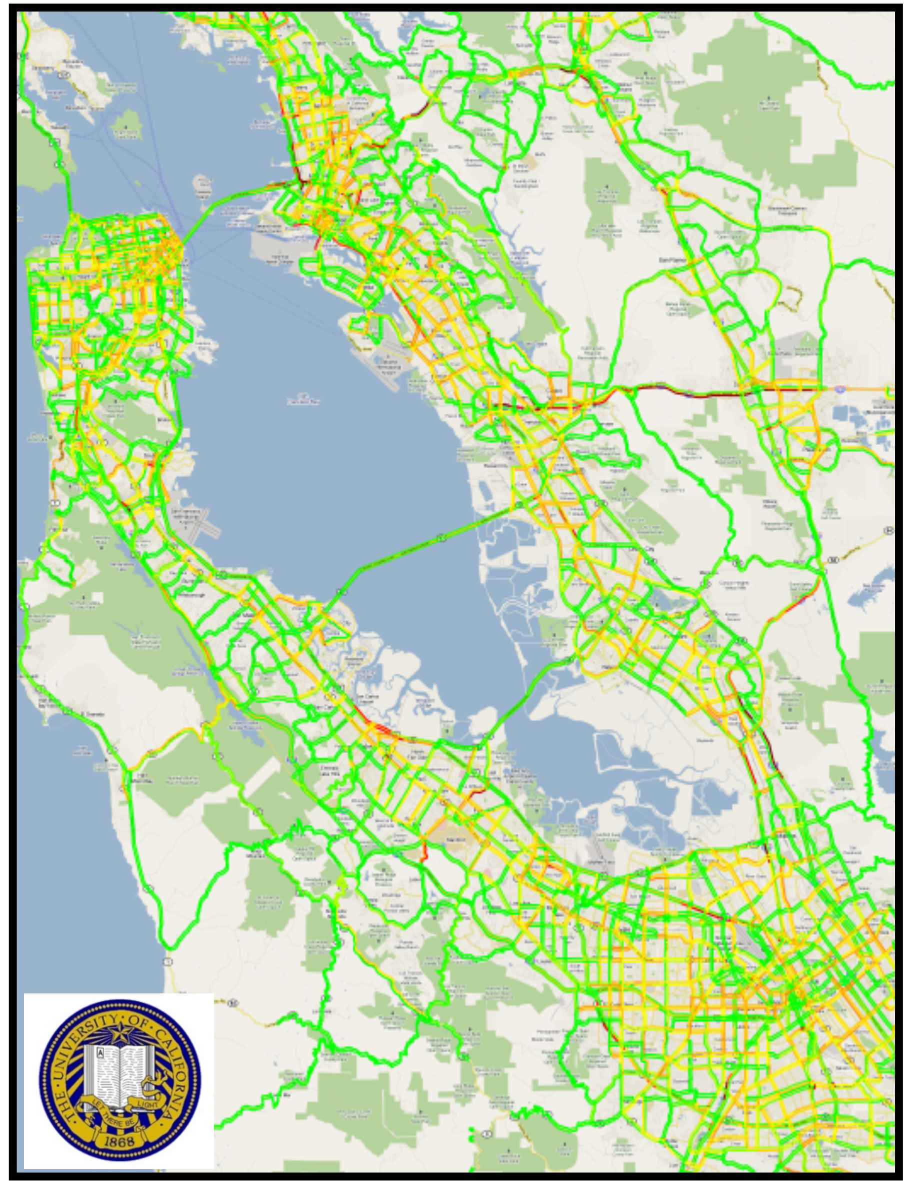
2. Mobile Millennium

3. Inverse modeling and data assimiliation
    1. A short introduction to traffic modeling
    2. The Moskowitz Hamilton-Jacobi equation
    3. Internal boundary conditions using the inf-morphism property
    4. Data assimilation in a privacy aware environment

4. Beyond Mobile Millennium
    1. Air
    2. Earthquakes
    3. Water

511.org 2010 (UC Berkeley)

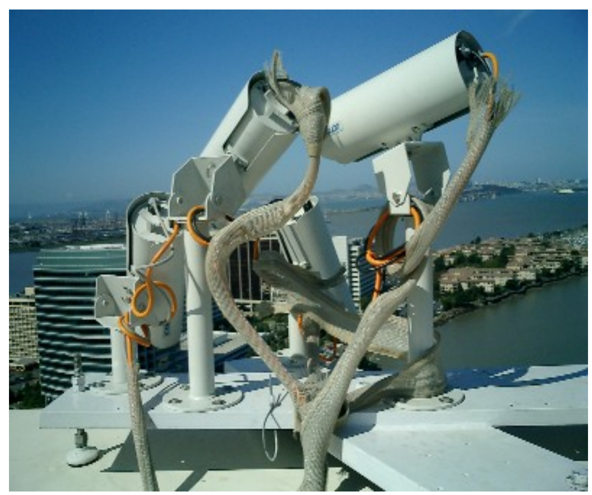




“Classical” source of traffic information

Dedicated traffic monitoring infrastructure:
- Self inductive loops
- Wireless pavement sensors
- FasTrak, EZ-pass transponders
- Cameras
- Radars
- License plate readers

Issues of today’s dedicated infrastructure
- Installation costs
- Maintenance costs
- Reliability
- Coverage
- Privacy intrusion



Web 2.0 on wheels

Emergence of the mobile internet

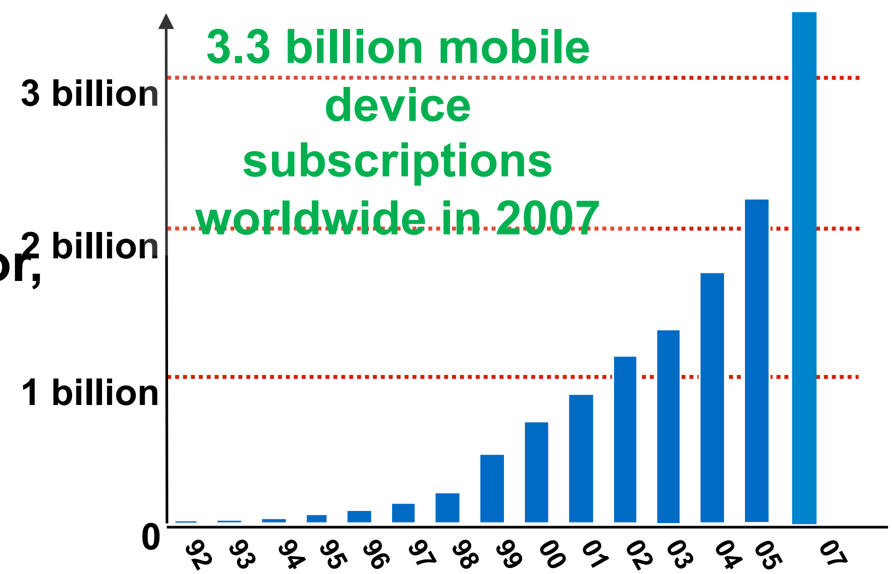
- Internet accesses from mobile devices skyrocketing
- Mobile devices outnumber PCs by 5:1
- 1. 5 million devices/day (Nokia)
- Redefining the mobile market: Google, Apple, Nokia, Microsoft, Intel, IBM, etc.
- Open source computing: Symbian Foundation, Android, Linux

Sensing and communication suite

- GSM, GPRS, WiFi, bluetooth, infrared
- GPS, accelerometer, light sensor, camera, microphone

Smartphones and Web 2.0

- Context awareness
- Sensing based user generated content



[Courtesy J. Shen, Nokia Research Center Palo Alto]

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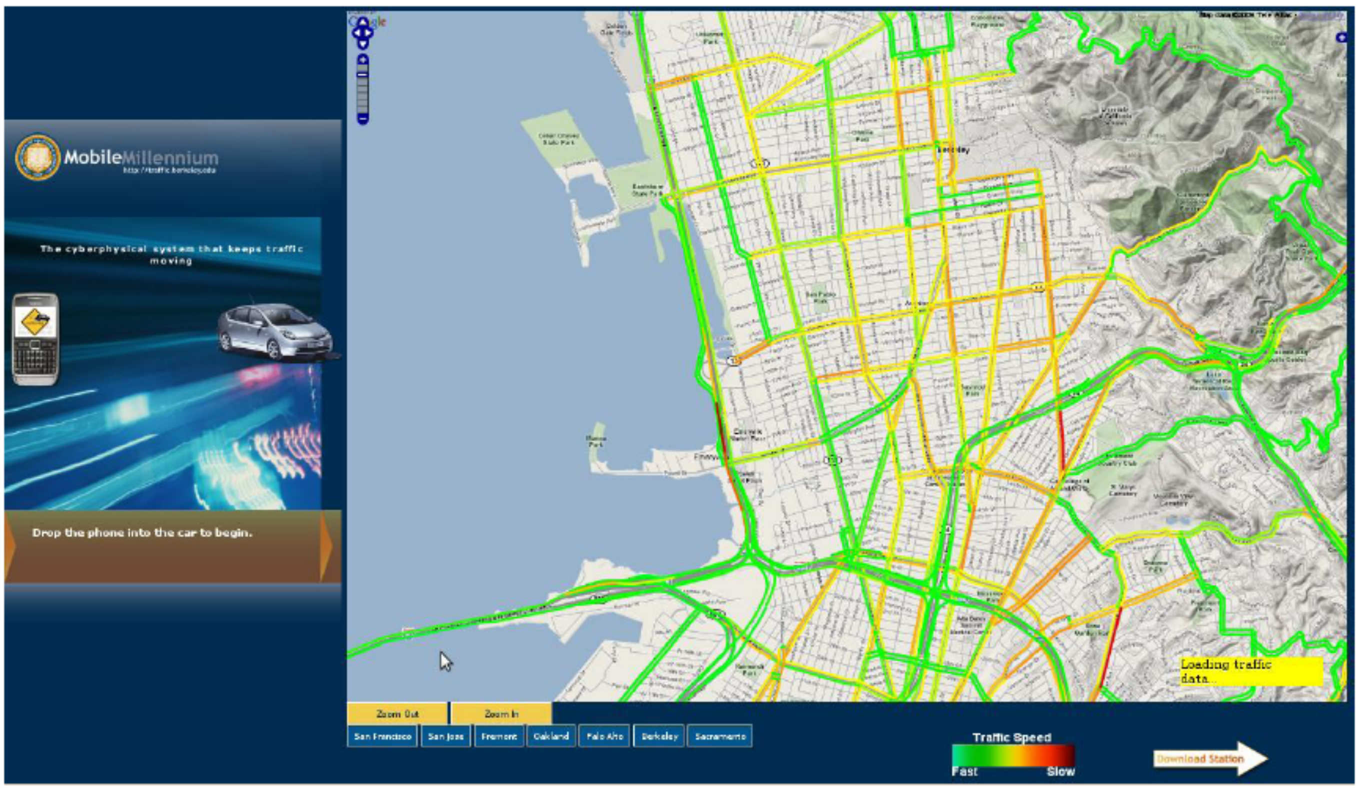
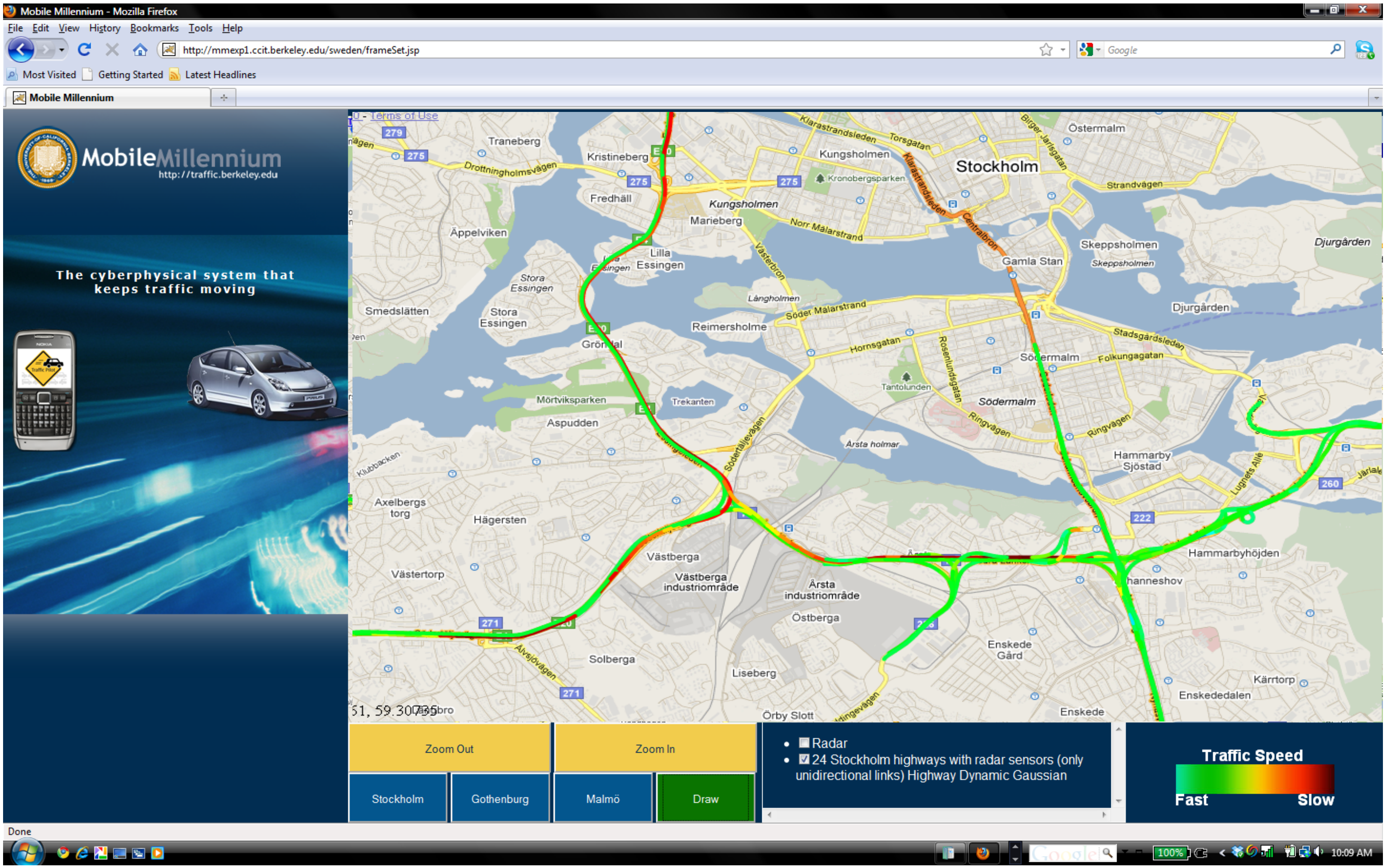
Mobile Millennium today

Current features of the system

- Initially, 5000 downloads of the FIRST Nokia traffic app worldwide
- Gathers about 60 million data points / day from dozen of sources (smartphones, taxis, fleets, static sensors, public feeds, etc.)
- Provides real-time nowcast (soon forecast) of highway and arterial traffic, provide routing and data fusion tools.
- Provides integration platform for any mobile data stream



Millennium Stockholm online since March 2011



Example of 500 vehicles in SF (taxis)

Mobile Millennium infrastructure

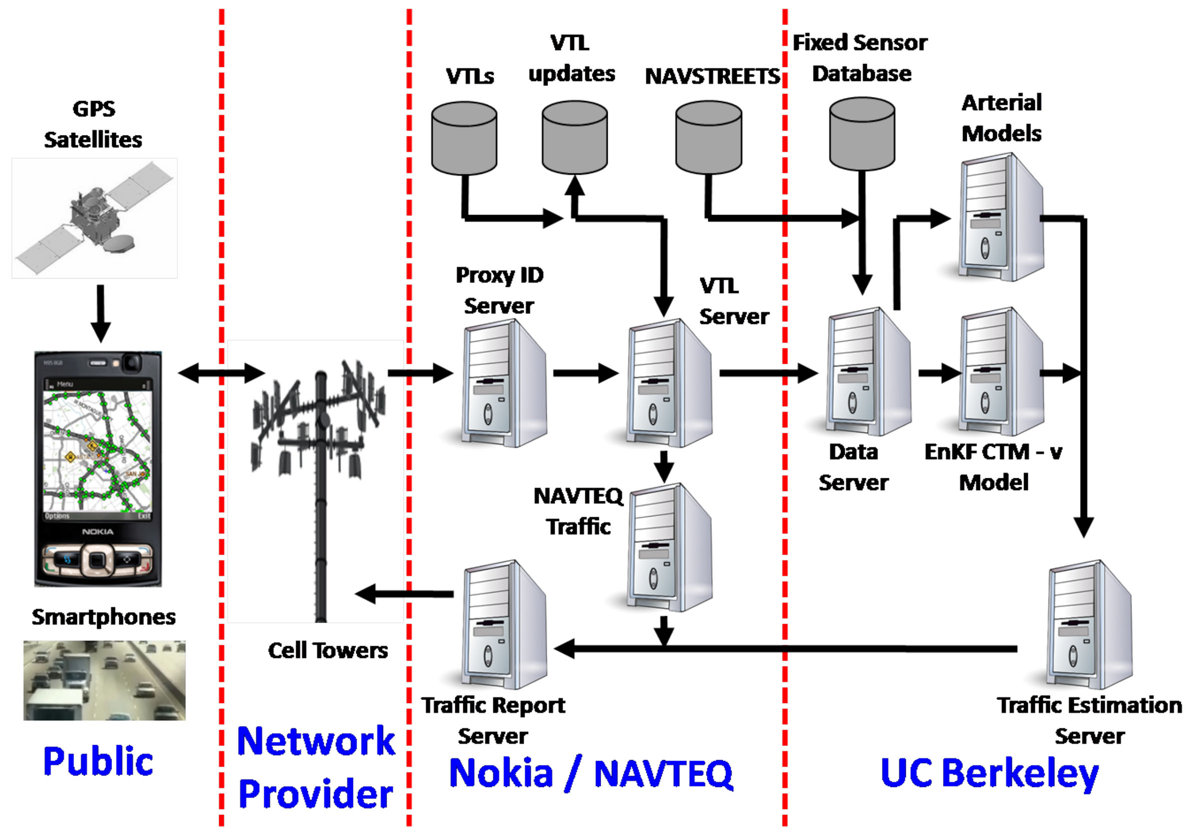
Sensing
- Millions of mobile devices as new sources for data

Communication
- Existing cell phone infrastructure to collect raw data and receive traffic information

Data assimilation
- Real-time, online traffic estimation

Privacy Management
- Encrypted transactions
- Client authentication
- Data anonymization

A cyberphysical system for participatory sensing



Data transmission

Data assimilation

Data generation, info distribution

Data aggregation

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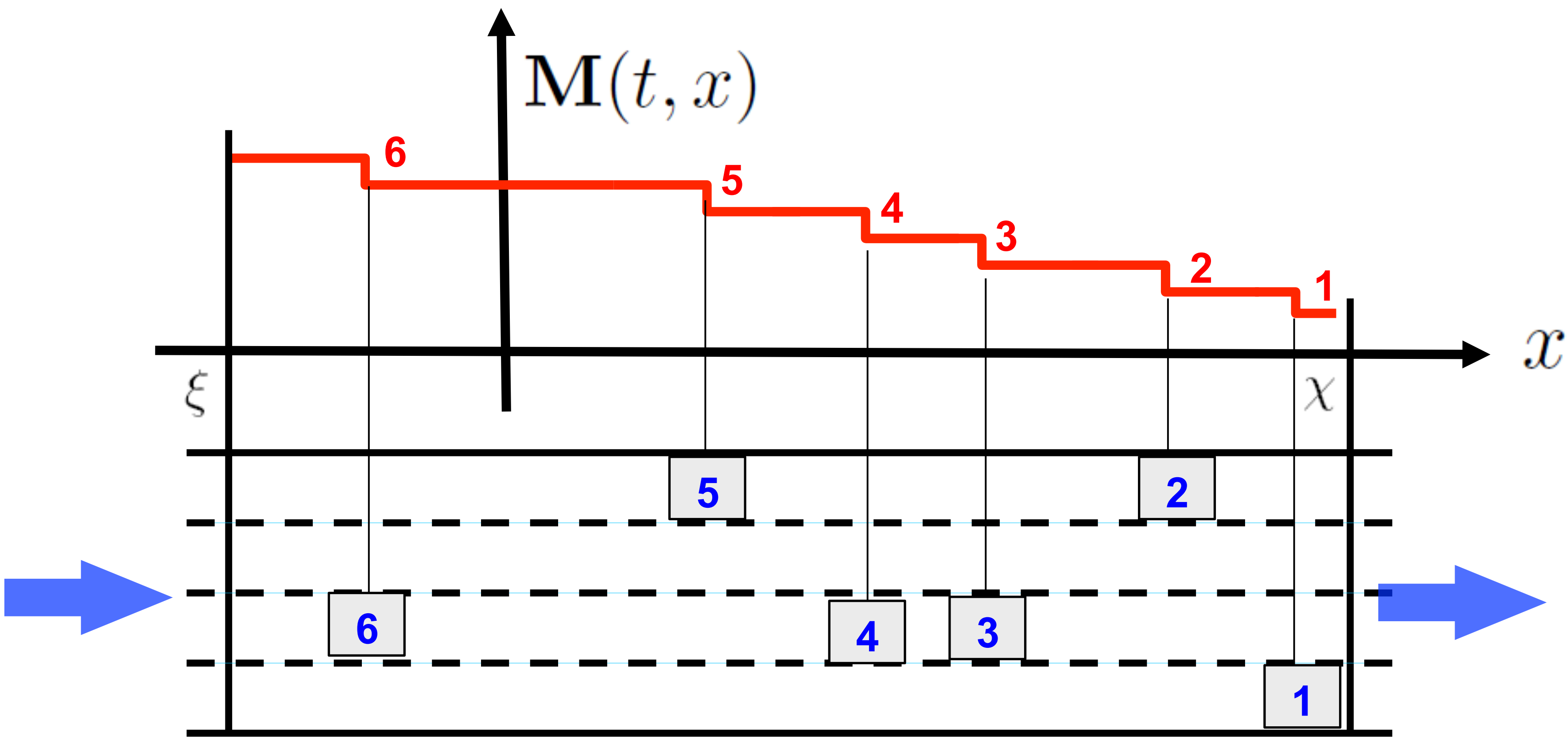
Definition of the Moskowitz function

State of traffic can be described by the Moskowitz function M(t,x)

- **Attribute consecutive labels n to the vehicles entering a section of the highway.**
- **The Moskowitz function is a continuous function satisfying M (t,x)=n**

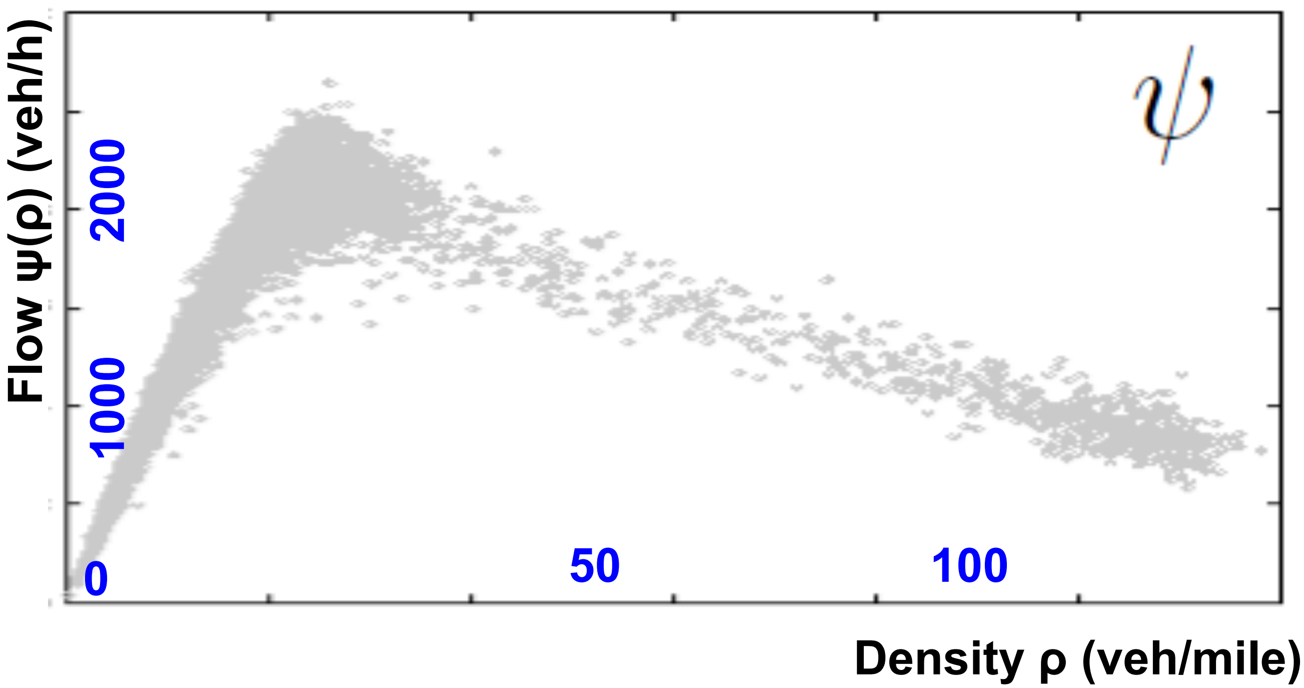


Mathematical model: Hamilton-Jacobi PDE

The Moskowitz satisfies the following Hamilton Jacobi PDE

- **It can be derived from the Lighthill Whitham Richards PDE**
- **The Hamiltonian of the Hamilton Jacobi PDE is the usual fundamental diagram known empirically, denoted ψ**

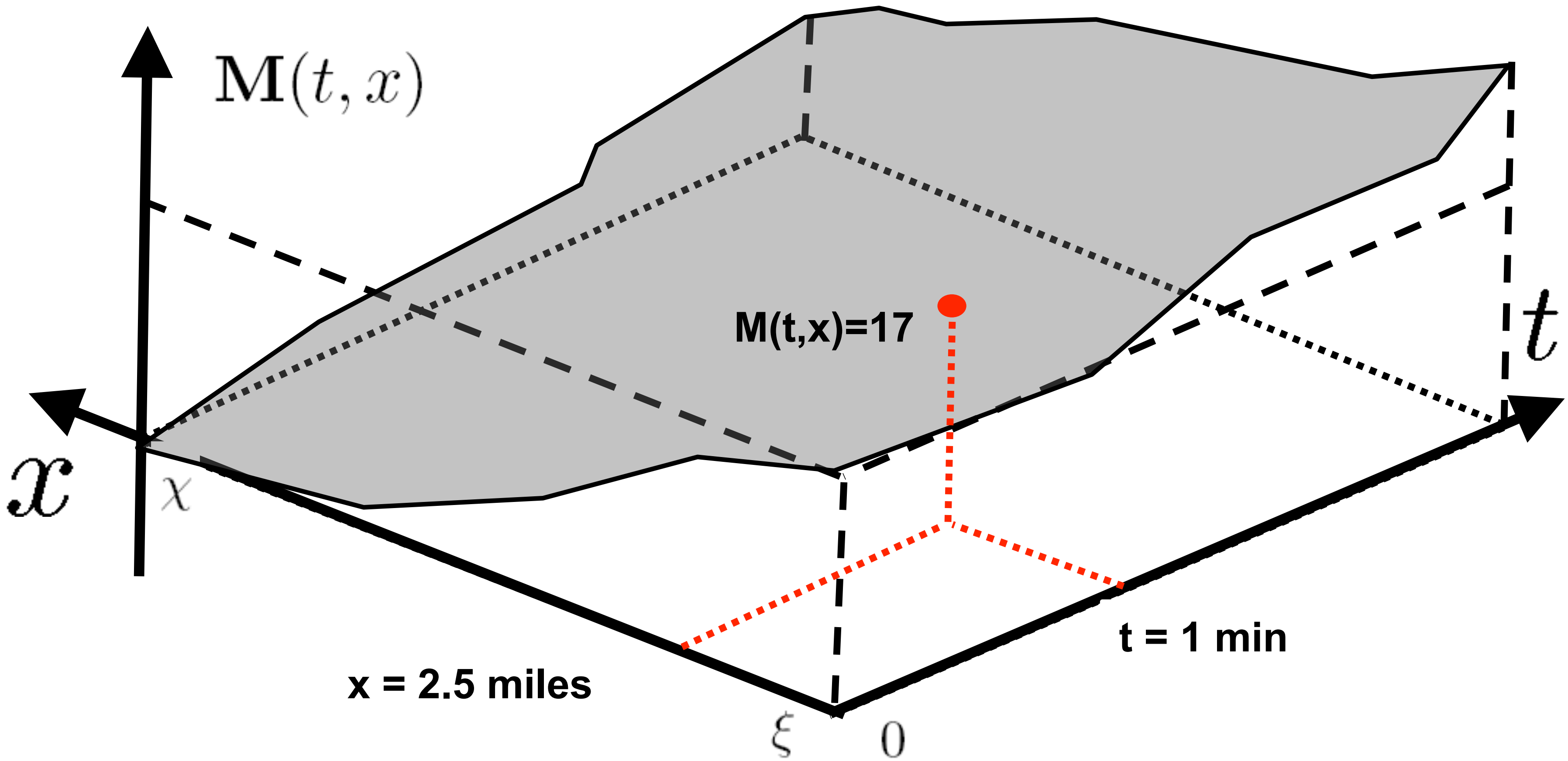
$$\frac{\partial \mathbf{M}(t,x)}{\partial t} - \psi\left(-\frac{\partial \mathbf{M}(t,x)}{\partial x}\right) = 0$$



Physical interpretation of the Moskowitz function

The Moskowitz function is the solution of the Hamilton Jacobi PDE

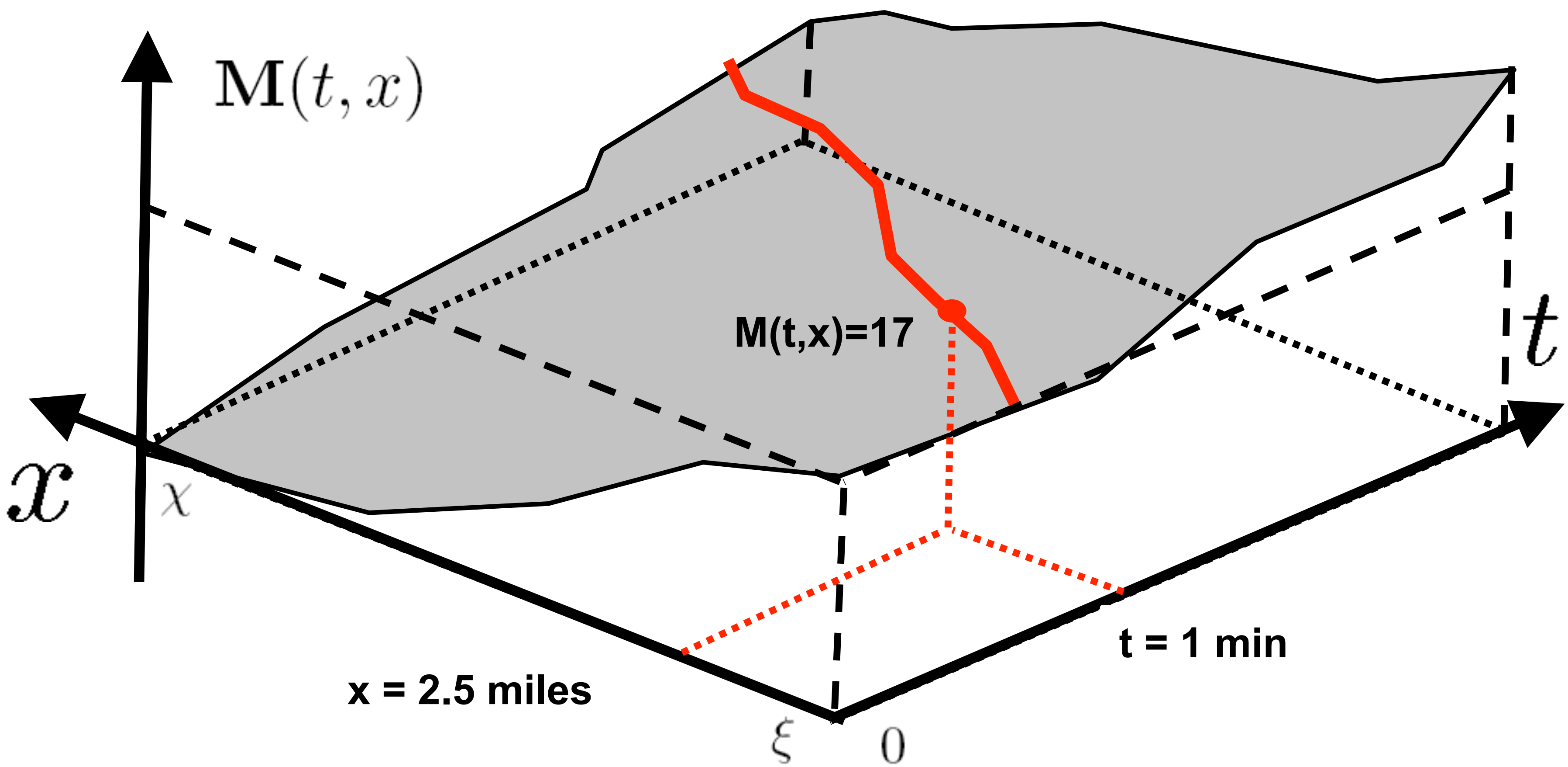
- **Its value at location x and time t represents the label of the vehicle at that location and at that time**
- **For example vehicle 17 is at postmile 2.5 at time 1 minute.**



Physical interpretation of the level sets

The Moskowitz function is the solution of the Hamilton Jacobi PDE

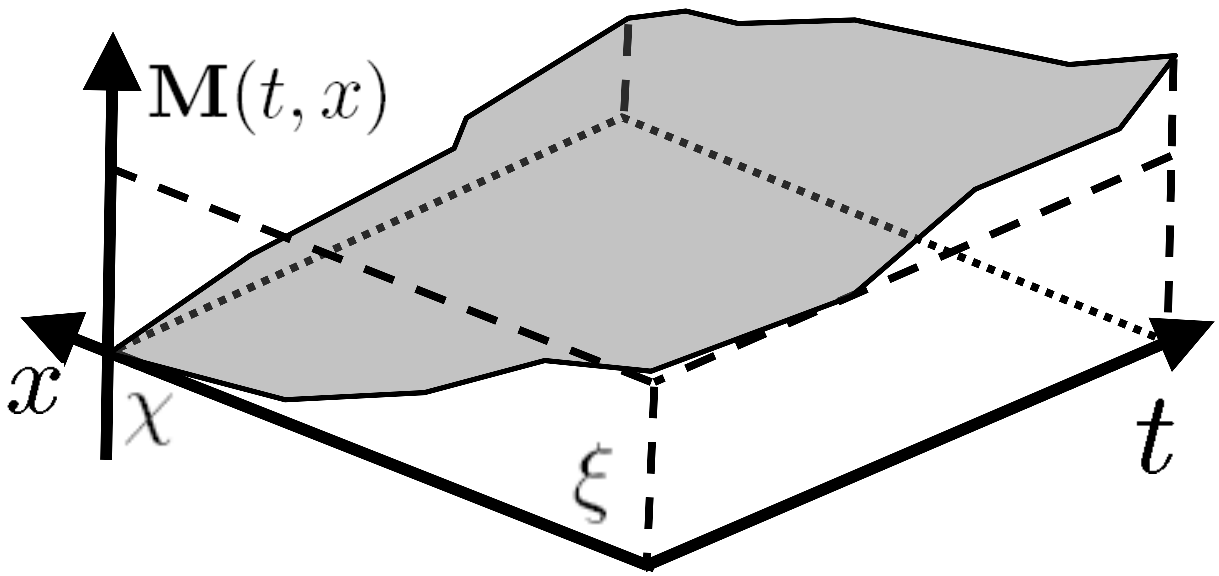
- **Its value at location x and time t represents the label of the vehicle at that location and at that time**
- **The set of points such that M(t,x)=17 is the trajectory of vehicle 17**



Solution of the forward problem

The solution of the forward problem relies on the notion of viscosity solution or its extensions (Frankowska solutions).

- **Solution of the forward problem (in gray) requires:**
- **Initial condition (in red)**
- **Boundary condition 1, inflow (in blue)**
- **Boundary condition 2, outflow (in green)**



$$\boxed{\frac{\partial \mathbf{M}}{\partial t} - \psi\left(\frac{\partial \mathbf{M}}{\partial t}\right) = 0}$$

$$\begin{aligned}
\mathbf{M}(0,x) &= \mathbf{M}_0(0,x) \\
\mathbf{M}(t,\xi) &= \gamma(t,\xi) \\
\mathbf{M}(t,\chi) &= \beta(t,\chi)
\end{aligned}$$

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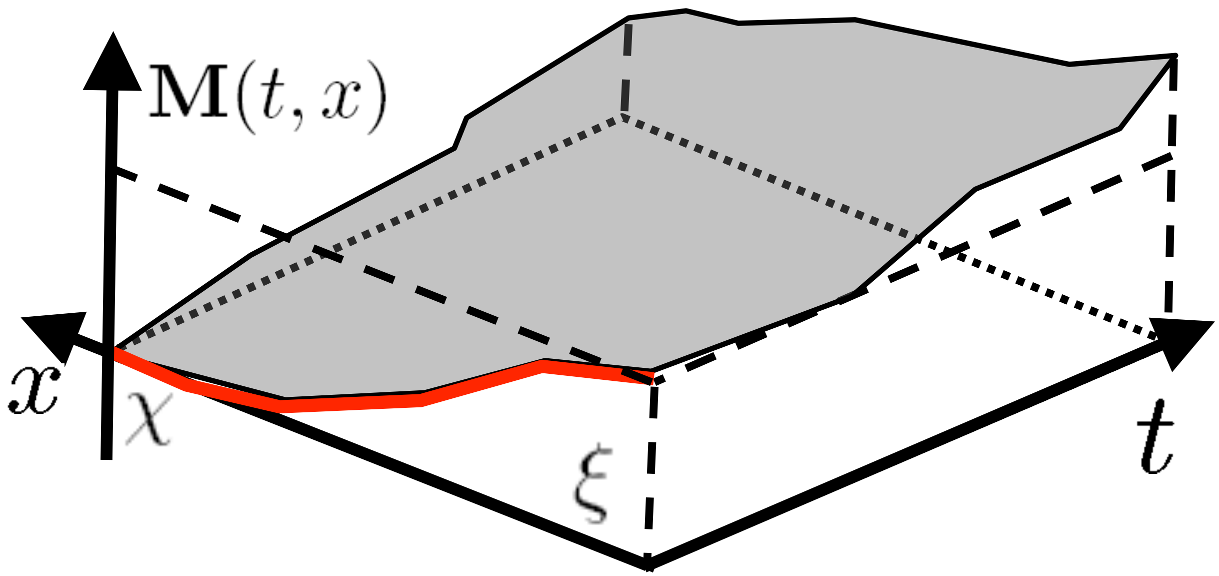

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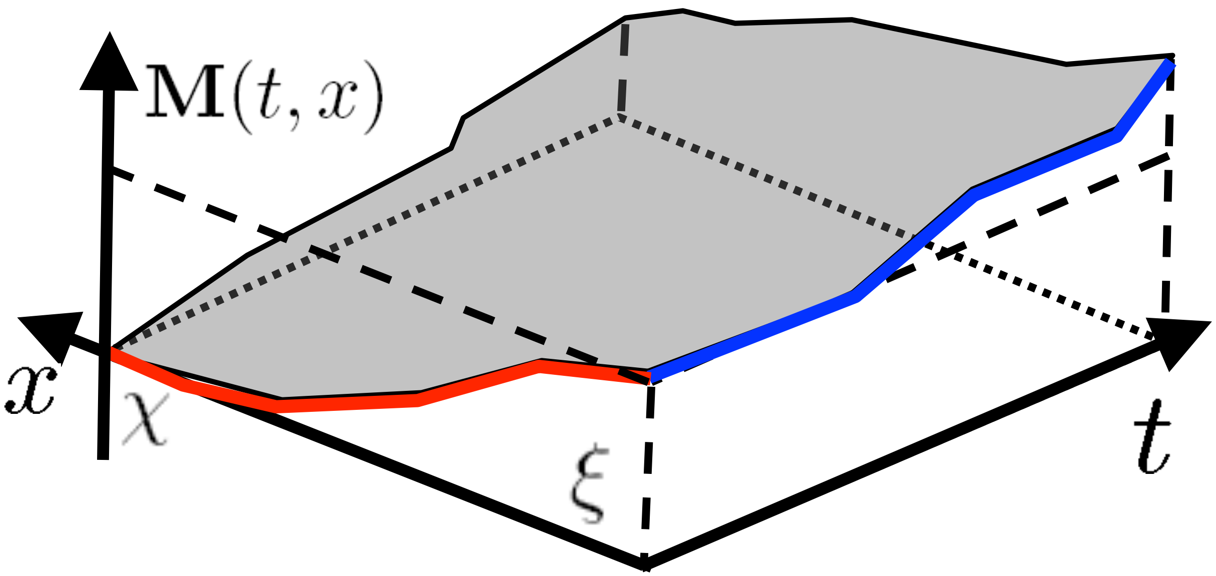

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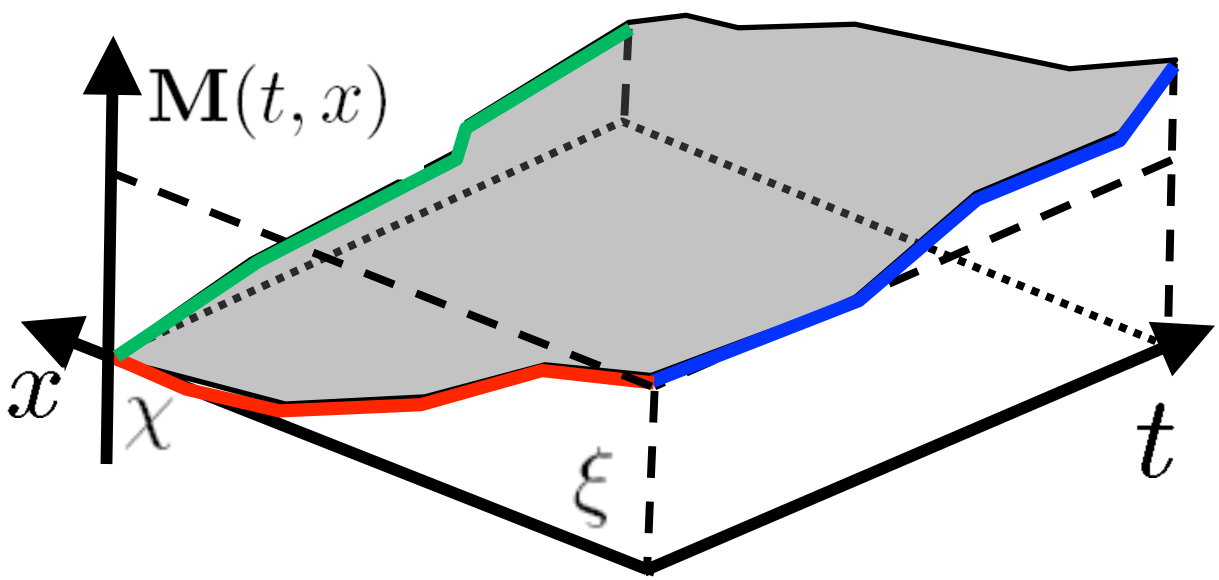

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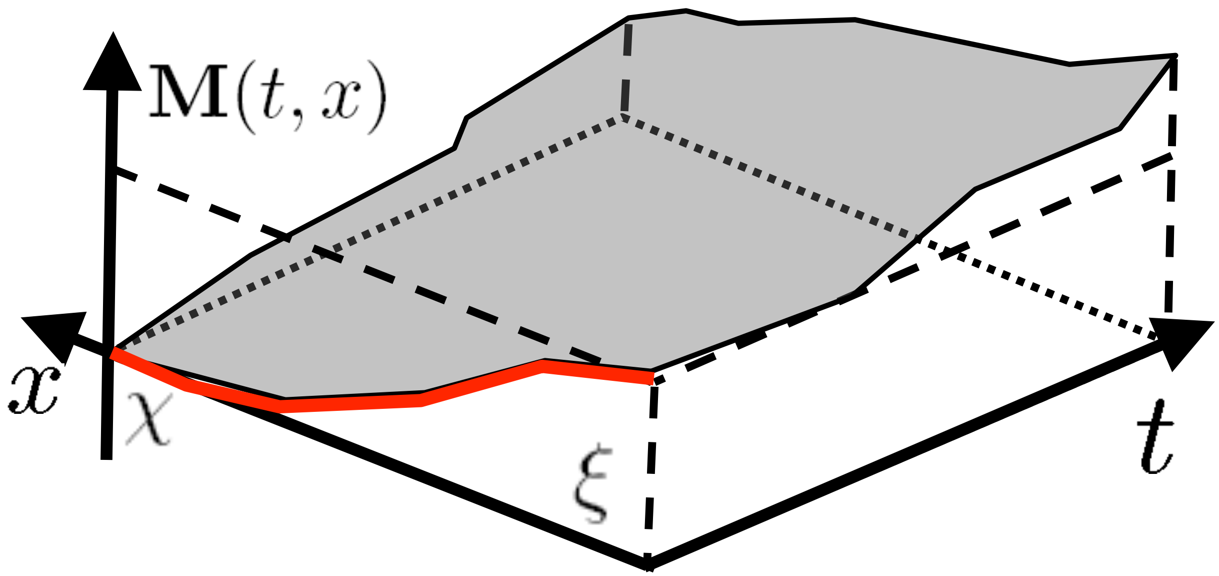
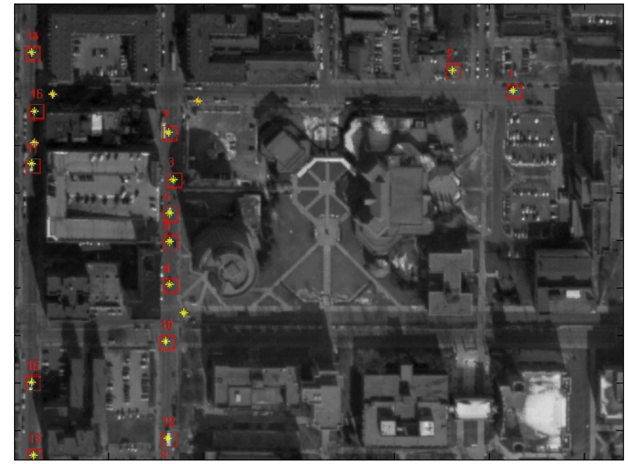
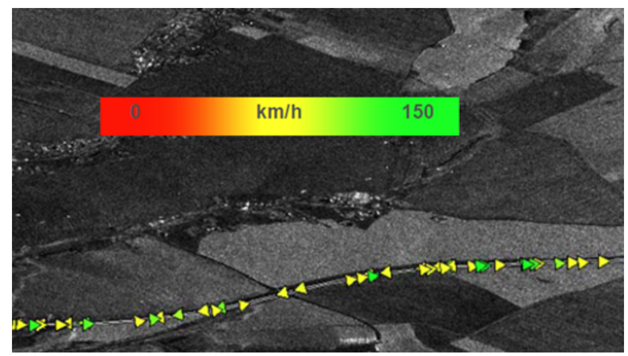
[In general] unavailable initial conditions

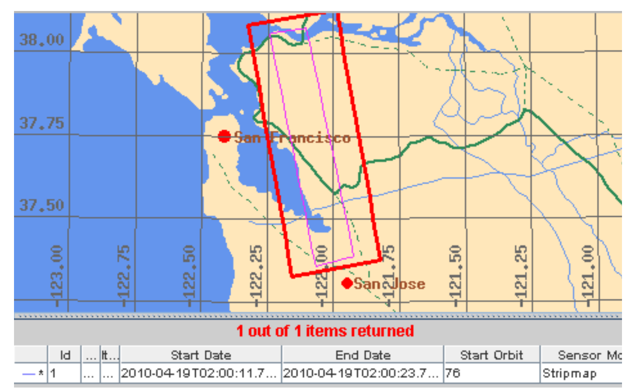
Initial conditions: initial state of highway at start of experiment

- **Can be measured with UAVs (DARPA)**
- **Could be measured with satellite (DLR) low orbit TerraSAR-X satellite**
  - **15 mins latency**
  - **Orbits around California once a day**
  - **Provides 70% of vehicles (and speeds)**





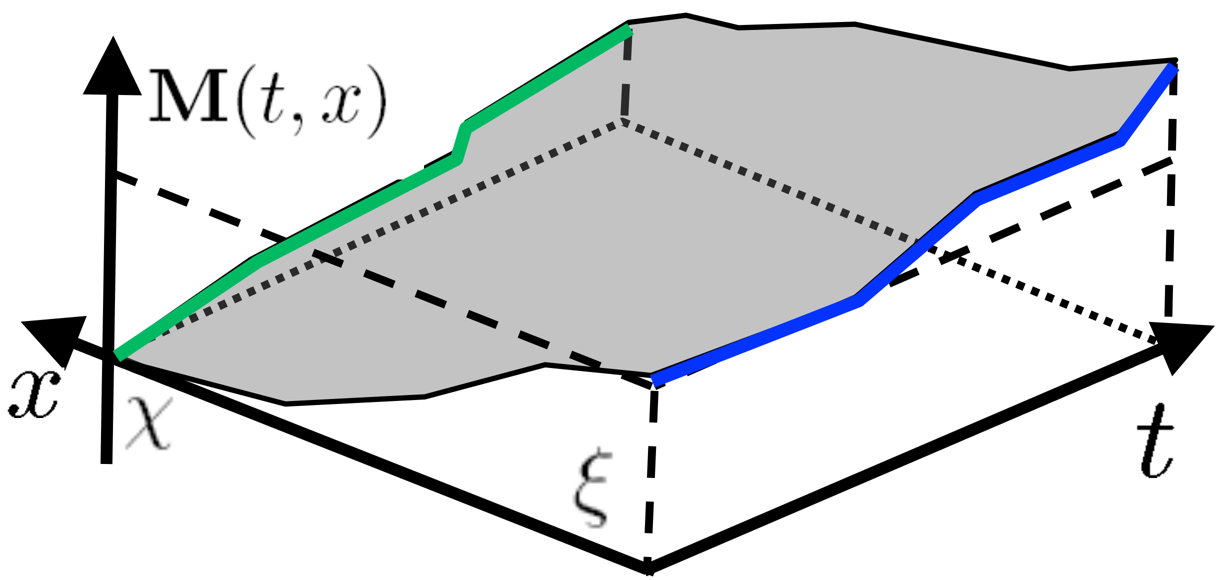


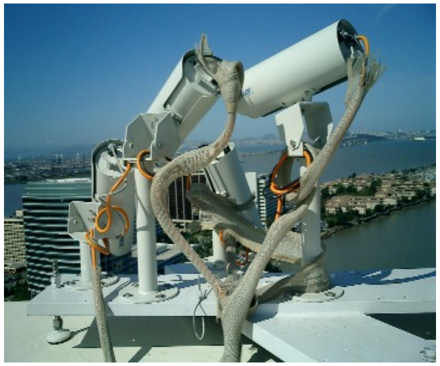


[Some erroneous] boundary conditions

Experimental boundary data:

- **Cameras, loop detectors, radar, etc.**
- **Noisy**
- **Inconsistent (up to 40% mass loss)**
- **Missing data**

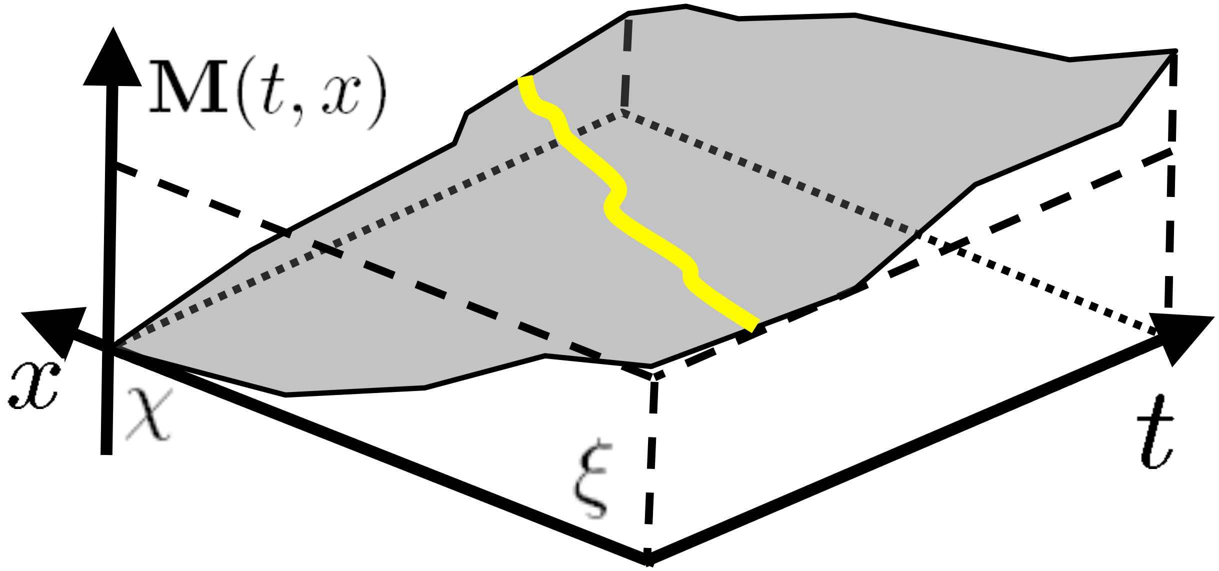


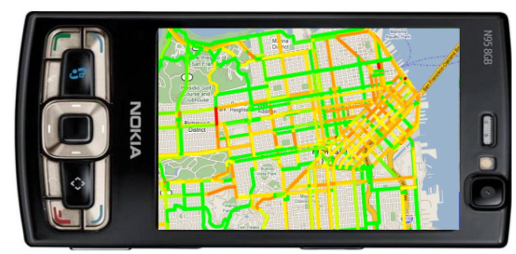


Lagrangian [internal] data of various types

Variety of a available probe data:

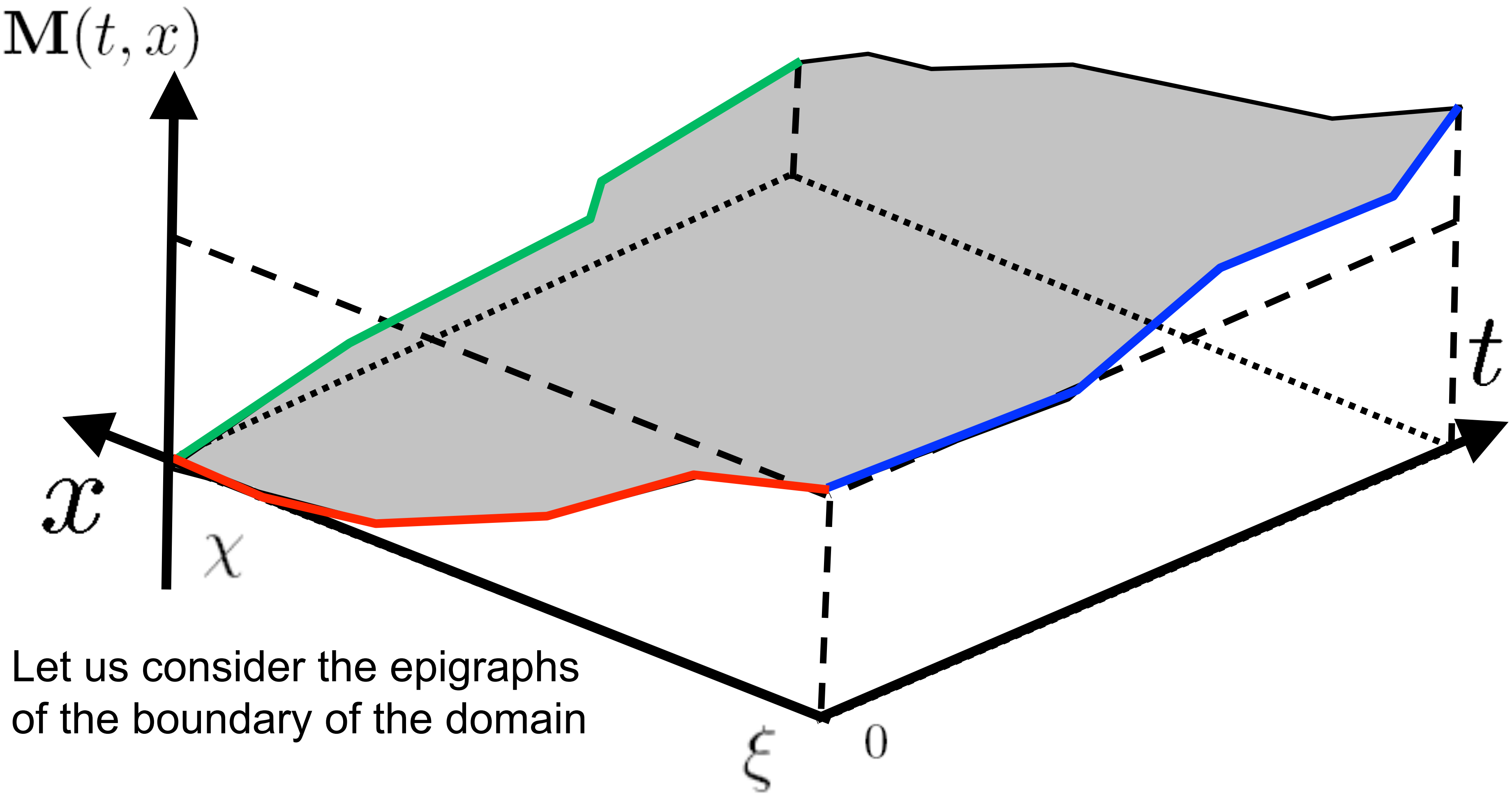
- **VTL data (Nokia)**
- **Full trajectory data**
- **Bread crumbs (trajectory subsets)**
- **Point-to-point**
- **Random samples**
- **Snail operations (police)**
- **Etc...**





Epigraphical characterization of the solution

Idea: characterize the Moskowitz surface as the lower envelope of a capture basin



Let us consider the epigraphs of the boundary of the domain

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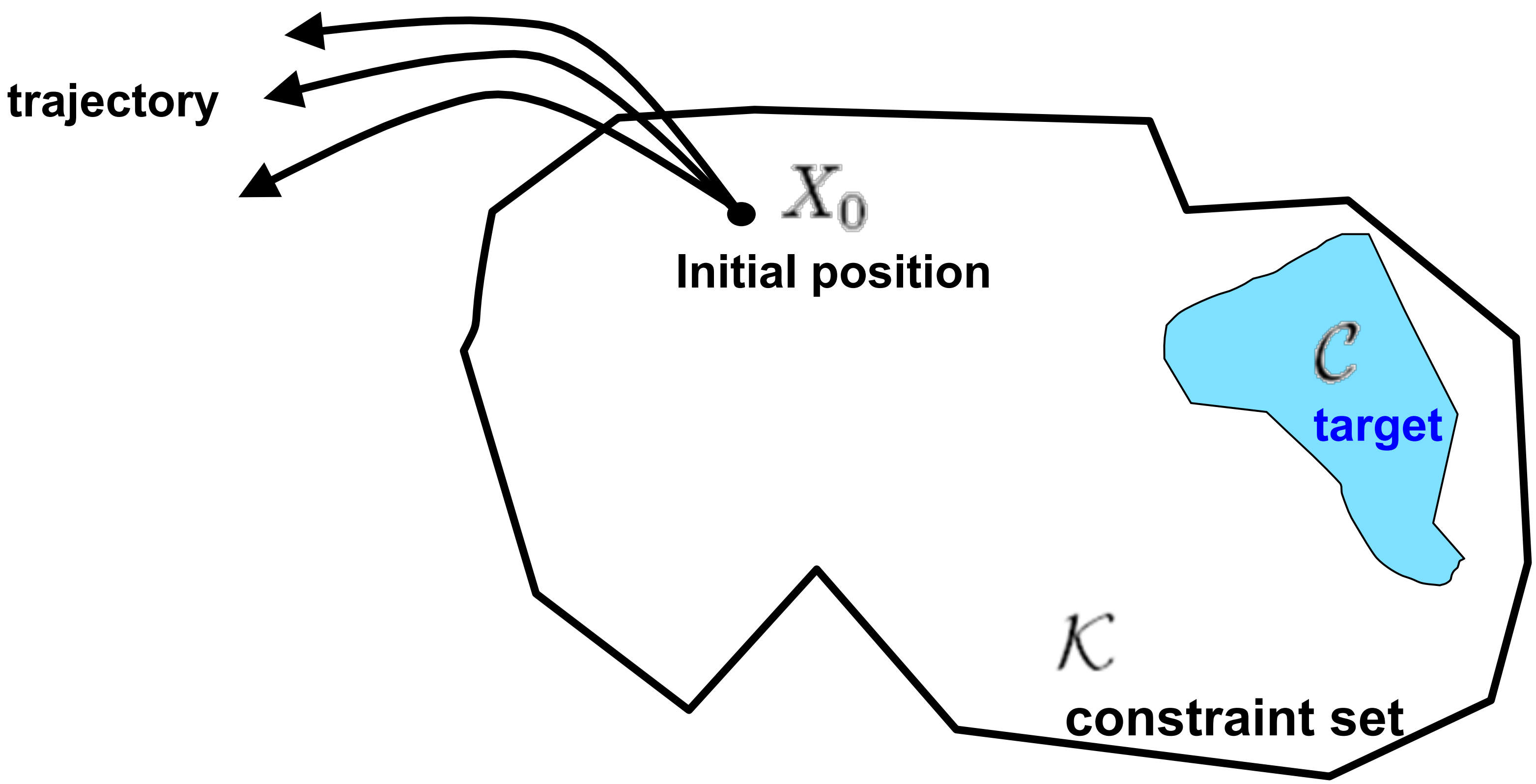
Construction of the solution to the HJB PDE

Main concept used: the Capture Basin

- **Consider a Differential inclusion** $\dot{x}(t) \in F(x(t))$
- **Solutions of this differential inclusion are trajectories.**
- **The capture basin of a target within a constraint set is the set of initial positions from which one can reach the target while staying in the constraint set.**



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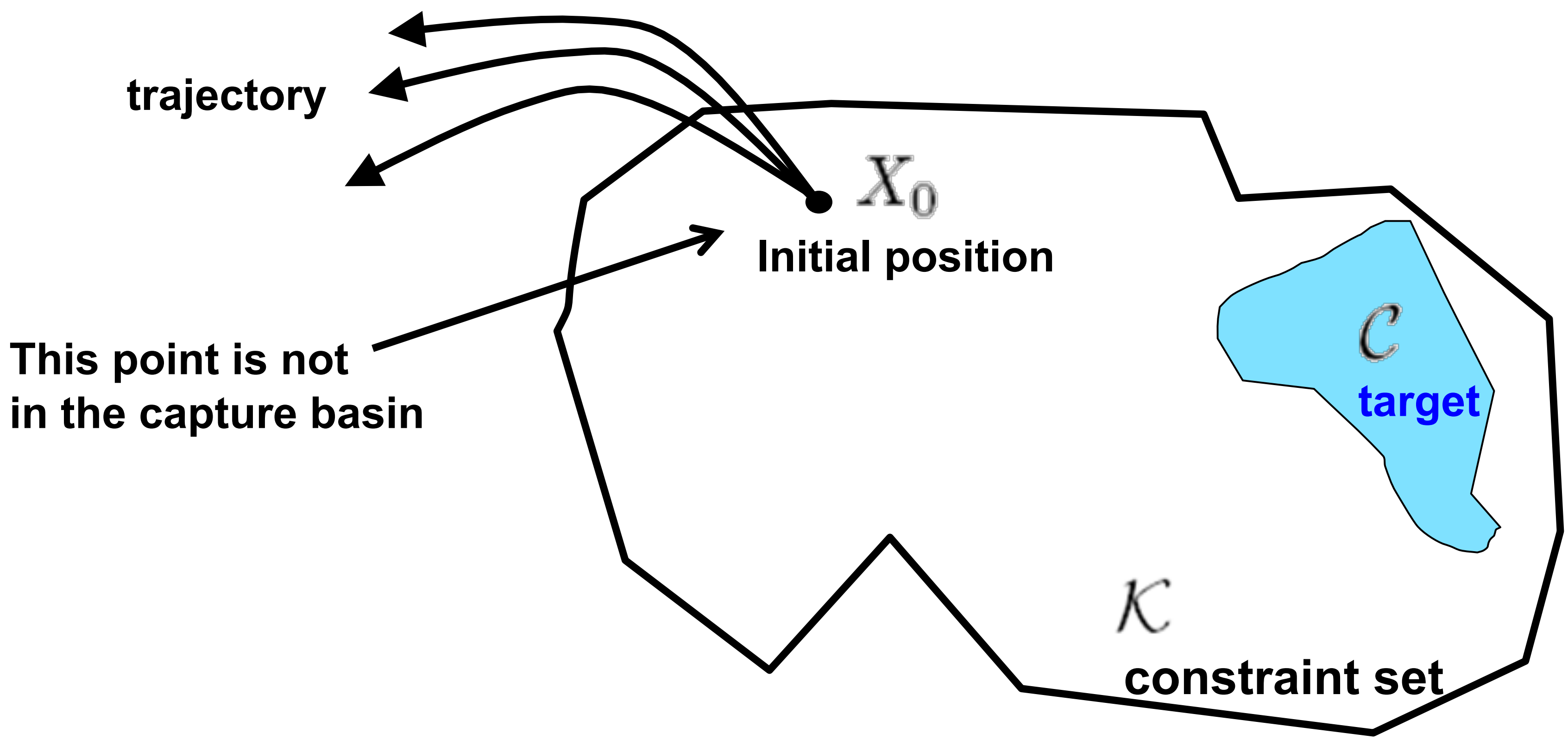

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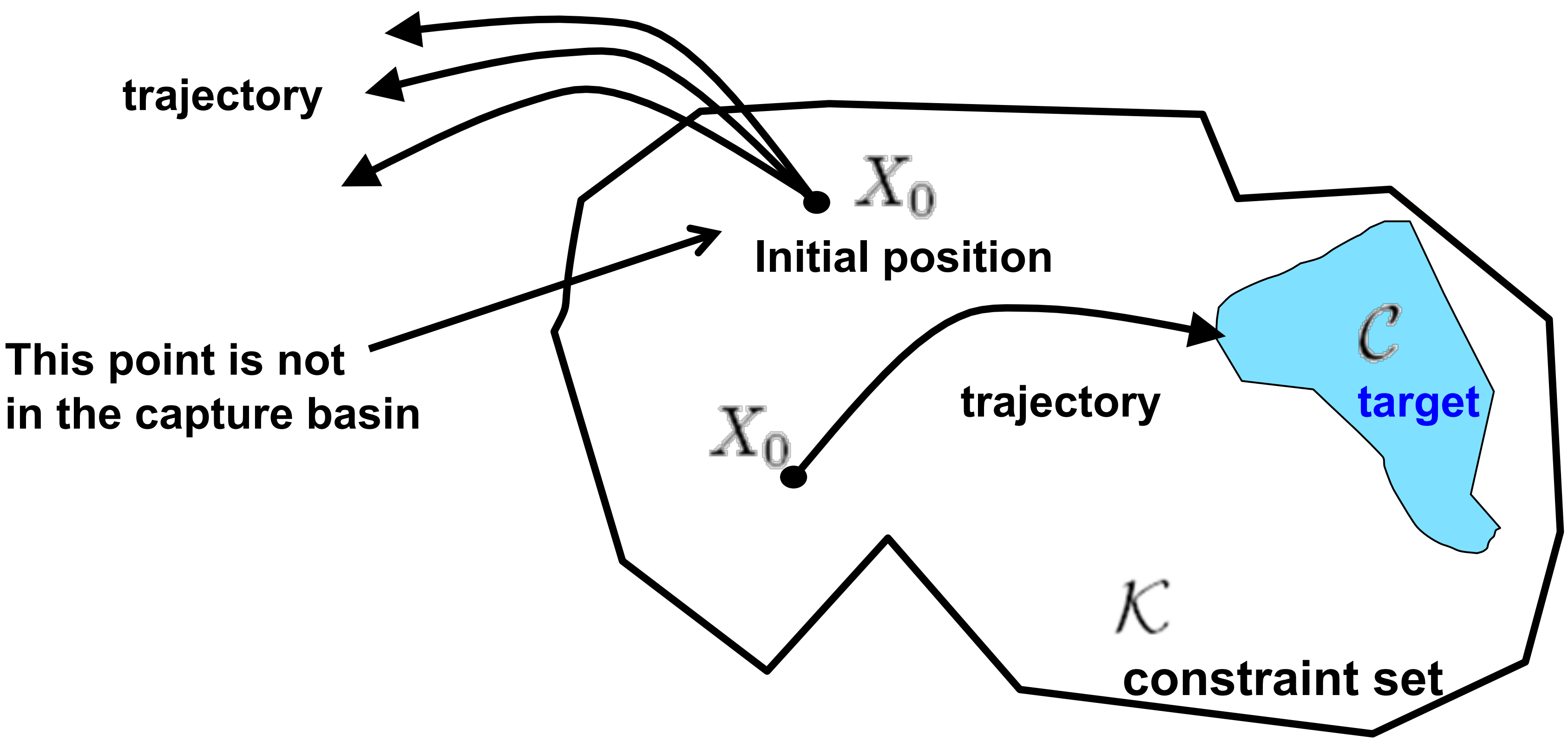

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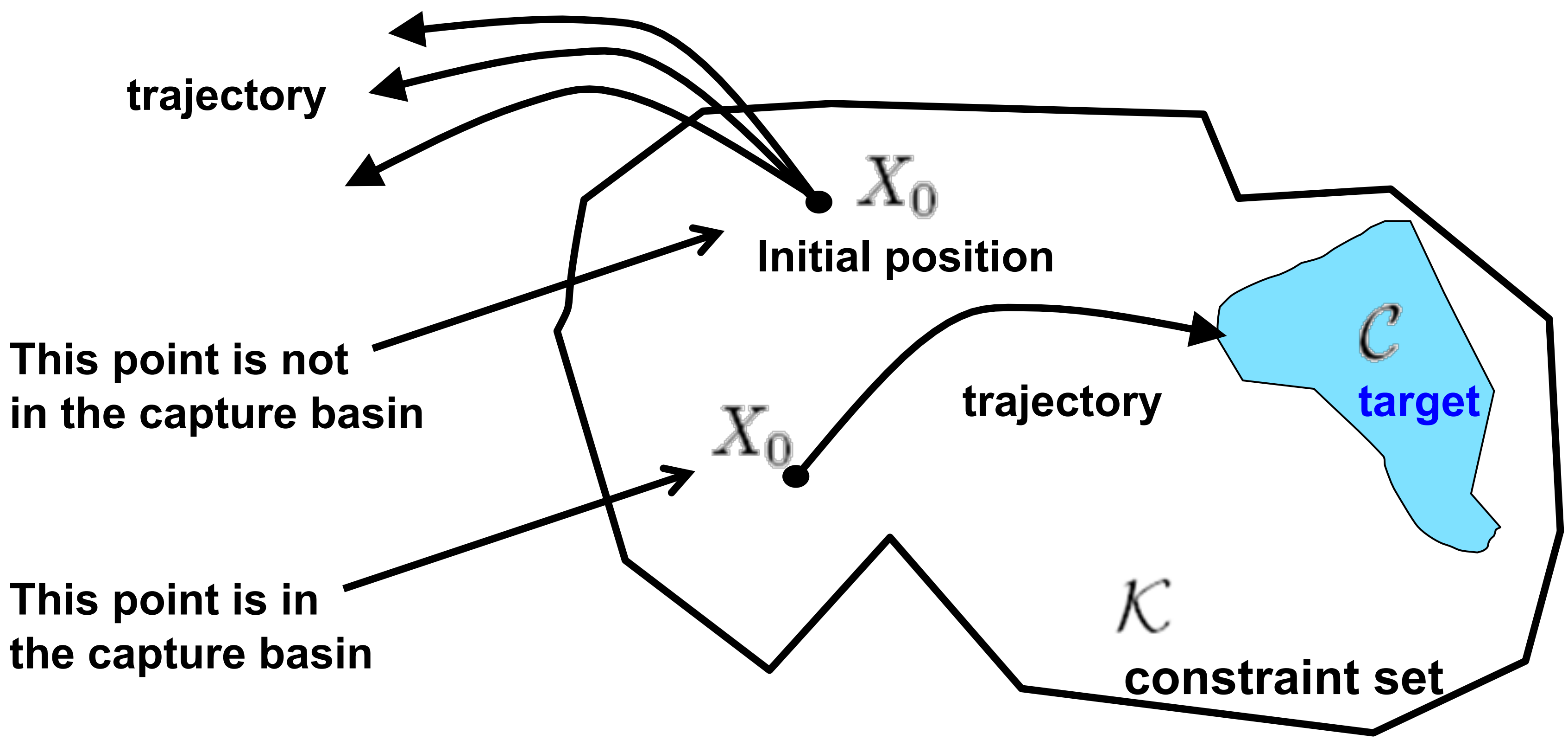

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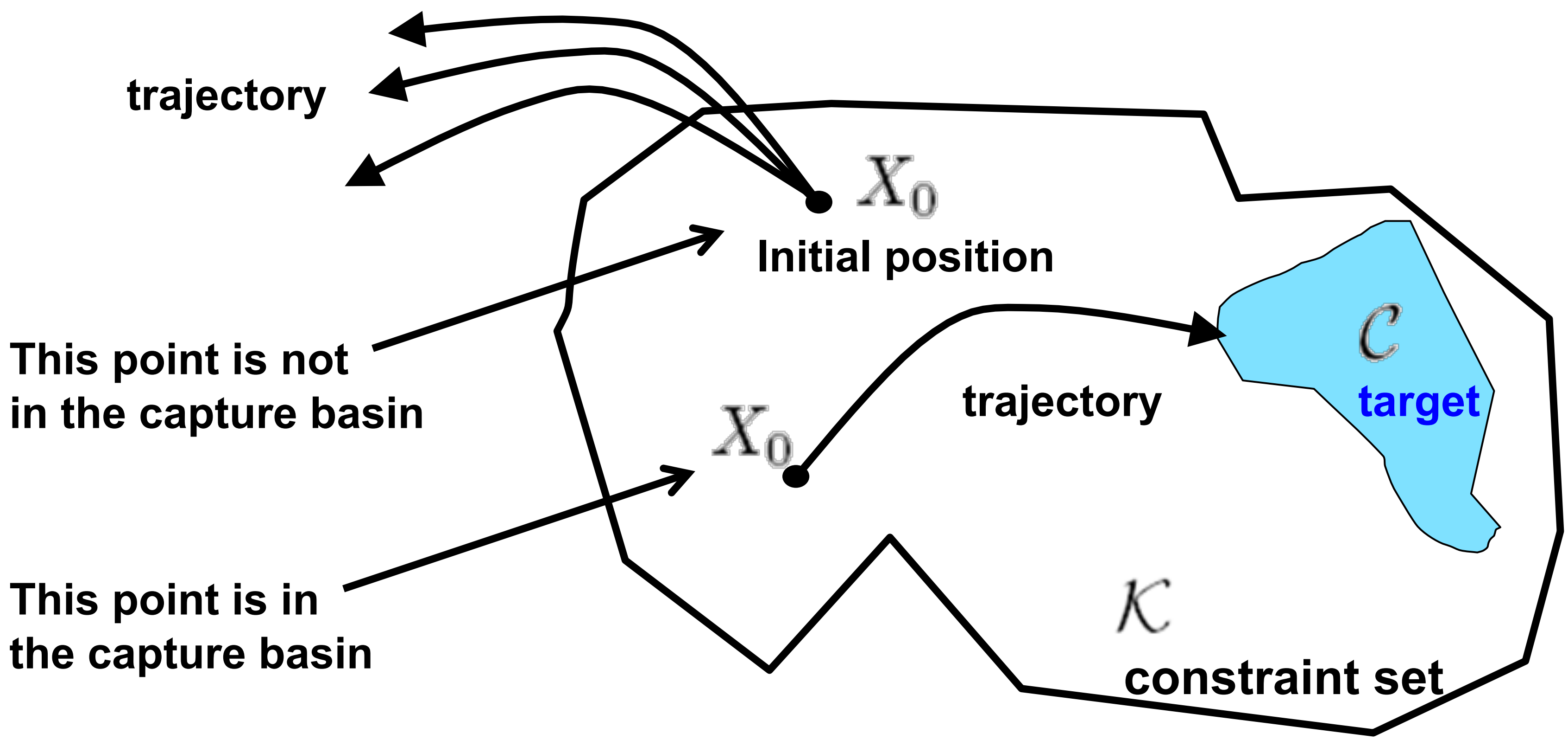

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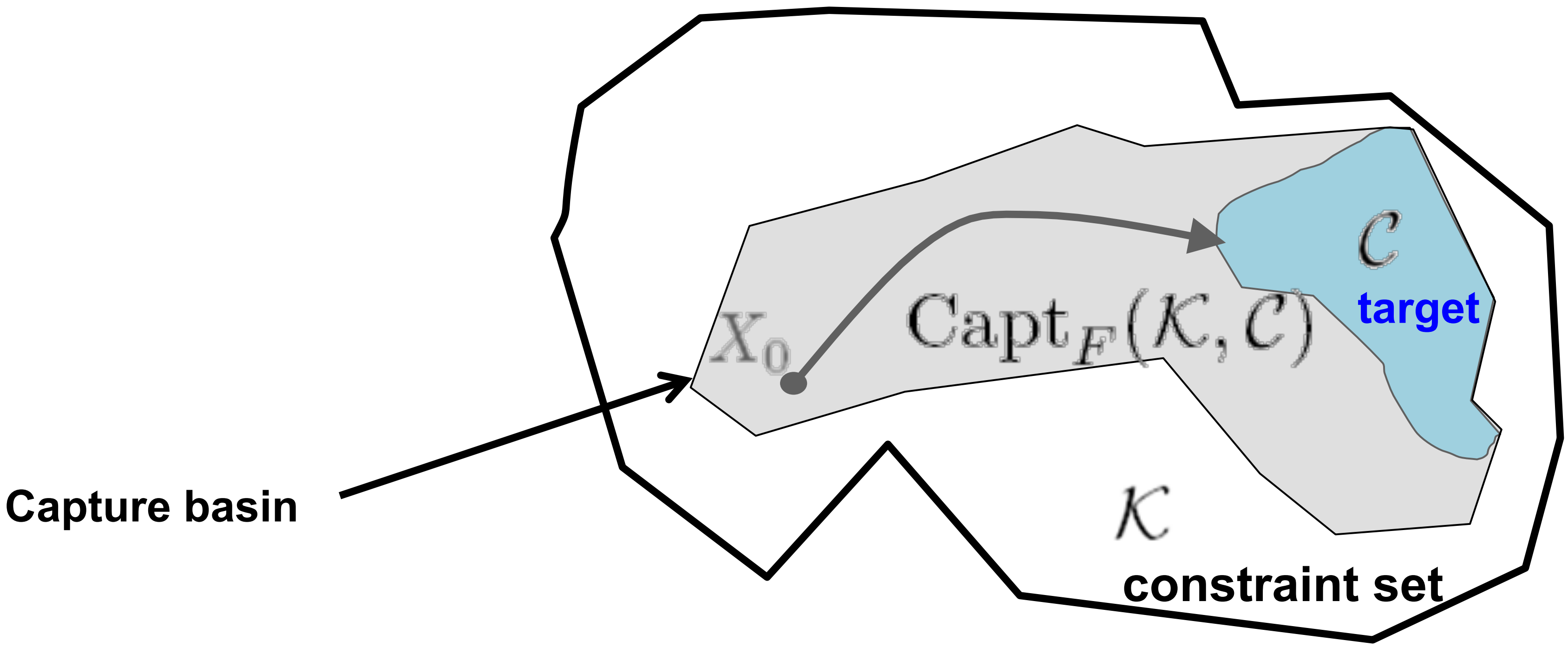
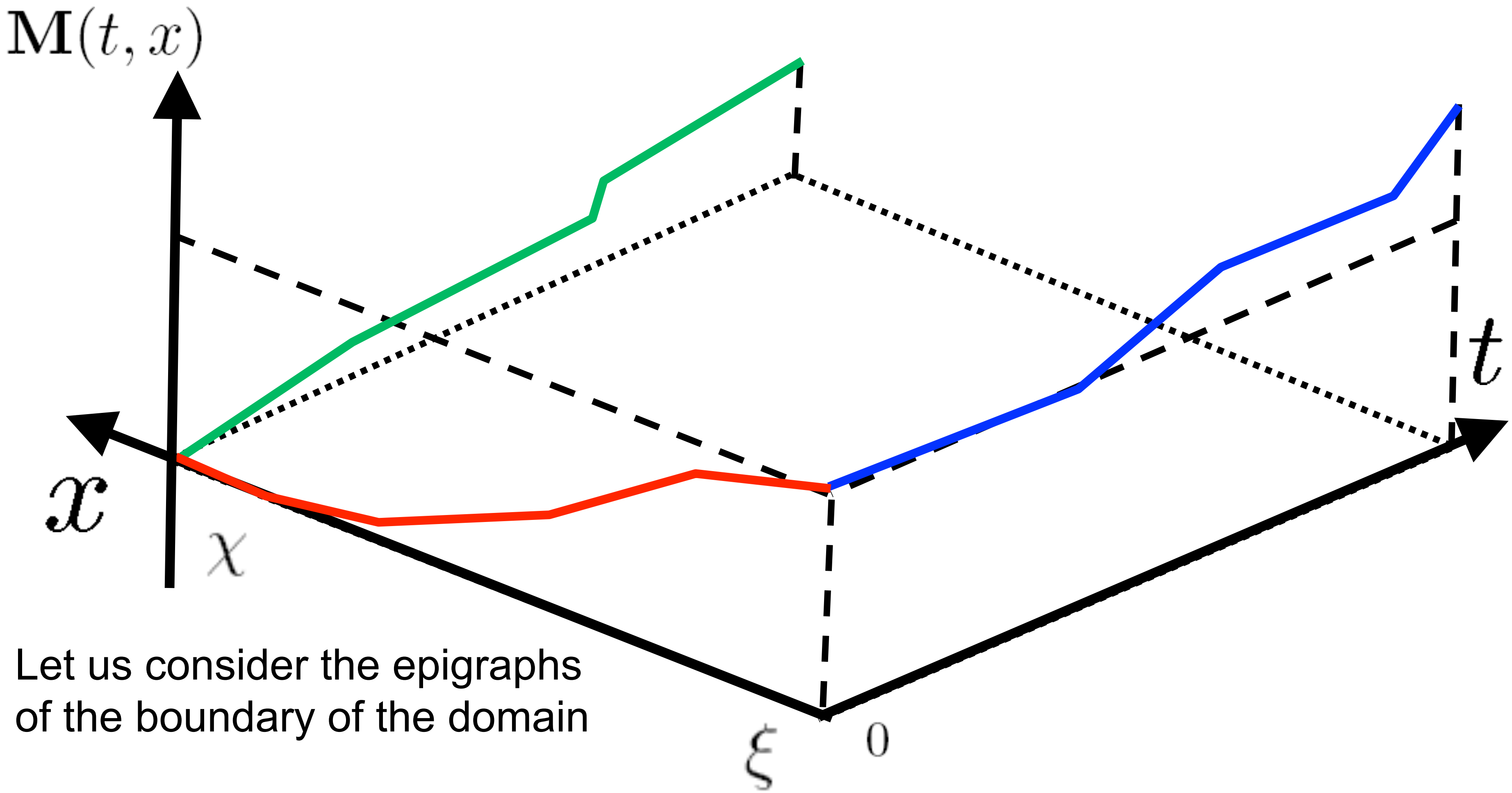
- **It is called** $\mathrm{Capt}_F(\mathcal{K}, \mathcal{C})$



Epigraphical characterization of the solution



Let us consider the epigraphs of the boundary of the domain

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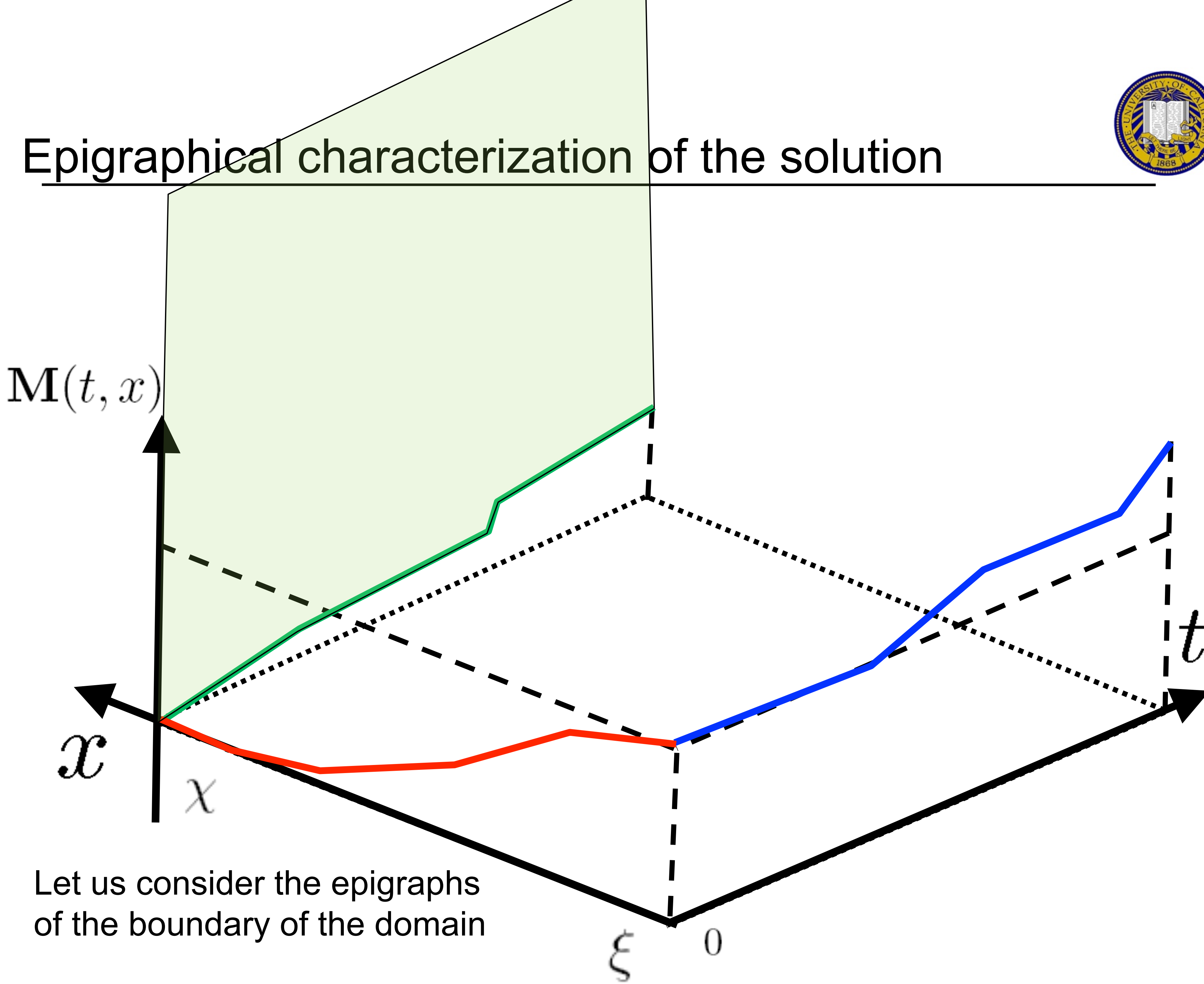

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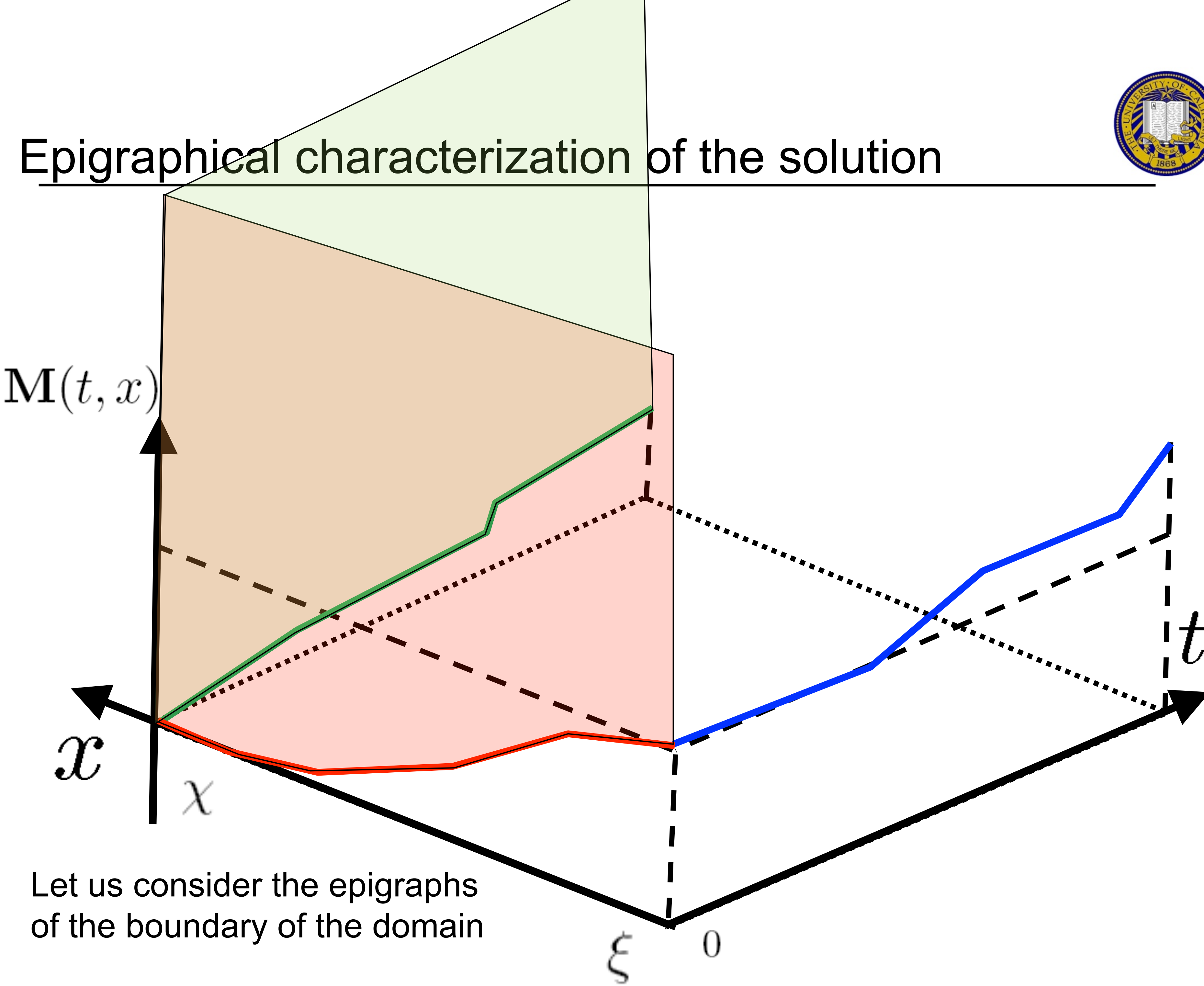

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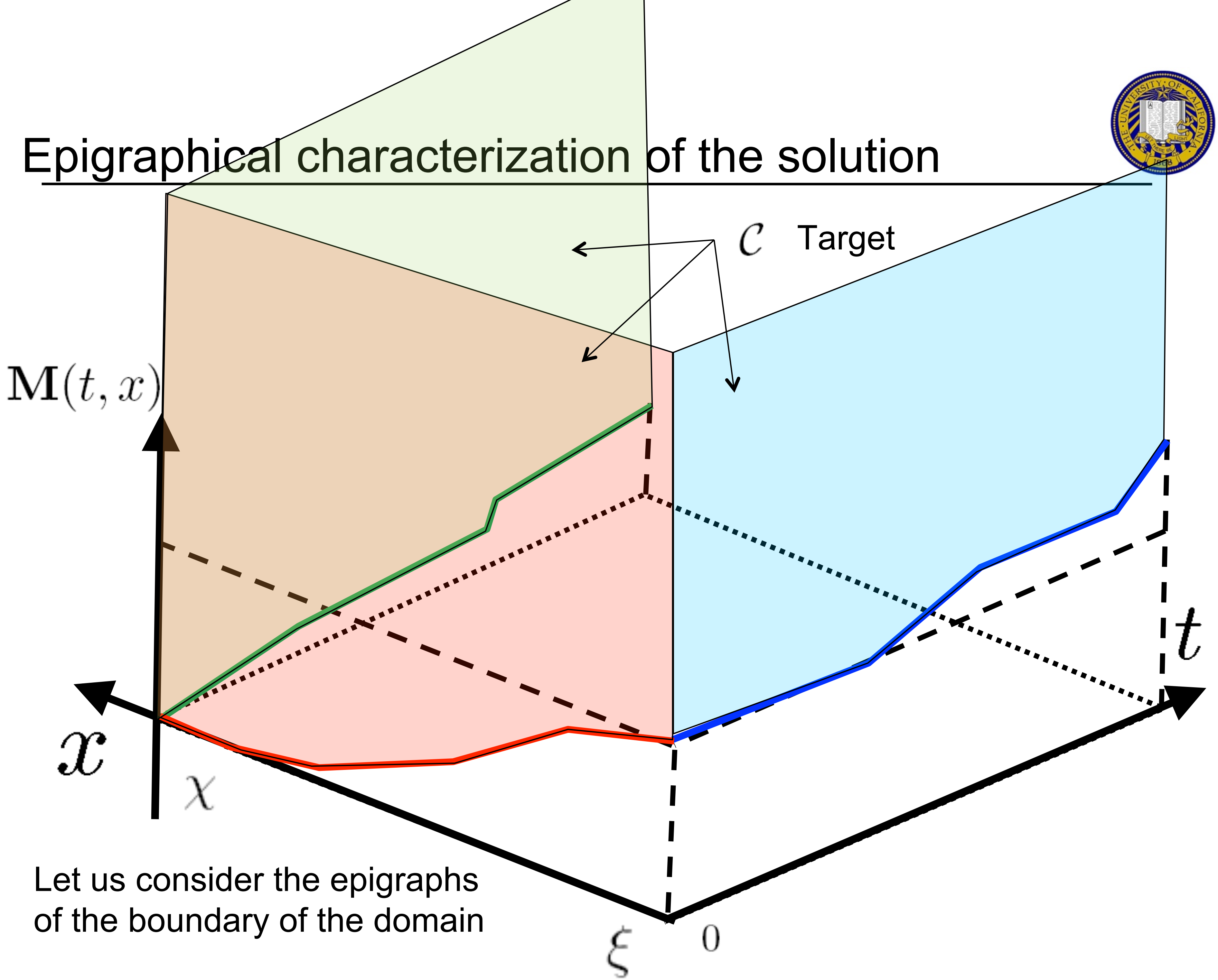

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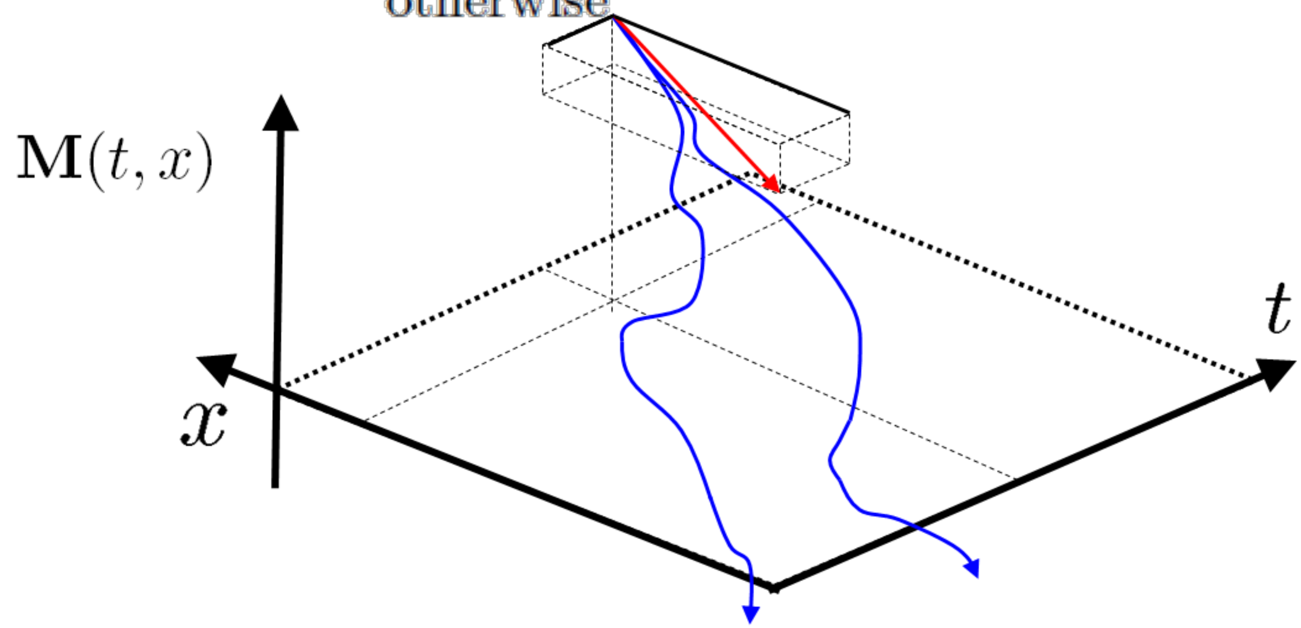
Construct an auxiliary dynamics

Consider the following set valued dynamics

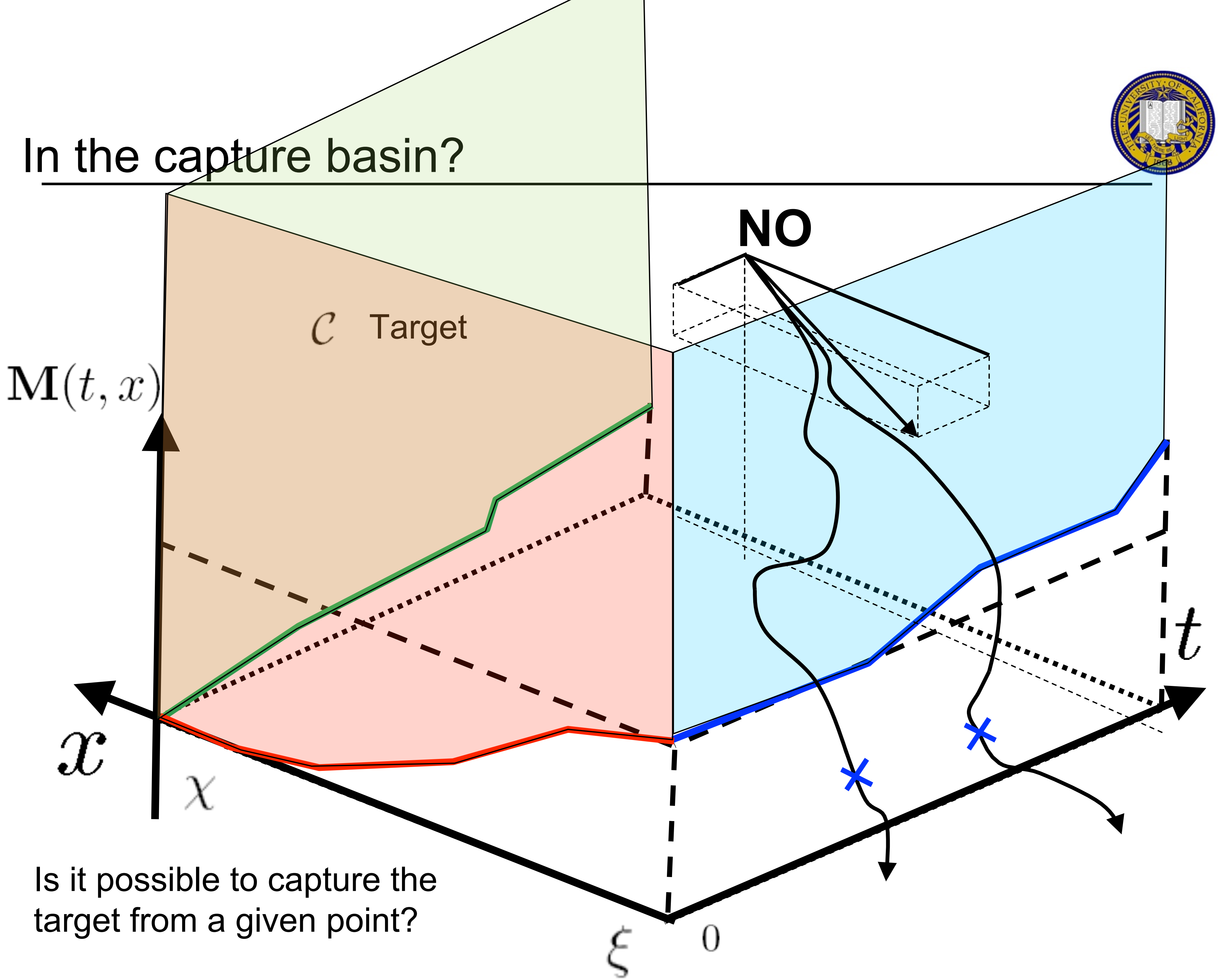
$$F := \begin{cases} \tau'(t) & = & -1 \\ x'(t) & = & u(t) \\ y'(t) & = & -\varphi^*(u(t)) \end{cases} \qquad \text{where } u(t) \in \mathrm{Dom}(\varphi^*)$$

Where the Fenchel transform of the Hamiltonian is given by:

$$\varphi^*(u) := \begin{cases} \sup\limits_{p \in \mathrm{Dom}(\psi)} [p \cdot u + \psi(p)] & \text{if } u \in [-\nu^\flat, \nu^\sharp] \\ +\infty & \text{otherwise} \end{cases}$$

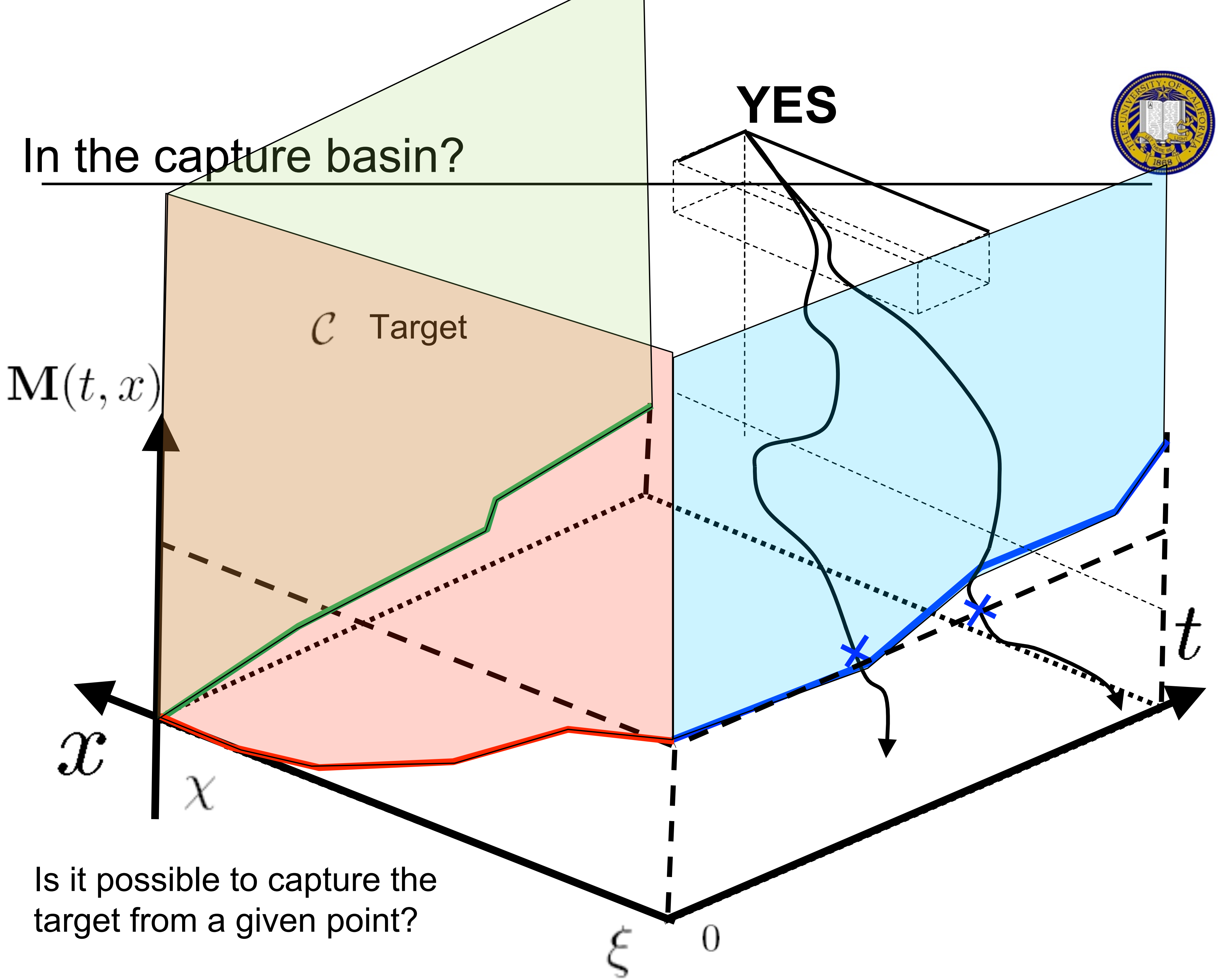


In the capture basin?



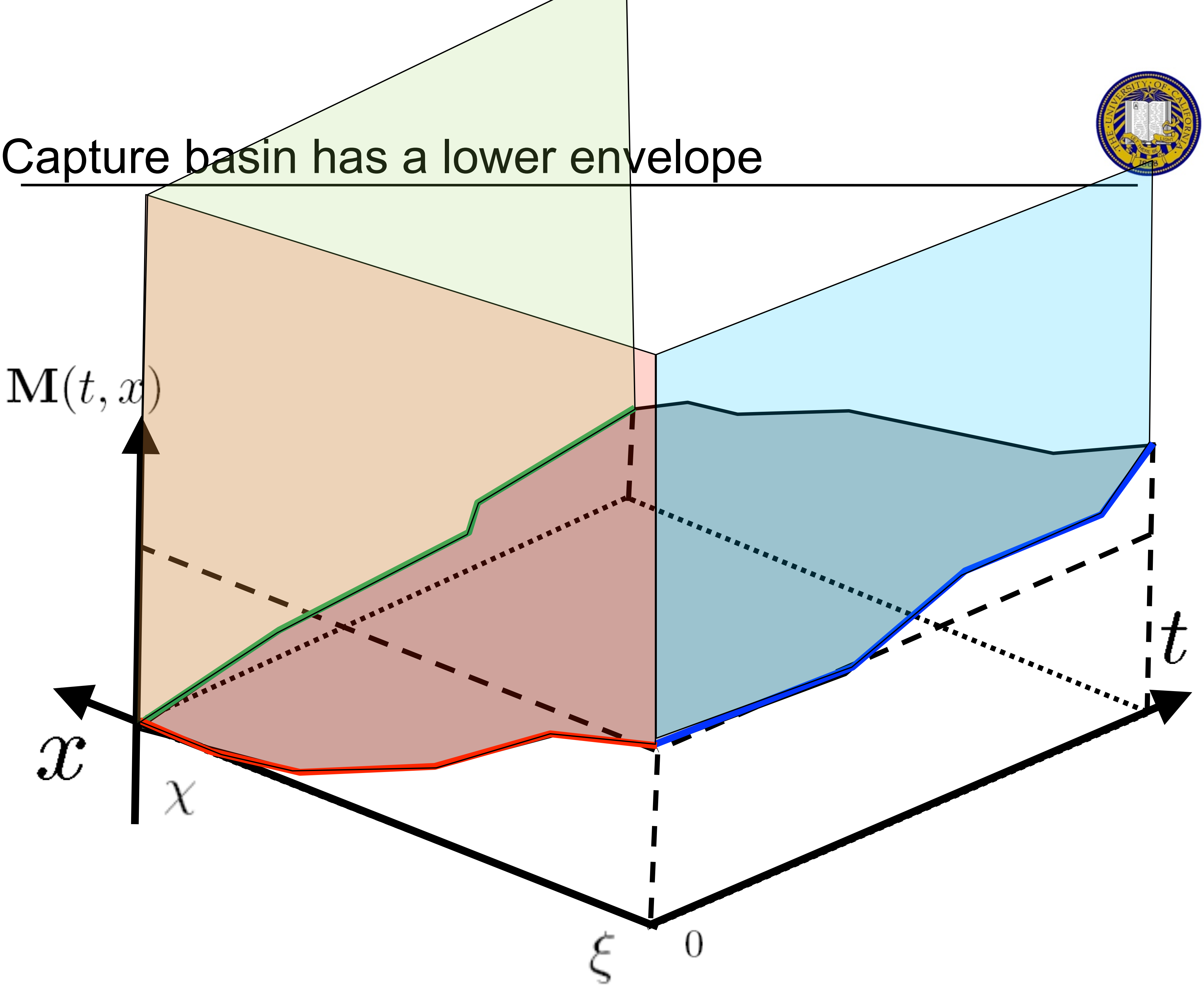
Is it possible to capture the target from a given point?

In the capture basin?

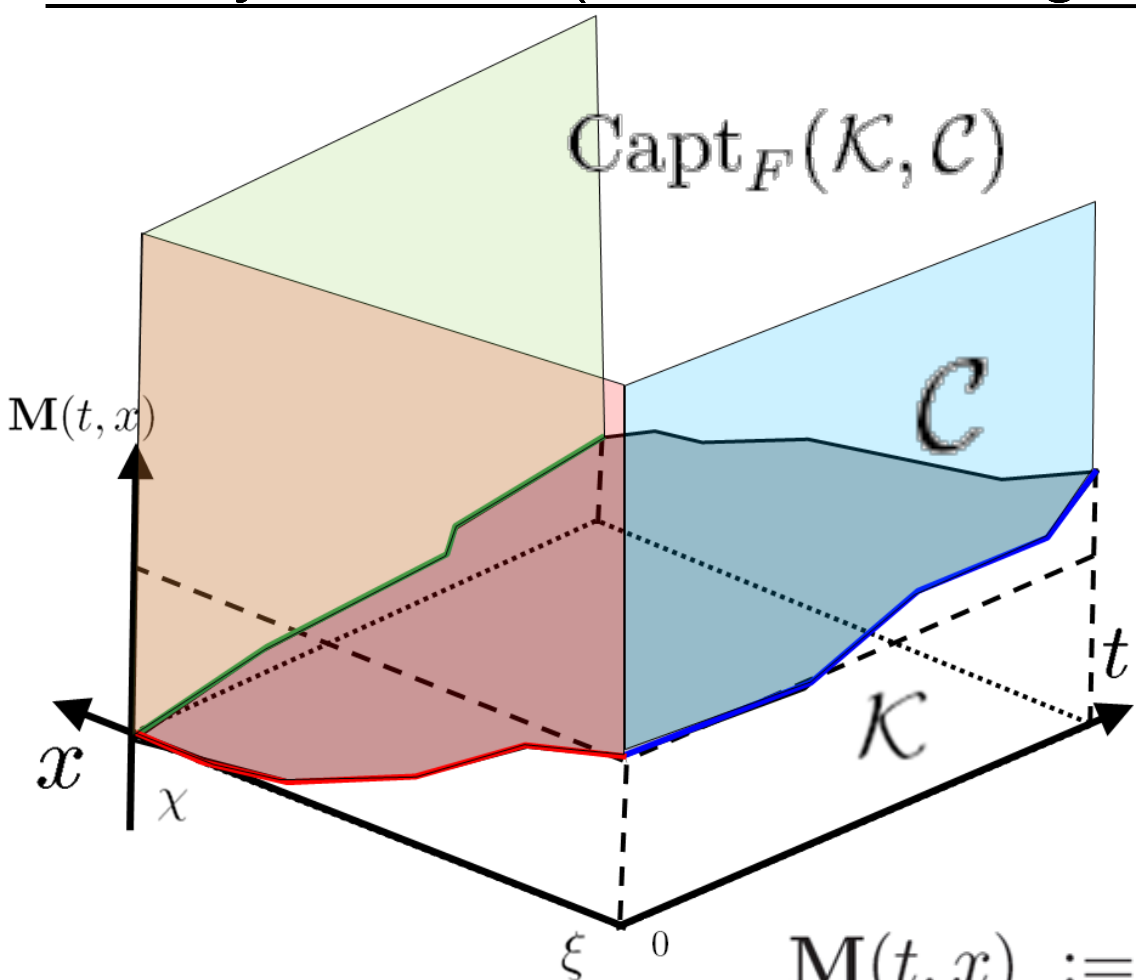


Is it possible to capture the target from a given point?

Capture basin has a lower envelope



Viability solution (definition using capture basin)



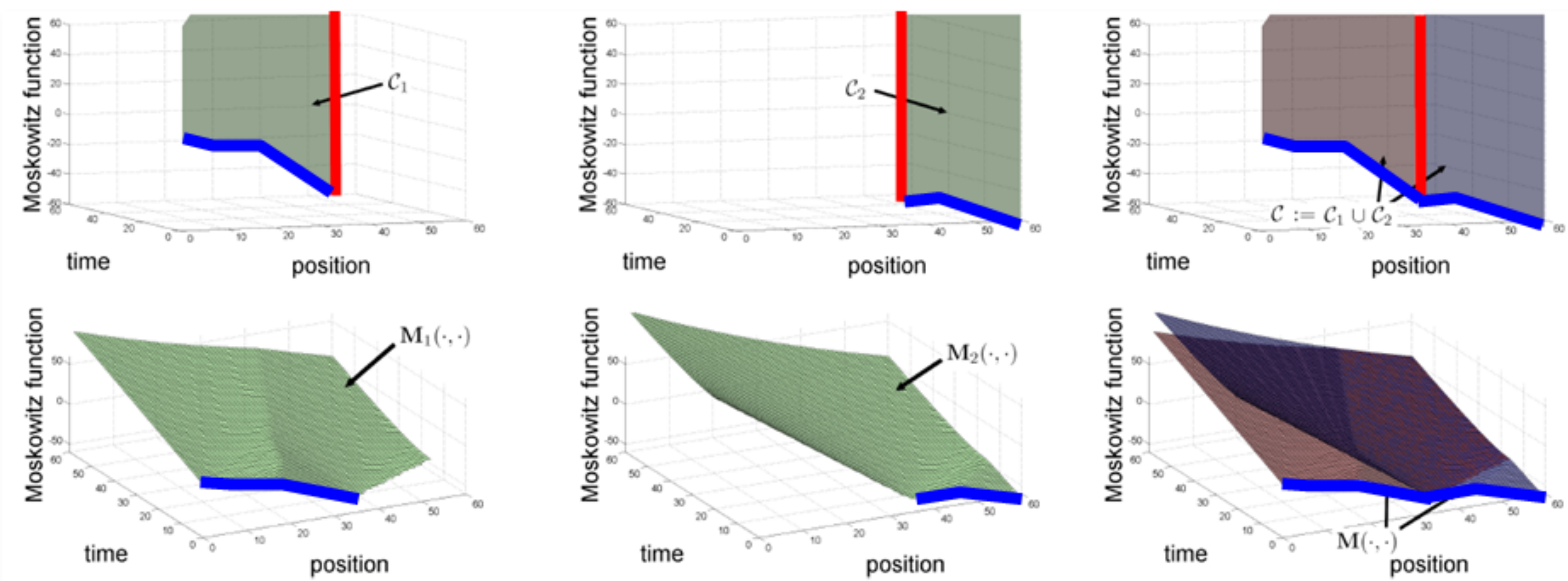
$$\mathbf{M}(t,x) := \inf_{(t,x,y)\in \mathrm{Capt}_F(\mathcal{K},\mathcal{C})} y$$

The inf-morphism property

The union property for capture basins $\mathrm{Capt}_F\left(\mathcal{K}, \bigcup_{i\in I} \mathcal{C}_i\right) = \bigcup_{i\in I} \mathrm{Capt}_F(\mathcal{K}, \mathcal{C}_i)$

translates into an inf-morphism property

$$\forall\, t \geq 0,\ x \in X,\quad \mathbf{M_c}(t,x) \ =\ \inf_{i\in I} \mathbf{M}_{\mathbf{c}_i}(t,x)$$



Tangential property of the capture basin

This defines a new class of solutions to the HJ PDE:

$$\mathbf{M}(t,x) \ := \inf_{(t,x,y)\in \mathrm{Capt}_{F}(\mathcal{K},\mathcal{C})} y$$

The solution provided by this formula is a lower semicontinuous function. It is the solution to the HJ PDE considered before, in a weaker sense than the viscosity solution. This solution is called the Barron/Jensen-Frankowska (B/J-F) solution.

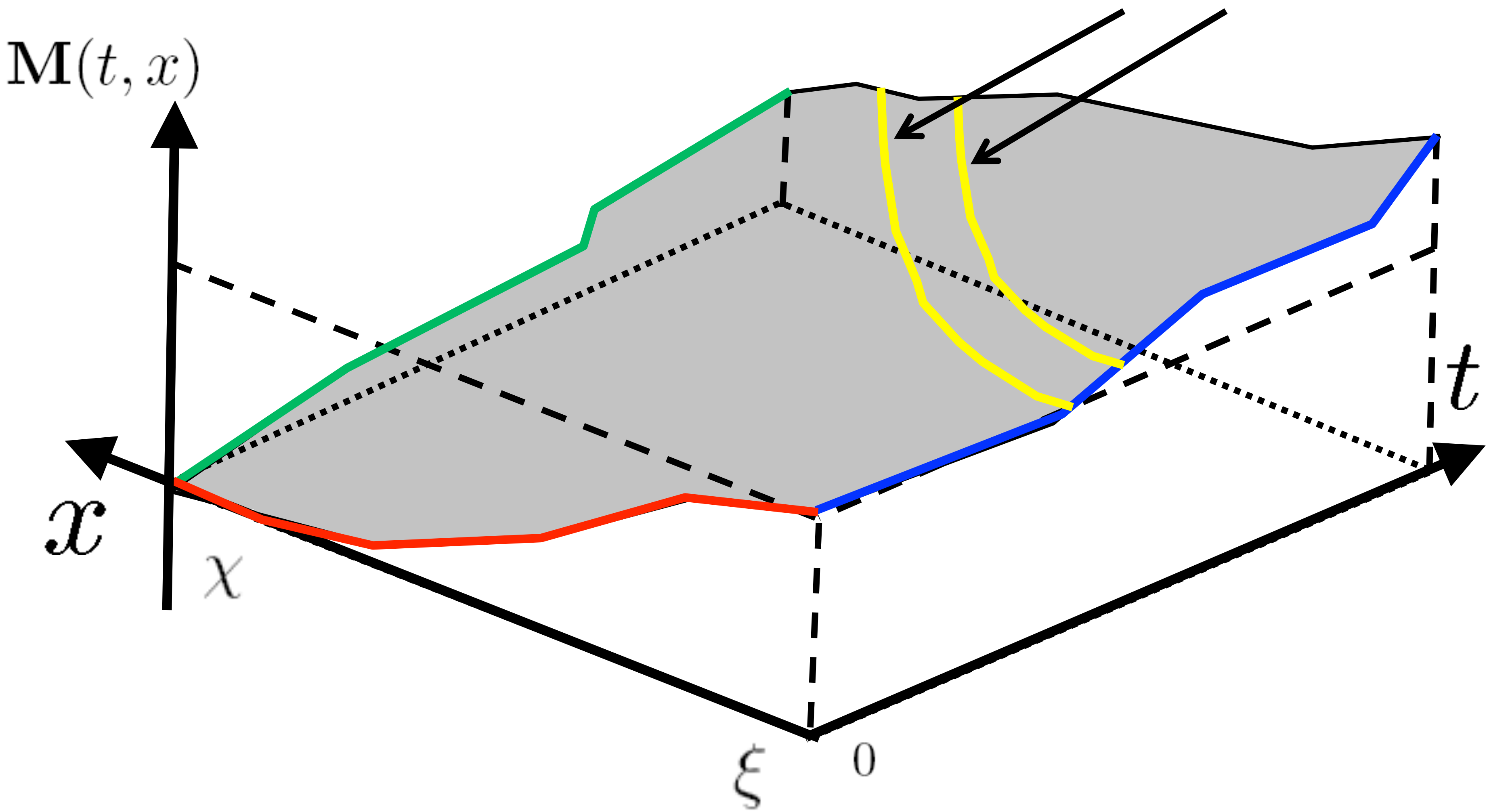
B/J-F solutions require only the lower semicontinuity of the solution.

In particular: whenever M is differentiable the tangential properties of the capture basin imply:

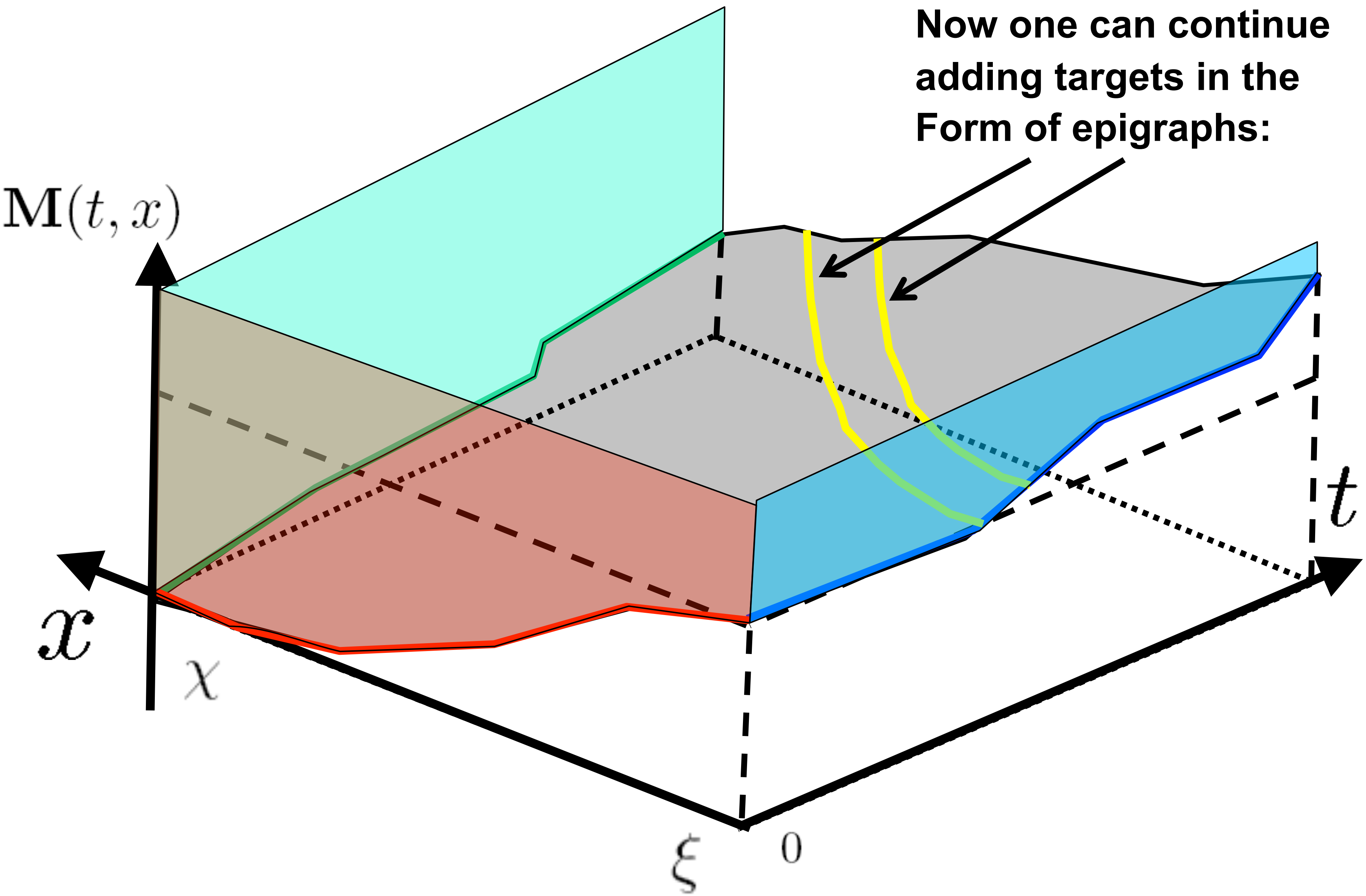
$$\forall (t,x) \in \mathrm{Dom}(\mathbf{M_c})\backslash \mathrm{Dom}(\mathbf{c}) \quad \frac{\partial \mathbf{M_c}(t,x)}{\partial t} - \psi\left(-\frac{\partial \mathbf{M_c}(t,x)}{\partial x}\right) \ = \ 0$$

Adding trajectories is equivalent to adding epigraphs

Very often, drivers drive in violation of the LWR HJ PDE model (because of external disturbances not included in the model: accidents, distraction, excessive speed, etc.). This can be measured:

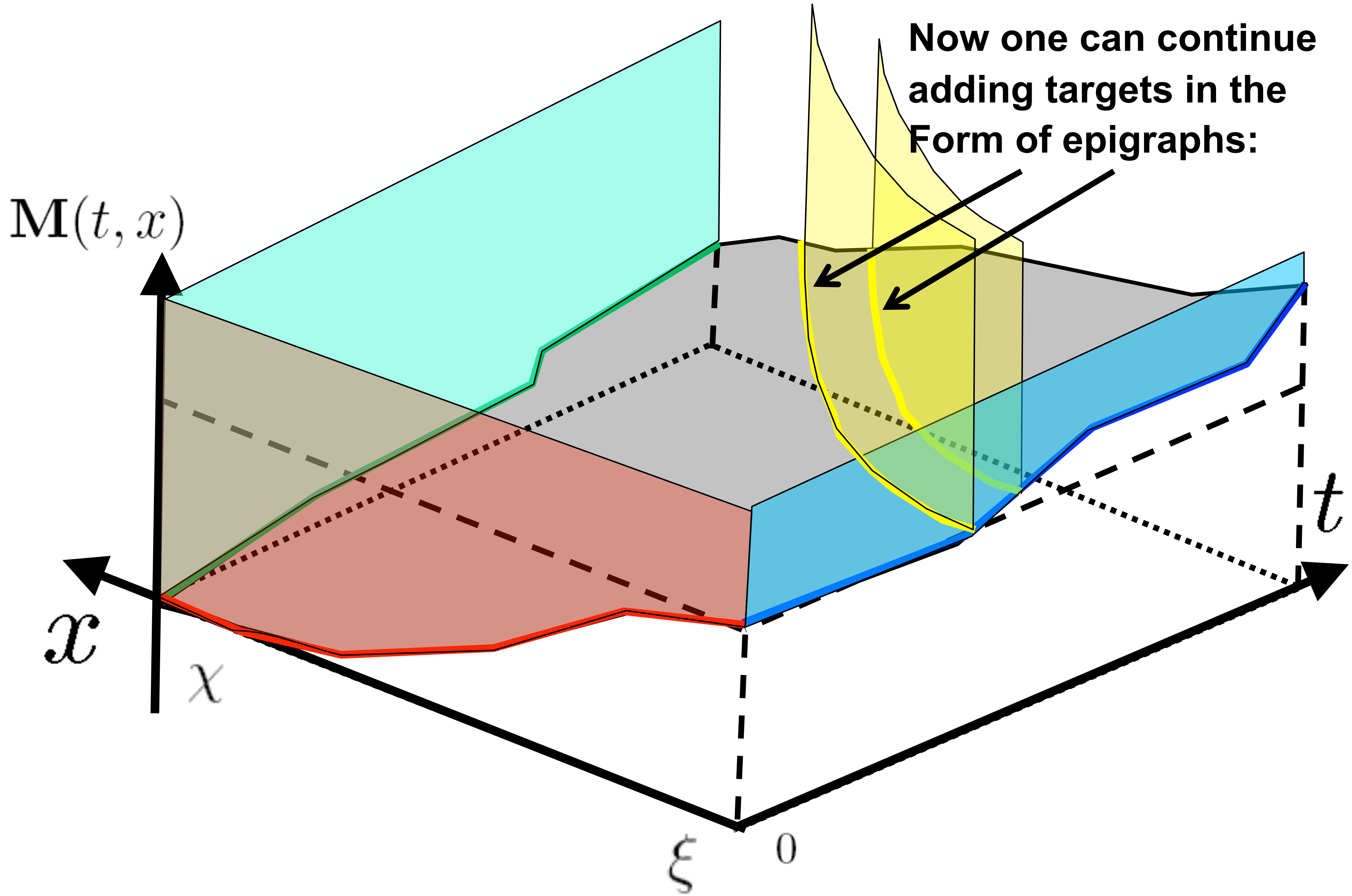


Adding trajectories is equivalent to adding epigraphs



Adding trajectories is equivalent to adding epigraphs

Now one can continue adding targets in the Form of epigraphs:



Outline

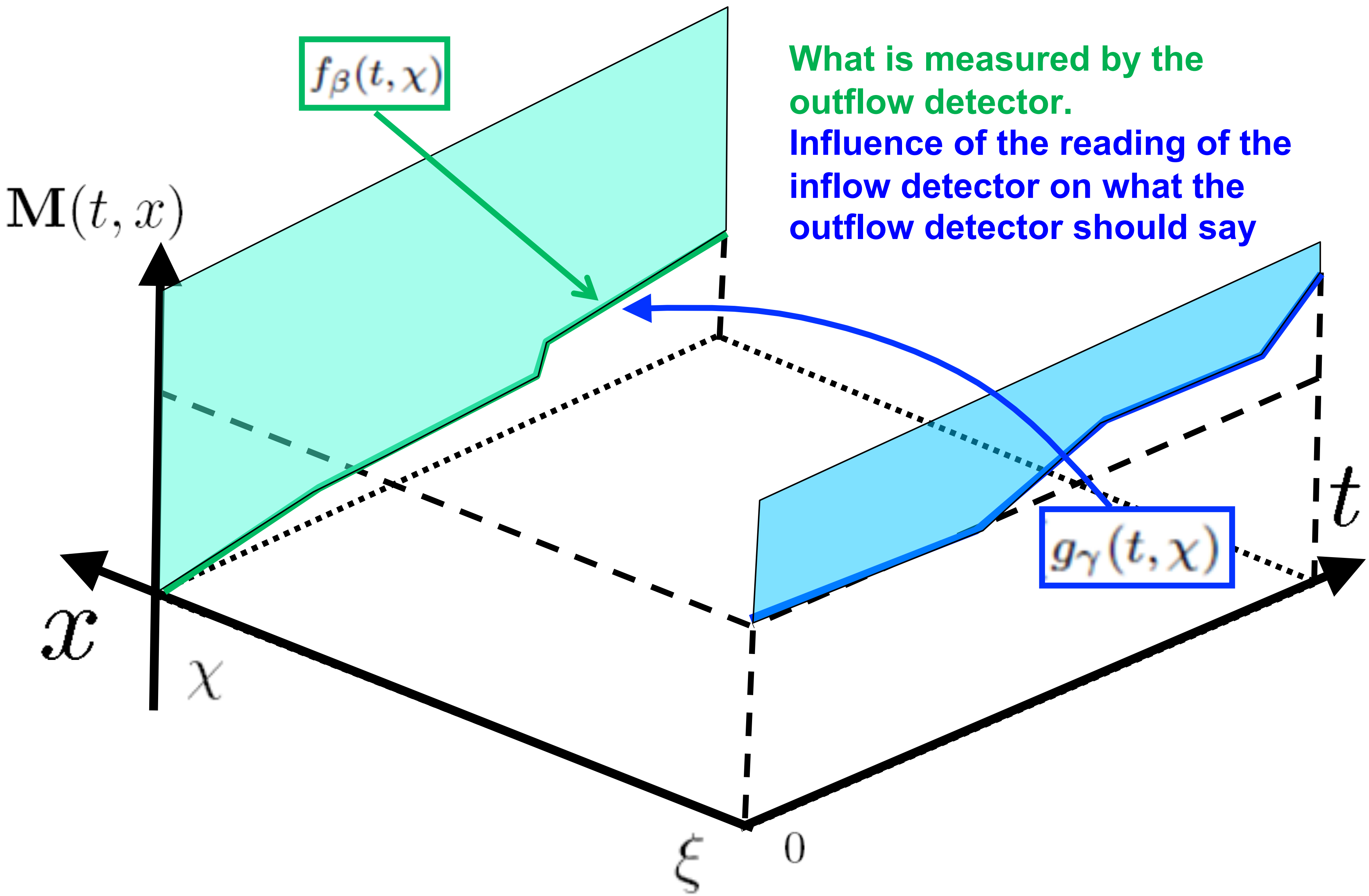
1. **Traffic information systems at the age of web 2.0**

2. **Mobile Millennium**

3. **Inverse modeling and data assimiliation**
    1. **A short introduction to traffic modeling**
    2. **The Moskowitz Hamilton-Jacobi equation**
    3. **Internal boundary conditions using the inf-morphism property**
    4. **Data assimilation in a privacy aware environment**

4. **Beyond Mobile Millennium**
    1. **Air**
    2. **Earthquakes**
    3. **Water**

Comparing information



Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i unknown

$$\begin{cases} (i) \qquad \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta \\ \\ \\ \end{cases}$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Condition on the outflow due to the inflow measurement

$$\left\{ \begin{array}{ll} (i) & \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta \end{array} \right.$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Reading of our outflow sensor

$$\left\{ \begin{array}{ll} (i) & \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta \end{array} \right.$$

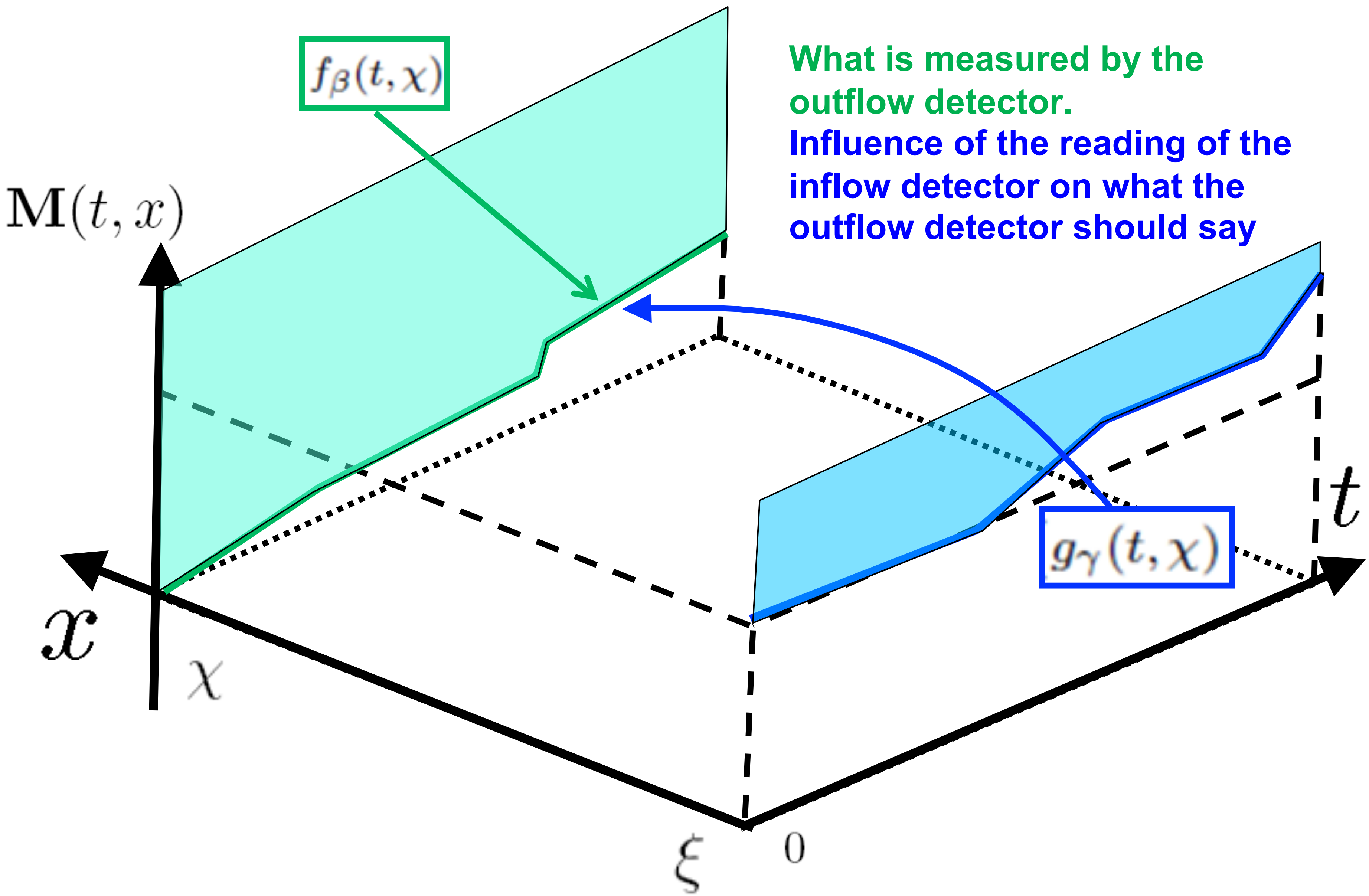
Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels M_i **unknown**

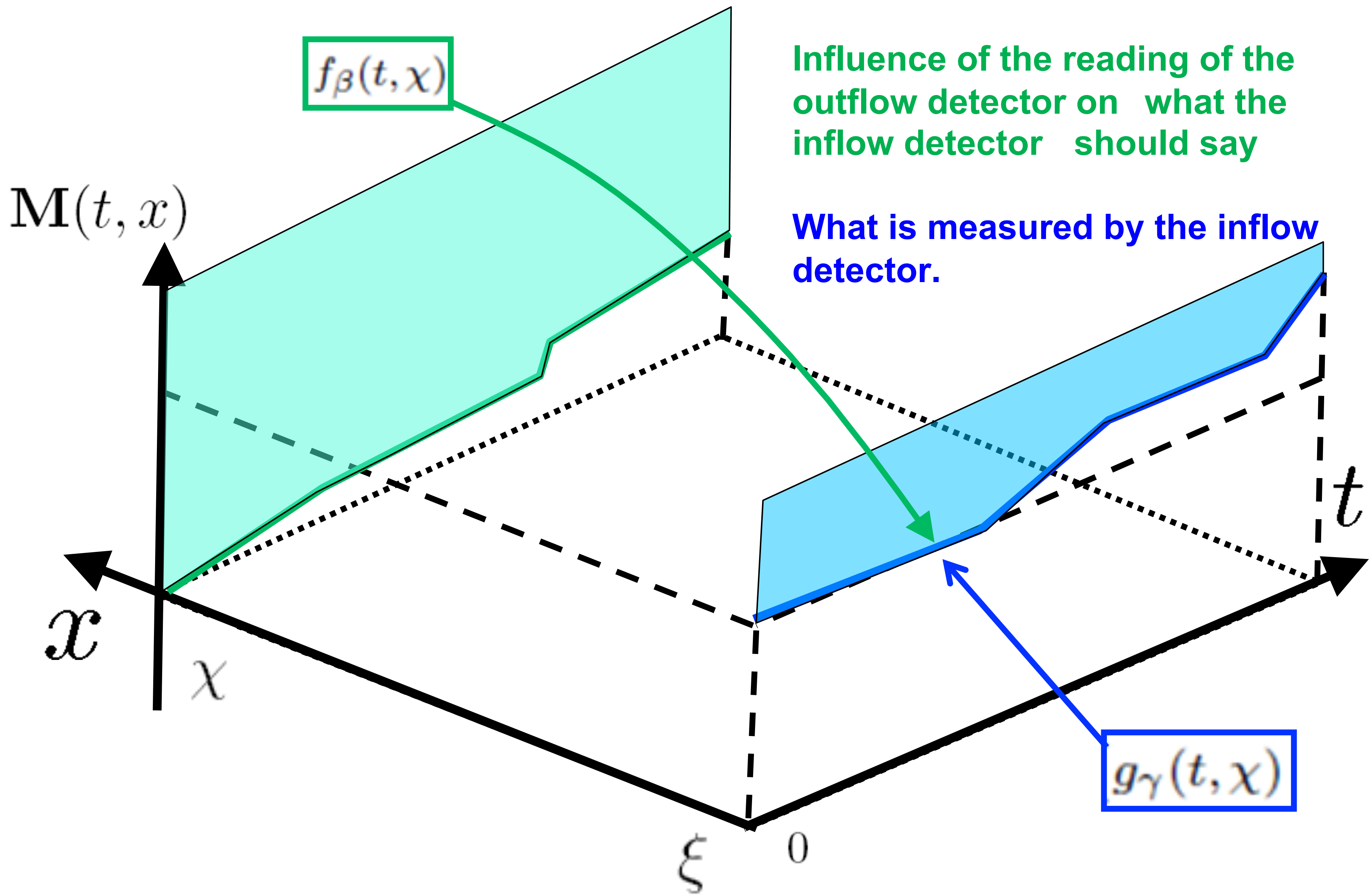
Initial number of vehicles (to be estimated)

$$\left\{ (i) \quad \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \boxed{\Delta} \right.$$

Comparing information



Comparing information



Data assimilation using linear programming

Initial condition unknown -Δ

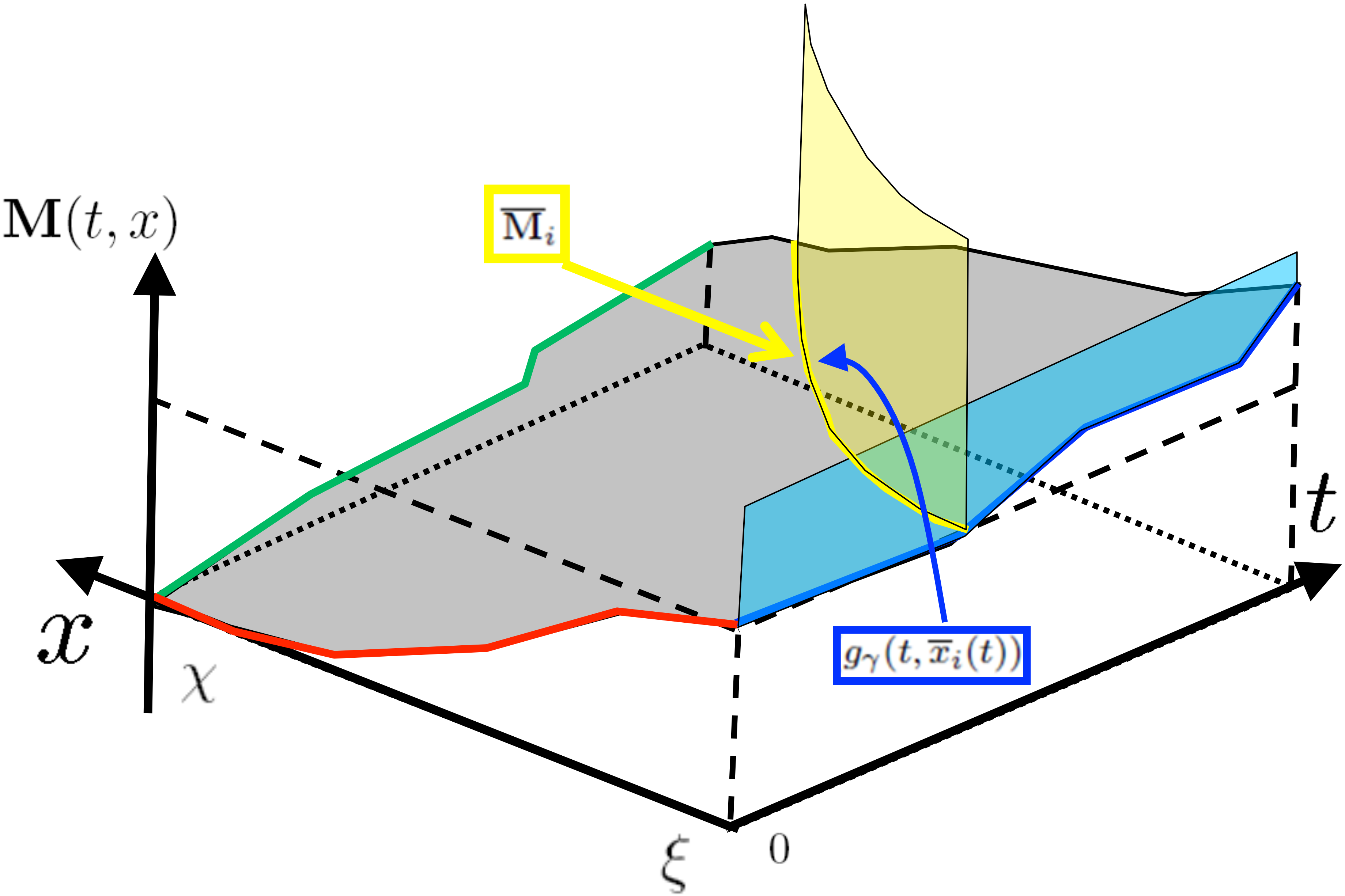
Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Symmetric condition

$$\begin{cases} (i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta \\ (ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) \end{cases}$$

Adding trajectories is equivalent to adding epigraphs



Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Influence of the inflow measurement on the label of the trajectory

$$\left\{\begin{array}{lll} (i) & \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\ (ii) & \Delta \geq \sup_{t \in \mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\ (iii) & \inf_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(g_\gamma(t, \overline{x}_i(t))\right) \geq \overline{\mathbf{M}}_i & \forall i \in I \end{array}\right.$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Label of vehicle i
(to be estimated)

$$
\left\{
\begin{array}{lll}
(i) & \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t \in \mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
(iii) & \inf_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(g_\gamma(t, \overline{x}_i(t))\right) \geq \boxed{\overline{\mathrm{M}}_i} & \forall i \in I
\end{array}
\right.
$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Similar condition between outflow and label estimated by the trajectory measurement

$$
\begin{cases}
(i) & \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t \in \mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi) \right) & \\
(iii) & \inf_{t \in [\underline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(g_\gamma(t, \overline{x}_i(t)) \right) \geq \overline{\mathrm{M}}_i & \forall i \in I \\
(iv) & \overline{\mathrm{M}}_i \geq \sup_{t \in \mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi) \right) & \forall i \in I
\end{cases}
$$

Data assimilation using linear programming

Initial condition unknown -Δ

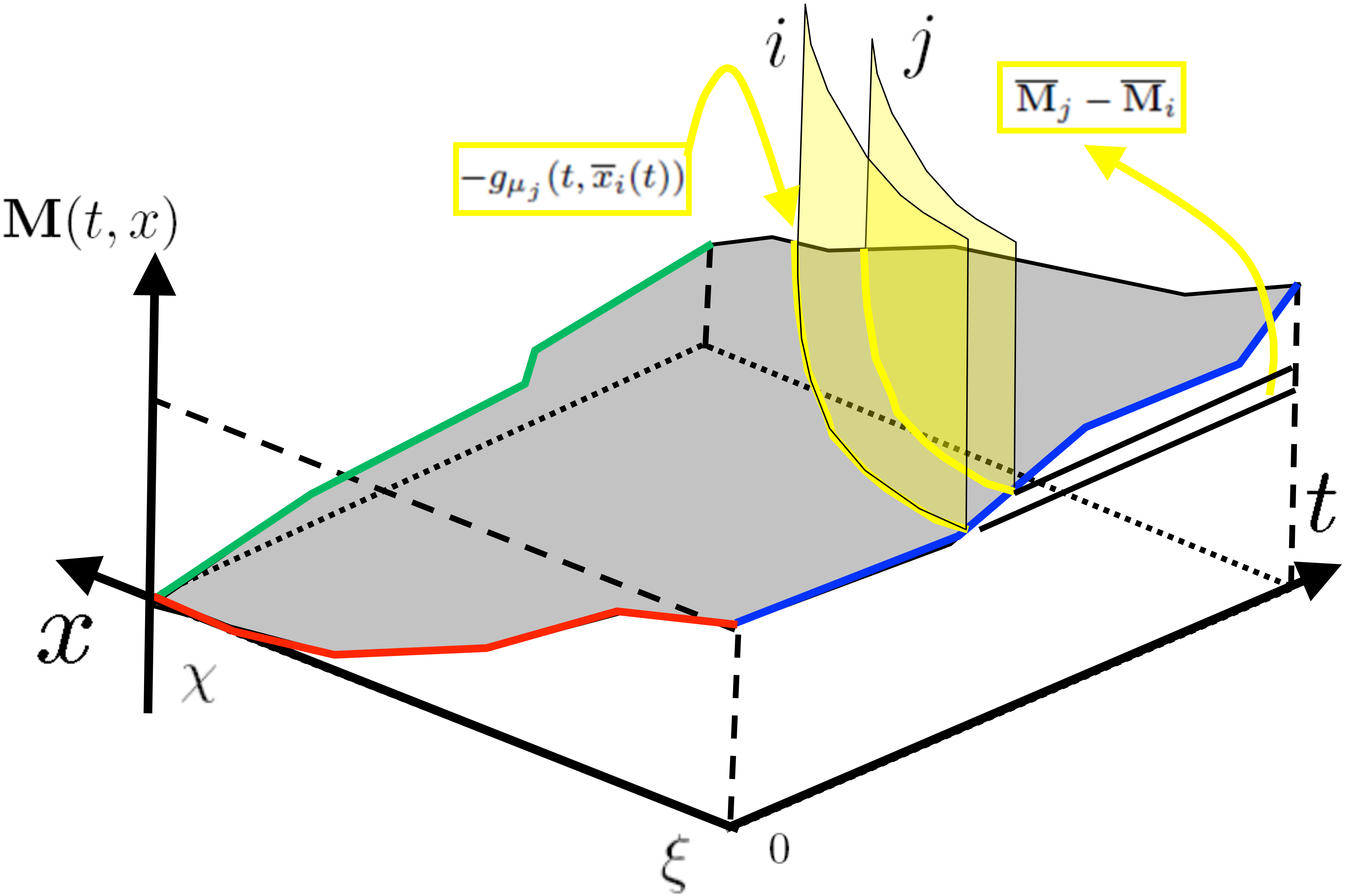
Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Similar conditions

$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathbf{M}}_i & \forall i \in I \\
(iv) & \overline{\mathbf{M}}_i \geq \sup_{t\in\mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi)\right) & \forall i \in I \\
(v) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\beta(t,\overline{x}_i(t))\right) \geq -\Delta + \overline{\mathbf{M}}_i & \forall i \in I \\
(vi) & \overline{\mathbf{M}}_i - \Delta \geq \sup_{t\in\mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi)\right) & \forall i \in I
\end{cases}
$$

Adding trajectories is equivalent to adding epigraphs



Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Difference in possible labels of vehicle i and j (unknown)

$$
\begin{cases}
(i) & \inf_{t \in \mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi) \right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t \in \mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi) \right) & \\
(iii) & \inf_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(g_\gamma(t, \overline{x}_i(t)) \right) \geq \overline{\mathrm{M}}_i & \forall i \in I \\
(iv) & \overline{\mathrm{M}}_i \geq \sup_{t \in \mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi) \right) & \forall i \in I \\
(v) & \inf_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(g_\beta(t, \overline{x}_i(t)) \right) \geq -\Delta + \overline{\mathrm{M}}_i & \forall i \in I \\
(vi) & \overline{\mathrm{M}}_i - \Delta \geq \sup_{t \in \mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi) \right) & \forall i \in I \\
(vii) & \boxed{\overline{\mathrm{M}}_j - \overline{\mathrm{M}}_i} \geq \sup_{t \in [\overline{t}_{\min_i}, \overline{t}_{\max_i}]} \left(-g_{\mu_j}(t, \overline{x}_i(t)) \right) & \\
 & & \forall i \in I, \ \forall j \in I \backslash \{i\}
\end{cases}
$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Constraint on the label of vehicle i based on the fact that vehicle j has a measured trajectory

$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathbf{M}}_i & \forall i \in I \\
(iv) & \overline{\mathbf{M}}_i \geq \sup_{t\in\mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi)\right) & \forall i \in I \\
(v) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\beta(t,\overline{x}_i(t))\right) \geq -\Delta + \overline{\mathbf{M}}_i & \forall i \in I \\
(vi) & \overline{\mathbf{M}}_i - \Delta \geq \sup_{t\in\mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi)\right) & \forall i \in I \\
(vii) & \overline{\mathbf{M}}_j - \overline{\mathbf{M}}_i \geq \sup_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(\boxed{-g_{\mu_j}(t,\overline{x}_i(t))}\right) & \\
 & & \forall i \in I,\ \ \forall j \in I\backslash\{i\}
\end{cases}
$$

Data assimilation using linear programming

Initial condition unknown -Δ

Left and right boundary conditions known

Internal conditions known, but labels M_i **unknown**

Grey: non linear analytical solution of the Hamilton Jacobi equation. Can be computed explicitly for piecewise affine functions, and semi-explicitly for general nonlinear functions

$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
(iii) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\gamma(t,\overline{x}_i(t))\right) \geq \overline{\mathbf{M}}_i & \forall i \in I \\
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 & & \forall i \in I,\ \forall j \in I\backslash\{i\}
\end{cases}
$$

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(v) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\beta(t,\overline{x}_i(t))\right) \geq -\Delta + \overline{\mathbf{M}}_i & \forall i \in I \\
(vi) & \overline{\mathbf{M}}_i - \Delta \geq \sup_{t\in\mathbb{R}_+} \left(f_\beta(t,\chi) - g_{\mu_i}(t,\chi)\right) & \forall i \in I \\
(vii) & \overline{\mathbf{M}}_j - \overline{\mathbf{M}}_i \geq \sup_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(-g_{\mu_j}(t,\overline{x}_i(t))\right) & \\
& & \forall i \in I,\ \forall j \in I\backslash\{i\}
\end{cases}
$$

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$$
\begin{cases}
(i) & \inf_{t\in\mathbb{R}_+} \left(g_\gamma(t,\chi) - f_\beta(t,\chi)\right) \geq \Delta & \\
(ii) & \Delta \geq \sup_{t\in\mathbb{R}_+} \left(-g_\beta(t,\xi) + f_\gamma(t,\xi)\right) & \\
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(iv) & \overline{\mathrm{M}}_i \geq \sup_{t\in\mathbb{R}_+} \left(f_\gamma(t,\xi) - g_{\mu_i}(t,\xi)\right) & \forall i \in I \\
(v) & \inf_{t\in[\overline{t}_{\min_i},\overline{t}_{\max_i}]} \left(g_\beta(t,\overline{x}_i(t))\right) \geq -\Delta + \overline{\mathrm{M}}_i & \forall i \in I \\
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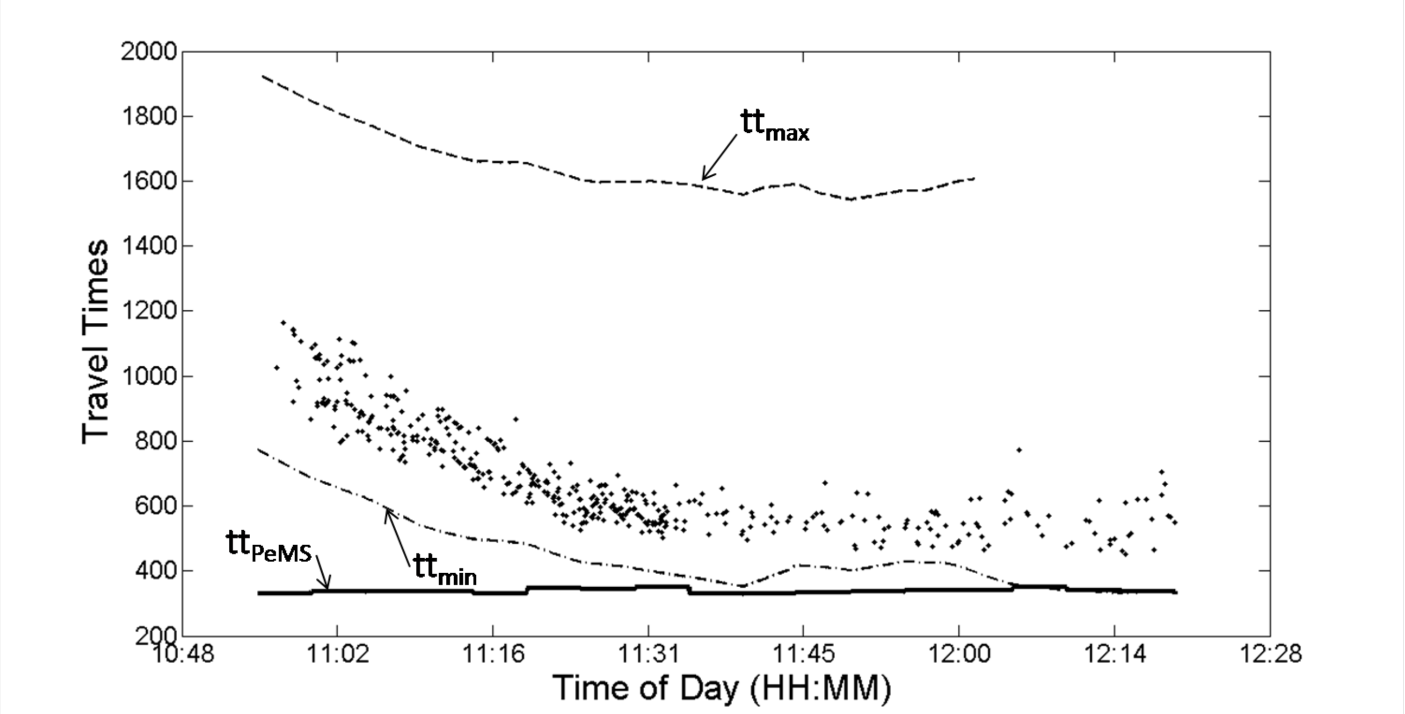
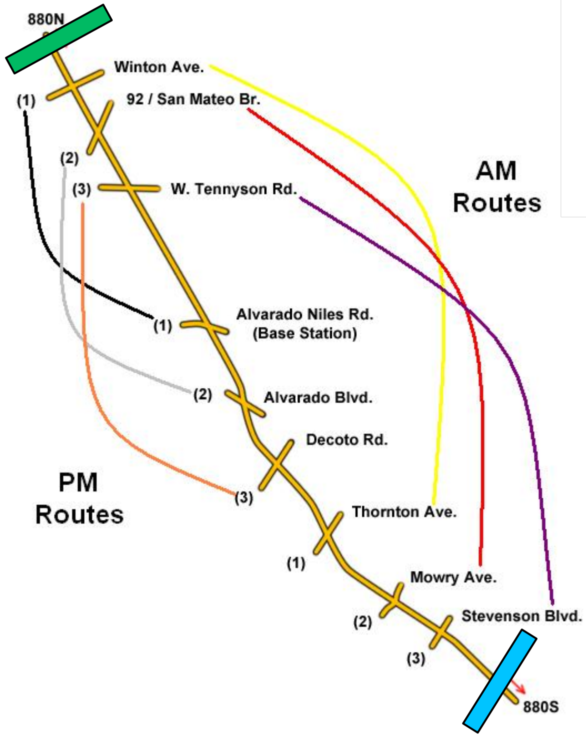
 & & \forall i \in I,\ \forall j \in I\setminus\{i\}
\end{cases}
$$

Bounds on travel time (PeMS)

Outflow loop

Inflow loop

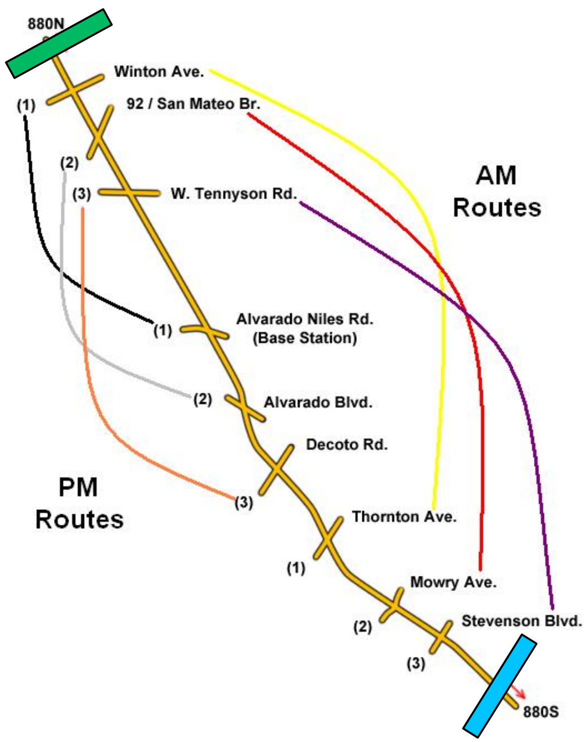


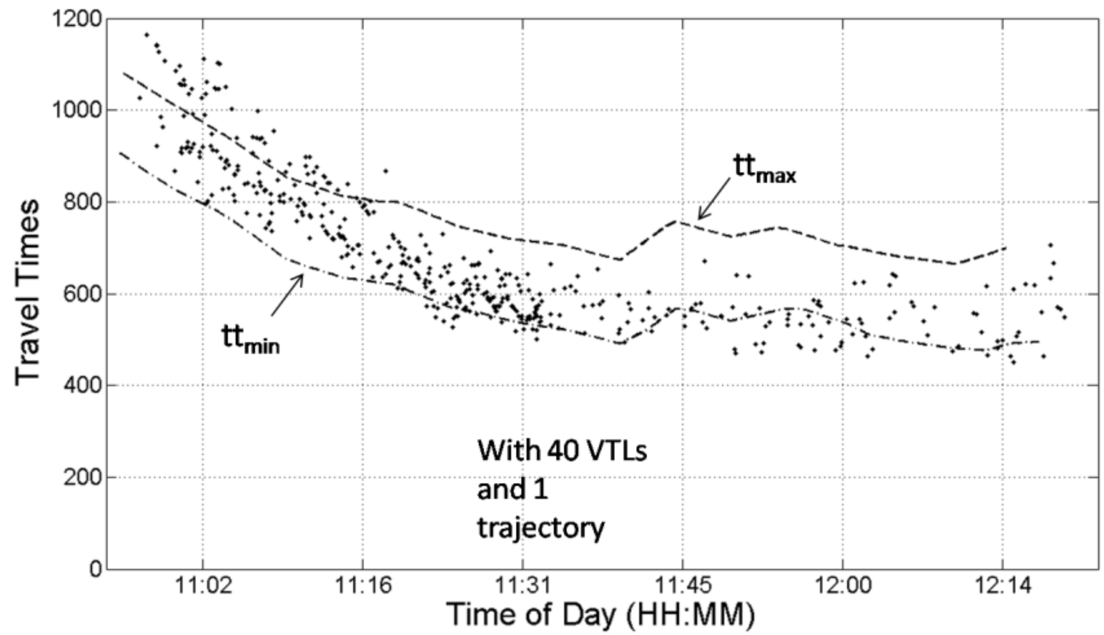


Bounds on travel time (PeMS and phones)

Outflow loop

Inflow loop

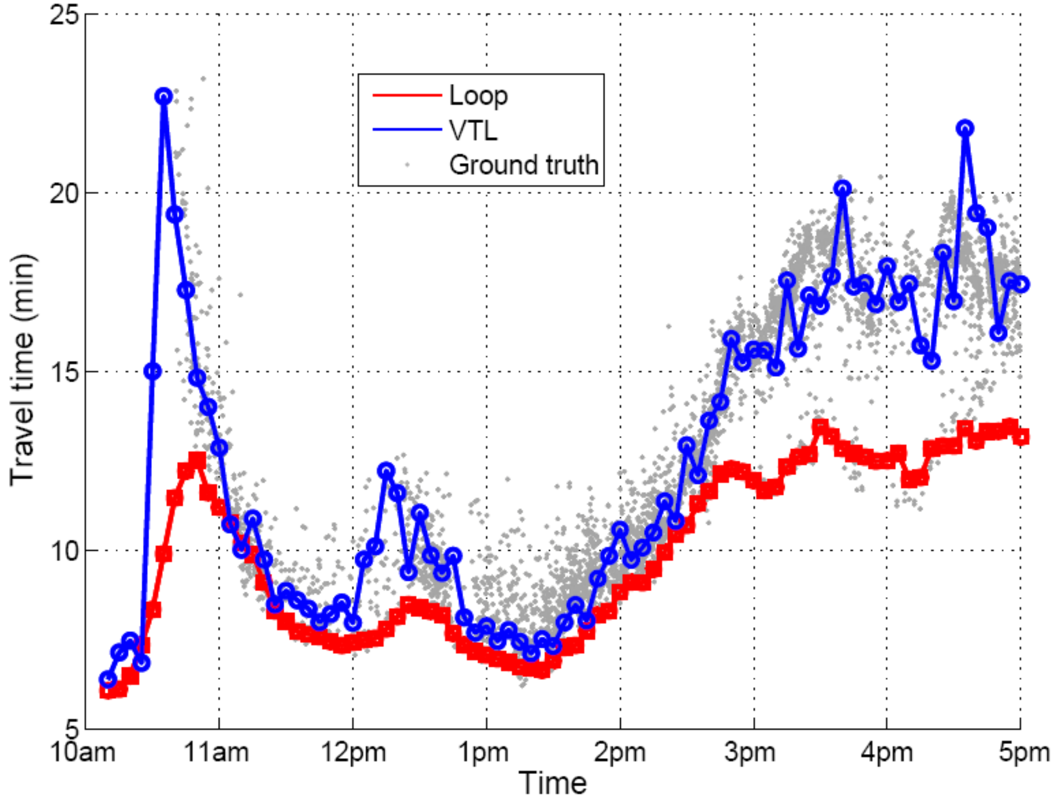


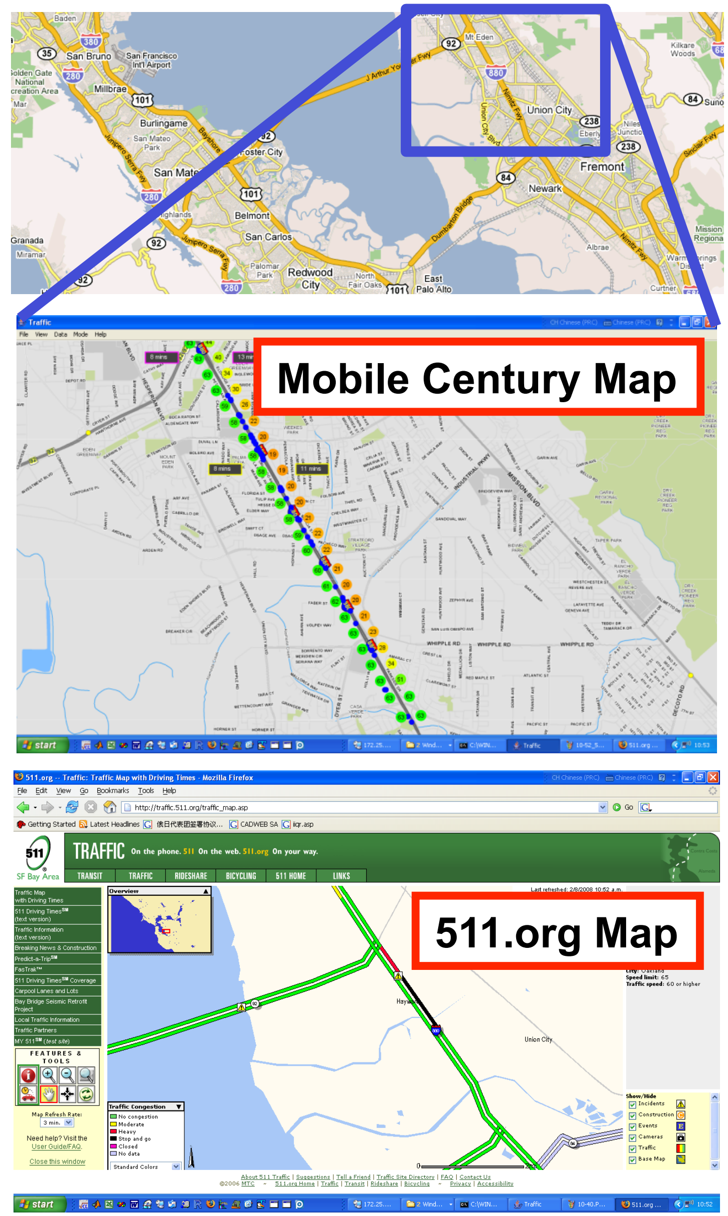


Validation of the data (video)

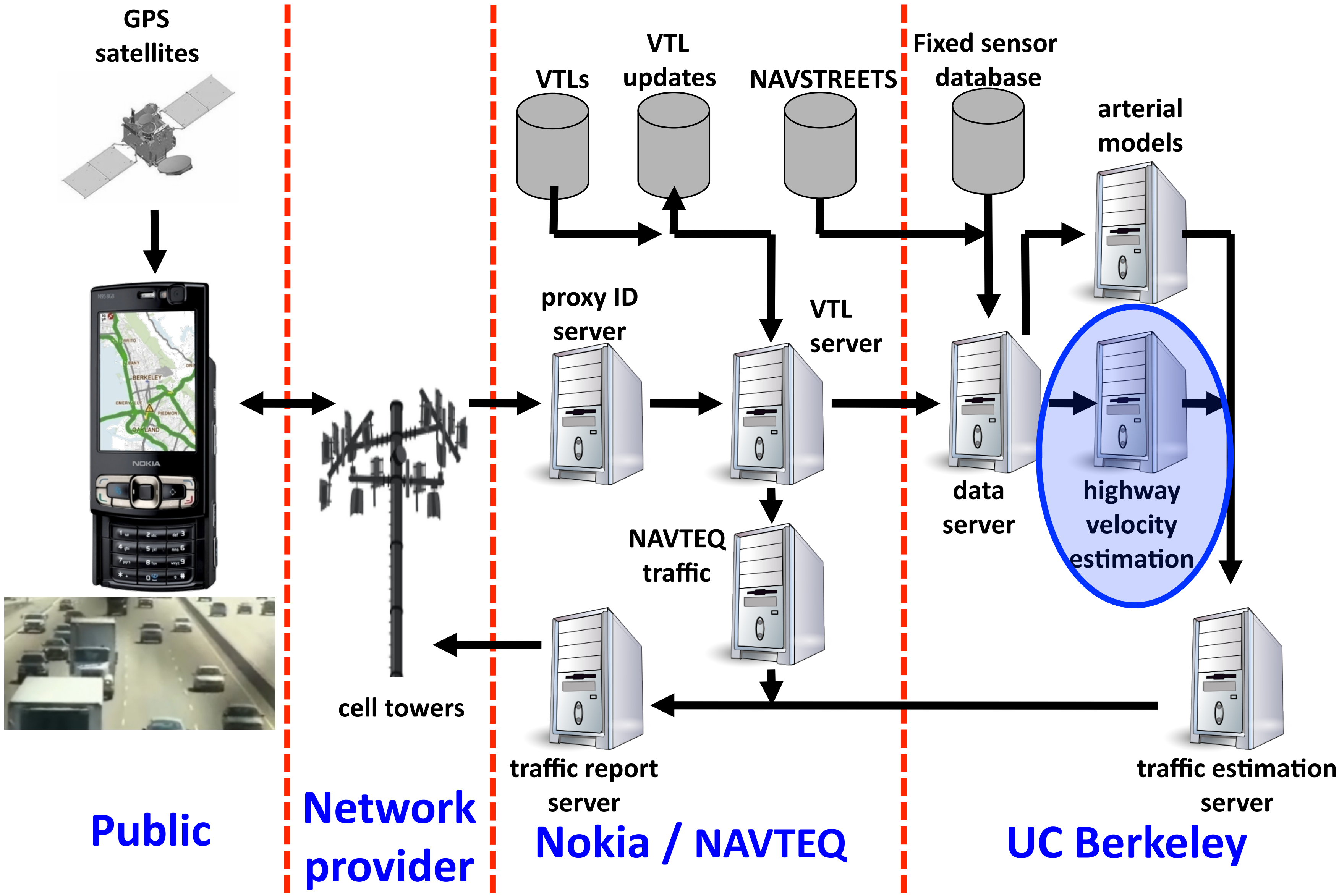
Travel time predictions

- **Can be done in real time at a 2% penetration rate of traffic**
- **Proved accurate against data from www.511.org, with higher degree of granularity**





Mobile Millennium system architecture



Outline

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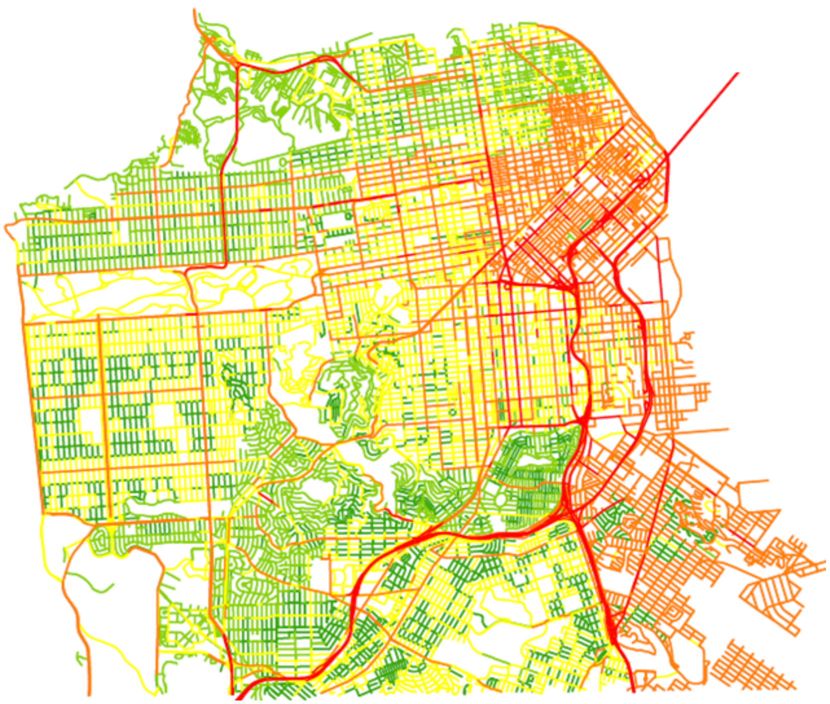
Mobile Millennium tomorrow: beyond traffic

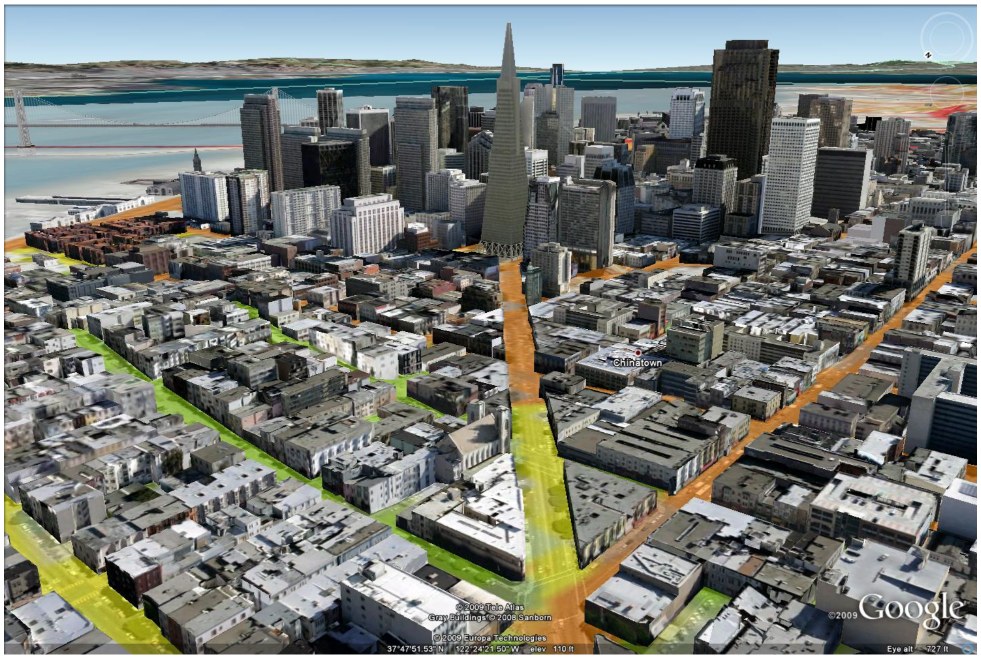
“e-Wellness”

- **Noise levels inferred from traffic: moving beyond the “average number of vehicles / year” paradigm: hour by hour noise levels.**

Today: noise map (static)



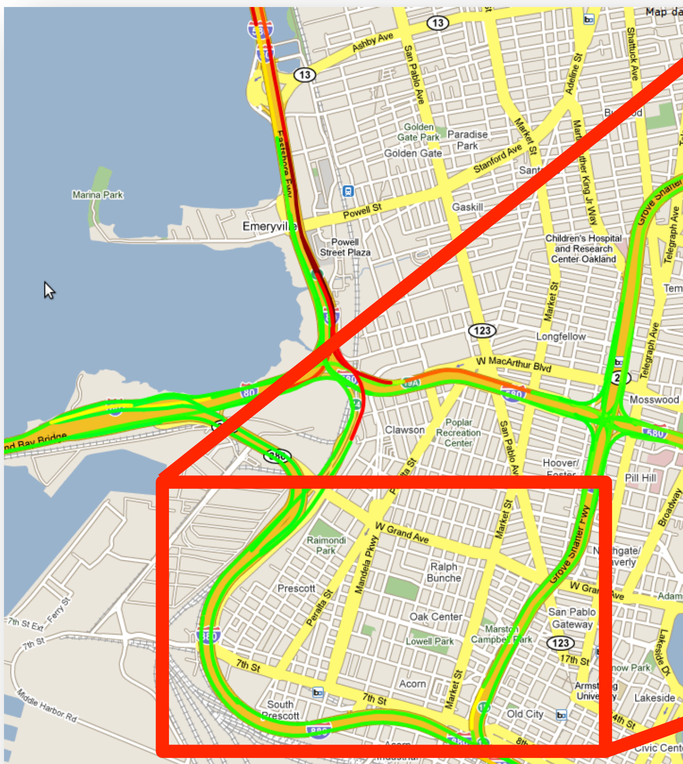
Tomorrow: hourly noise map



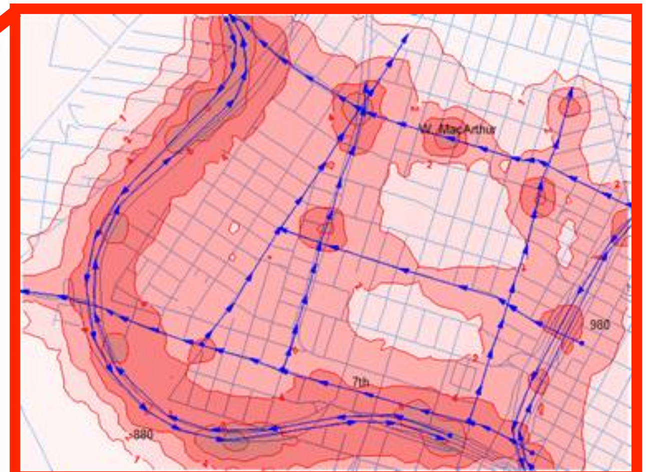
Mobile Millennium tomorrow: beyond traffic

"e-Wellness"

- **Noise levels inferred from traffic: moving beyond the "average number of vehicles / year" paradigm: hour by hour noise levels.**
- **Emission levels inferred from traffic, using emission and atmospheric dispersion models. Next gen: sensor based.**



Today: pollution map



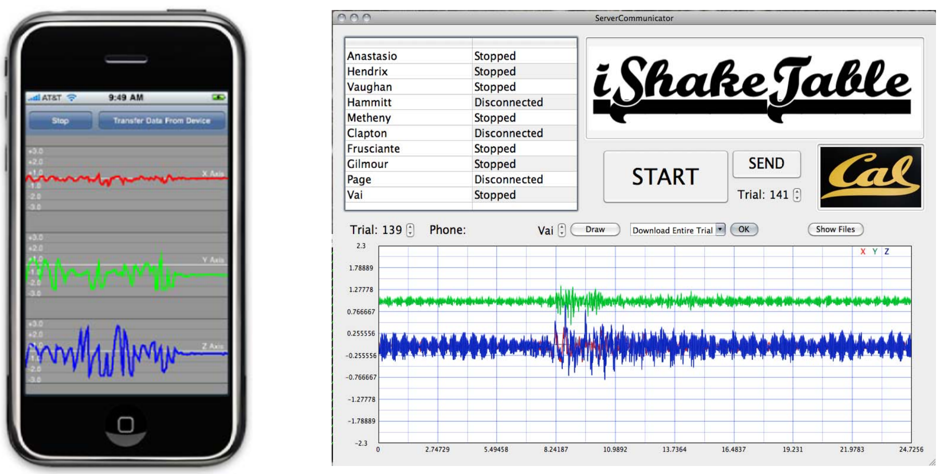
Tomorrow: sensor based data

Courtesy NASA/DHS

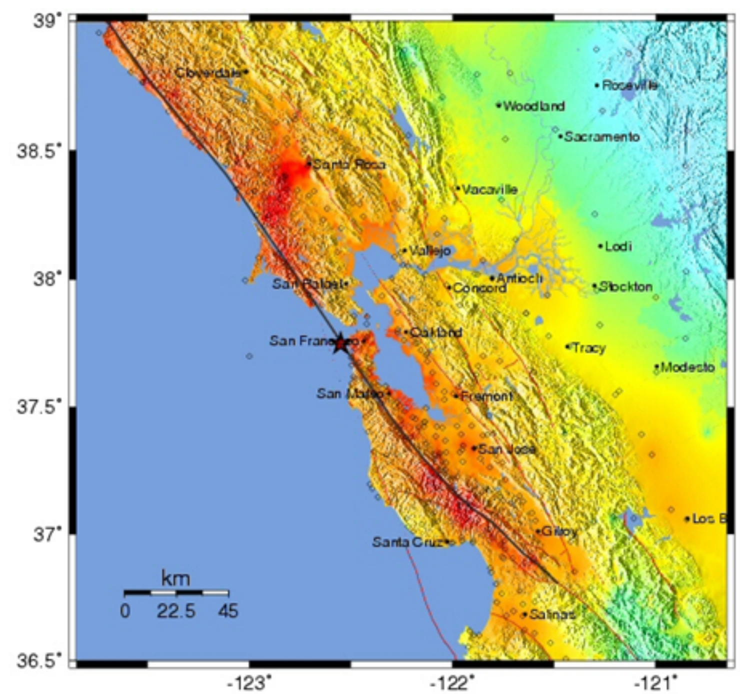
The emergence of the human as a sensor

Best known sensor for earthquakes: accelerometer

- USGS has dedicated array of embedded accelerometers
- Human is faster than USGS by posting on Twitter
- All smartphones have accelerometers, UCLA already succeeded in capturing a P-wave from a smartphone (CENS)
- Information could be enhanced by having additional accelerometer information available.



UC Berkeley iShake app and shake table testing procedure



USGS shakemap (from static USGS sensors)

Mobile Millennium tomorrow: beyond traffic

“e-Wellness”

- **Noise levels inferred from traffic: moving beyond the “average number of vehicles / year” paradigm: hour by hour noise levels.**
- **Emission levels inferred from traffic, using emission and atmospheric dispersion models. Next gen: sensor based.**
- **iShake, measuring earthquakes using cellphones while they charge or are at rest**

Already tested on the 140 most famous earthquakes on the UC Berkeley, UCSD and UCD shaketables

Closing the loop on the phone

Floating sensor network

- **Summer 2011: deployment of 100 floating / submersible units in the San Francisco Bay / Sacramento Delta**
- **All units include GSM (soon: Android), GPS, linux gumstix, Zigbee, water quality sensor platform**
- **Interfaced with static sensor infrastructure in the Delta**

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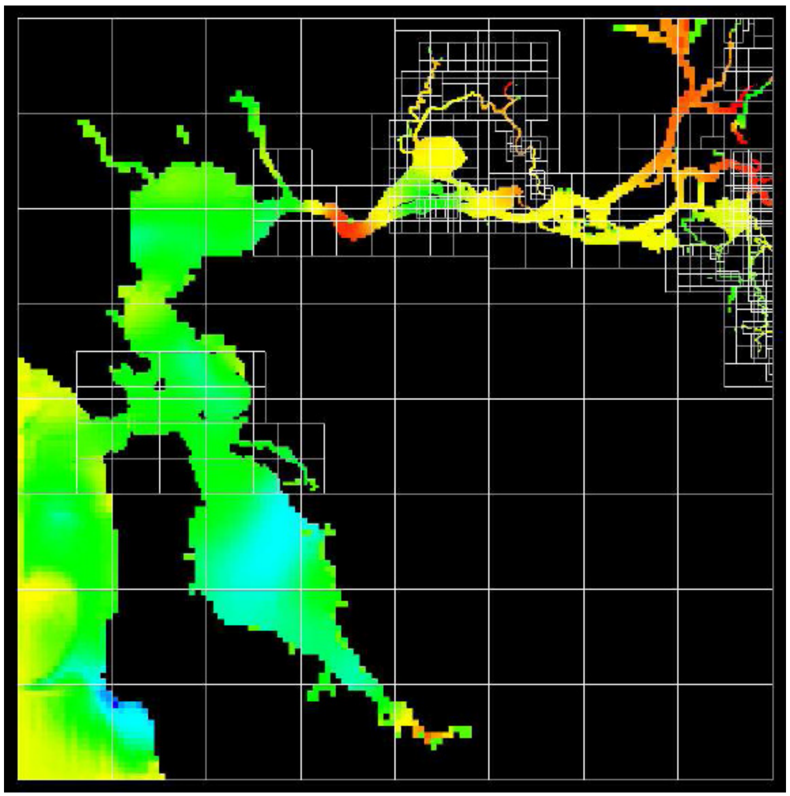
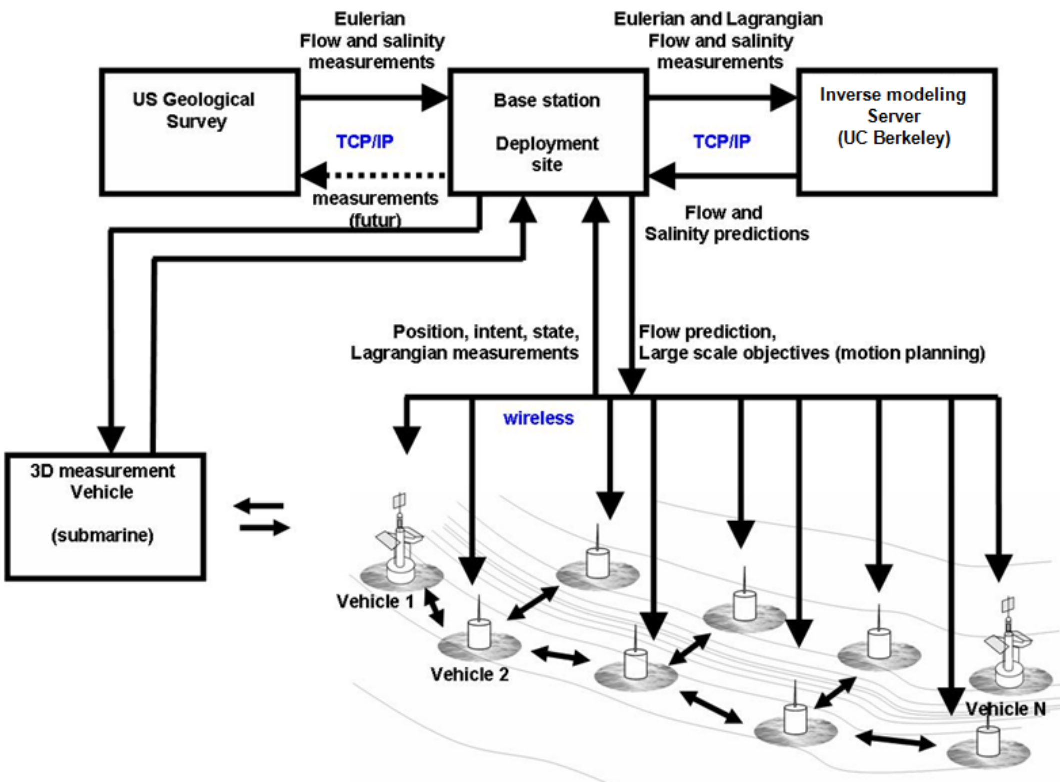
Putting water online (Google maps of water)

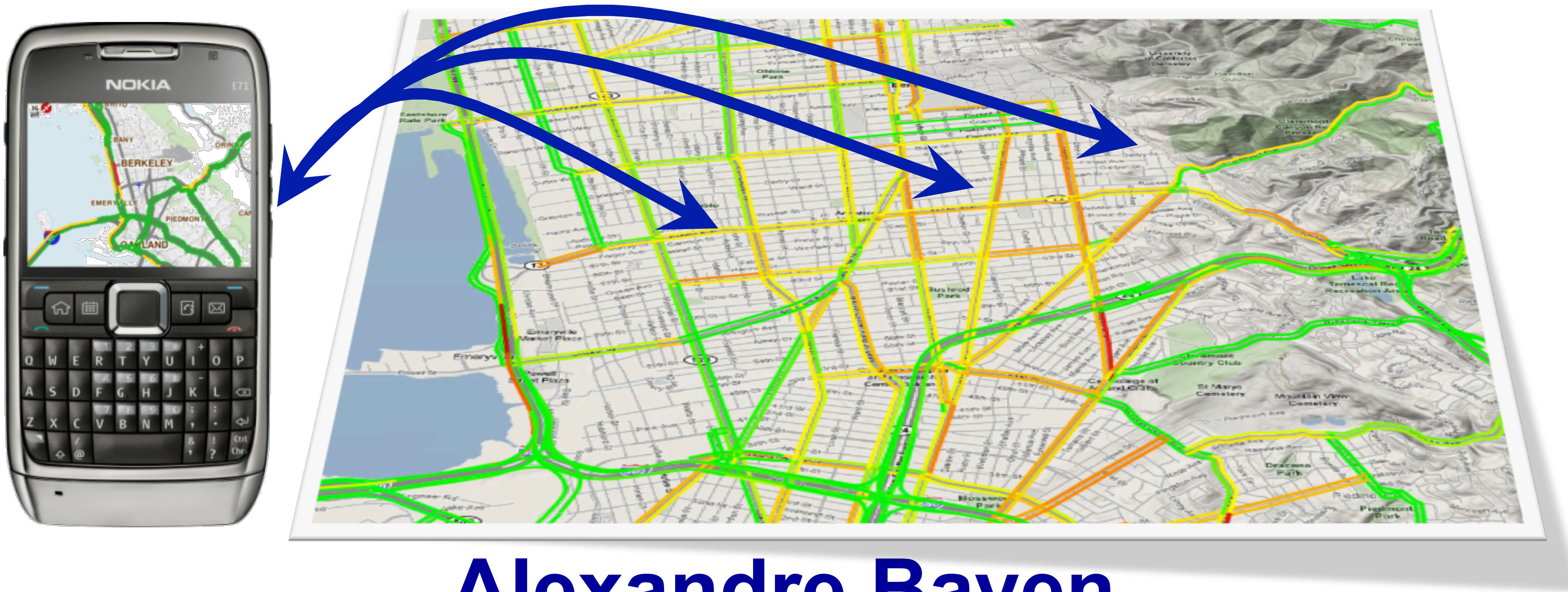
Inverse modeling, data assimilation, inference, estimation

- **Real-time, online (with streaming data)**
- **Running two dimensional shallow water models (LBNL REALM)**
- **Using Ensemble Kalman Filtering, statistical inference methods**
- **Running on 500 nodes of the Magellan / NERSC cluster at LBNL**
- **Will be live in a few months**





Real-time estimation of distributed parameters systems: application to large scale infrastructure systems



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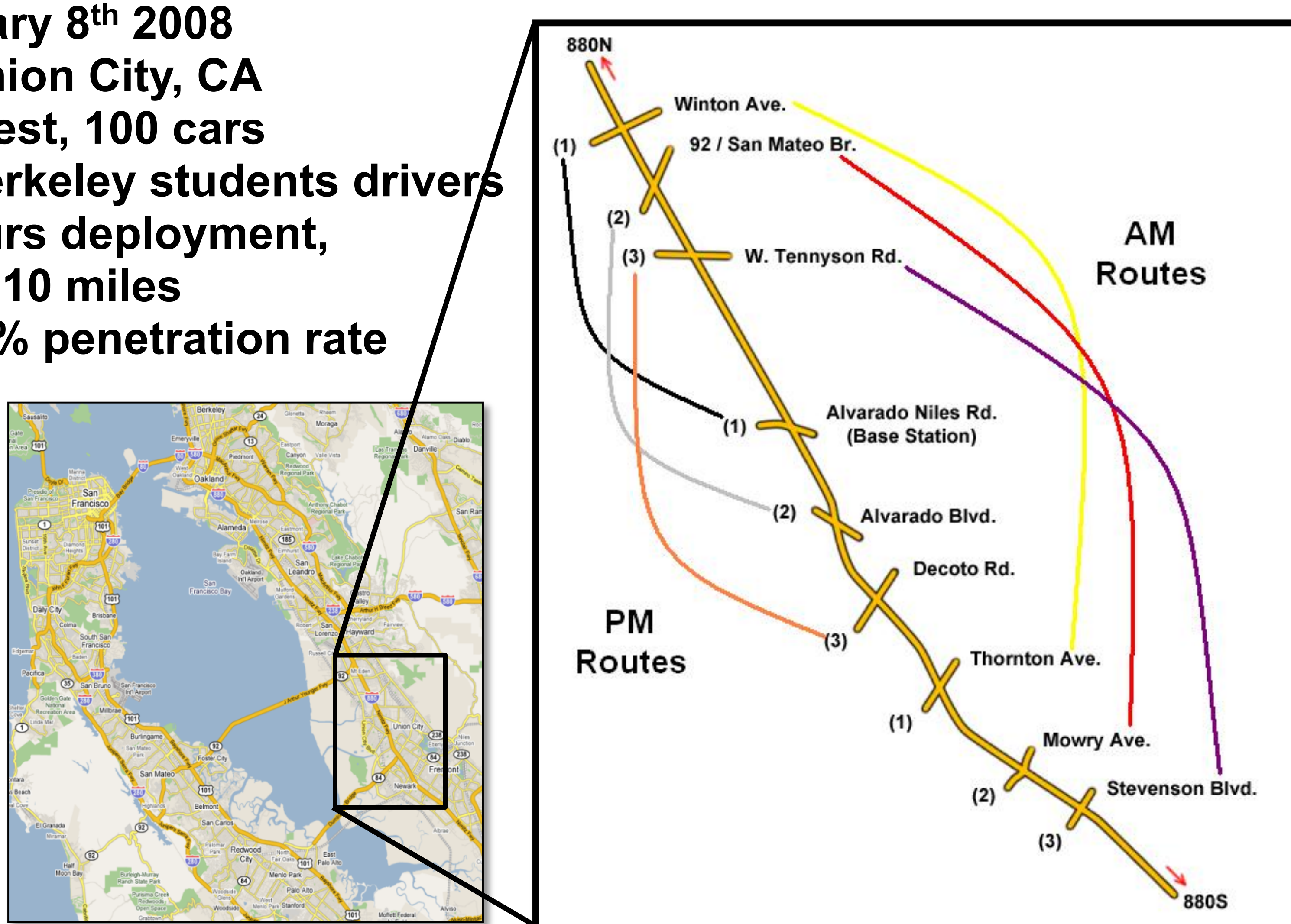
http://traffic.berkeley.edu **http://float.berkeley.edu**

Prototype experiment: *Mobile Century*

Experimental proof of concept: the *Mobile Century* field test

- **February 8th 2008**
- **I80, Union City, CA**
- **Field test, 100 cars**
- **165 Berkeley students drivers**
- **10 hours deployment,**
- **About 10 miles**
- **2% - 5% penetration rate**



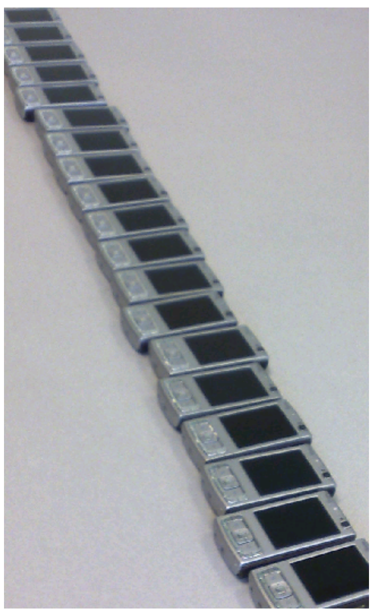
Mobile Century validation video data collection

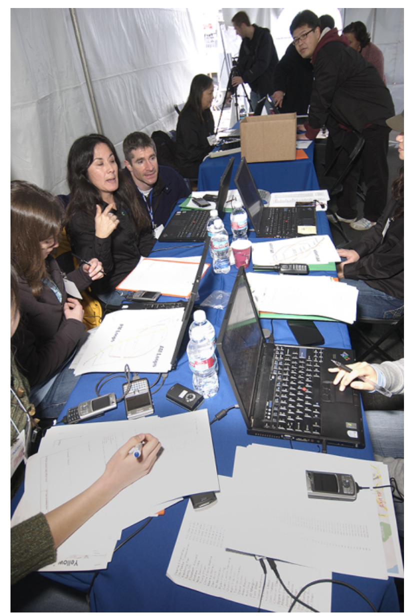
Video data:

- **Vehicles counts**
- **Travel time validation**

A glimpse of *Mobile Century* (February 8th, 2008)

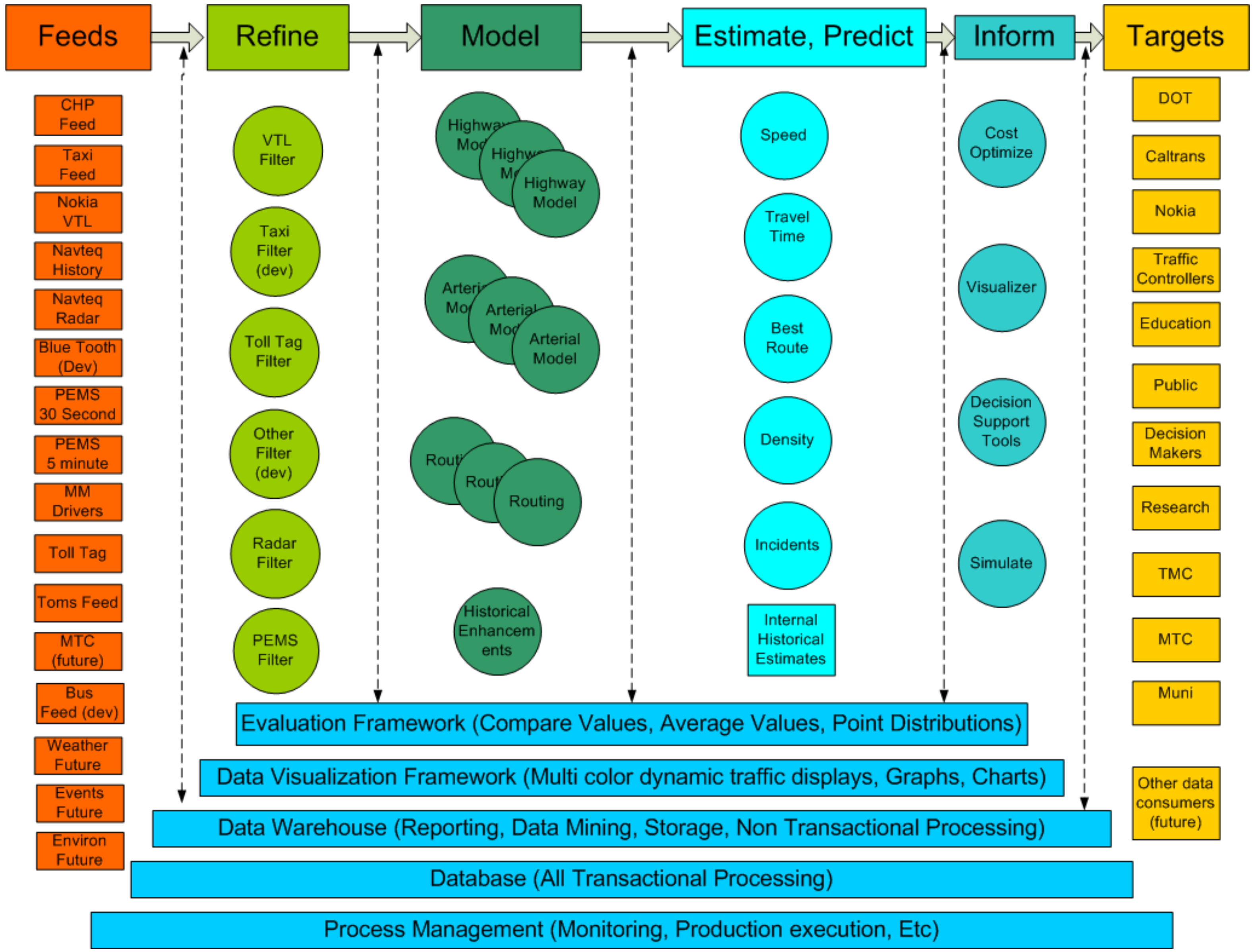





A glimpse of *Mobile Century* (February 8^{th}, 2008)

Data flow in the *Mobile Millennium* system



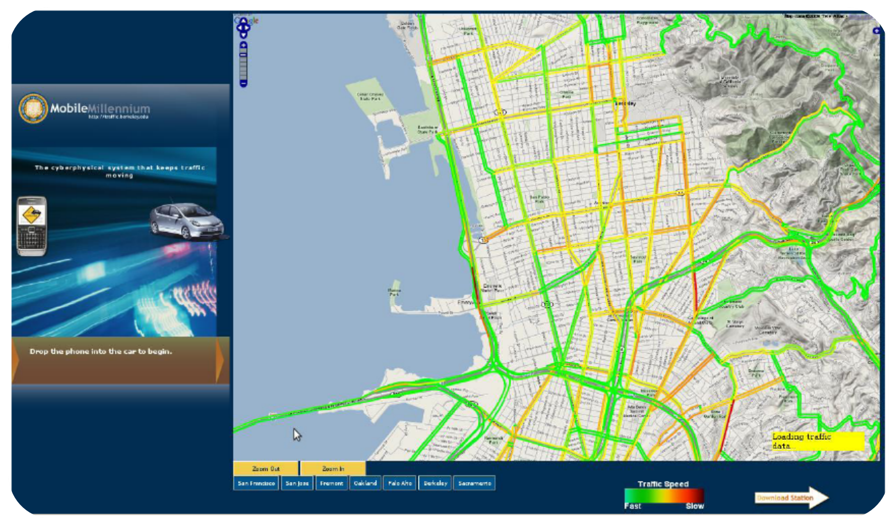
Google Maps vs. model driven estimation

Friday, March 20th, 2009

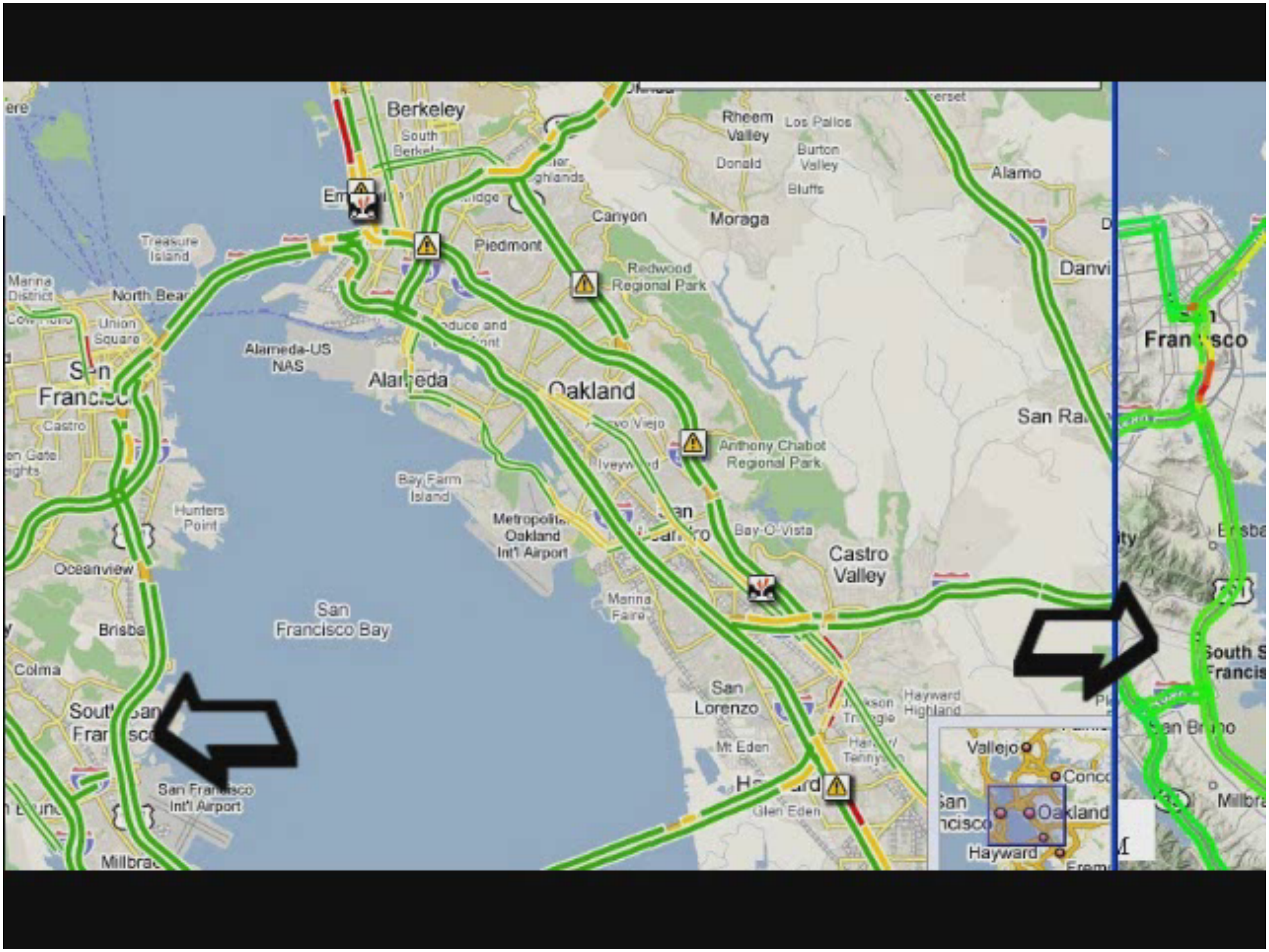
- 1:30pm (Friday afternoon congestion)
- Acceleration: 1 frame = 30 seconds of physical time
- Movies are synchronized

Google Maps

Mobile Millennium

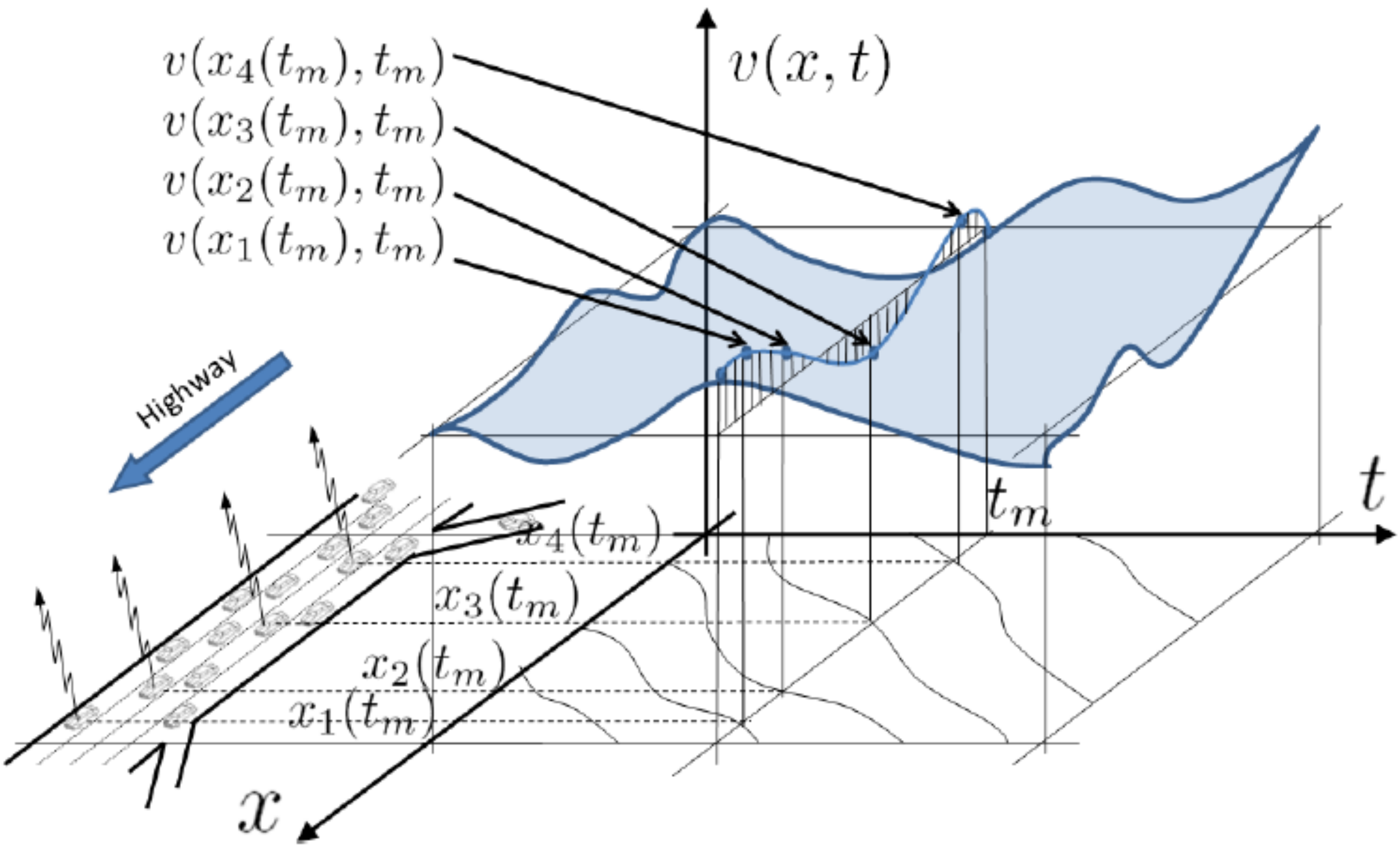


Google Maps vs. model driven estimation



Data assimilation / inverse modeling

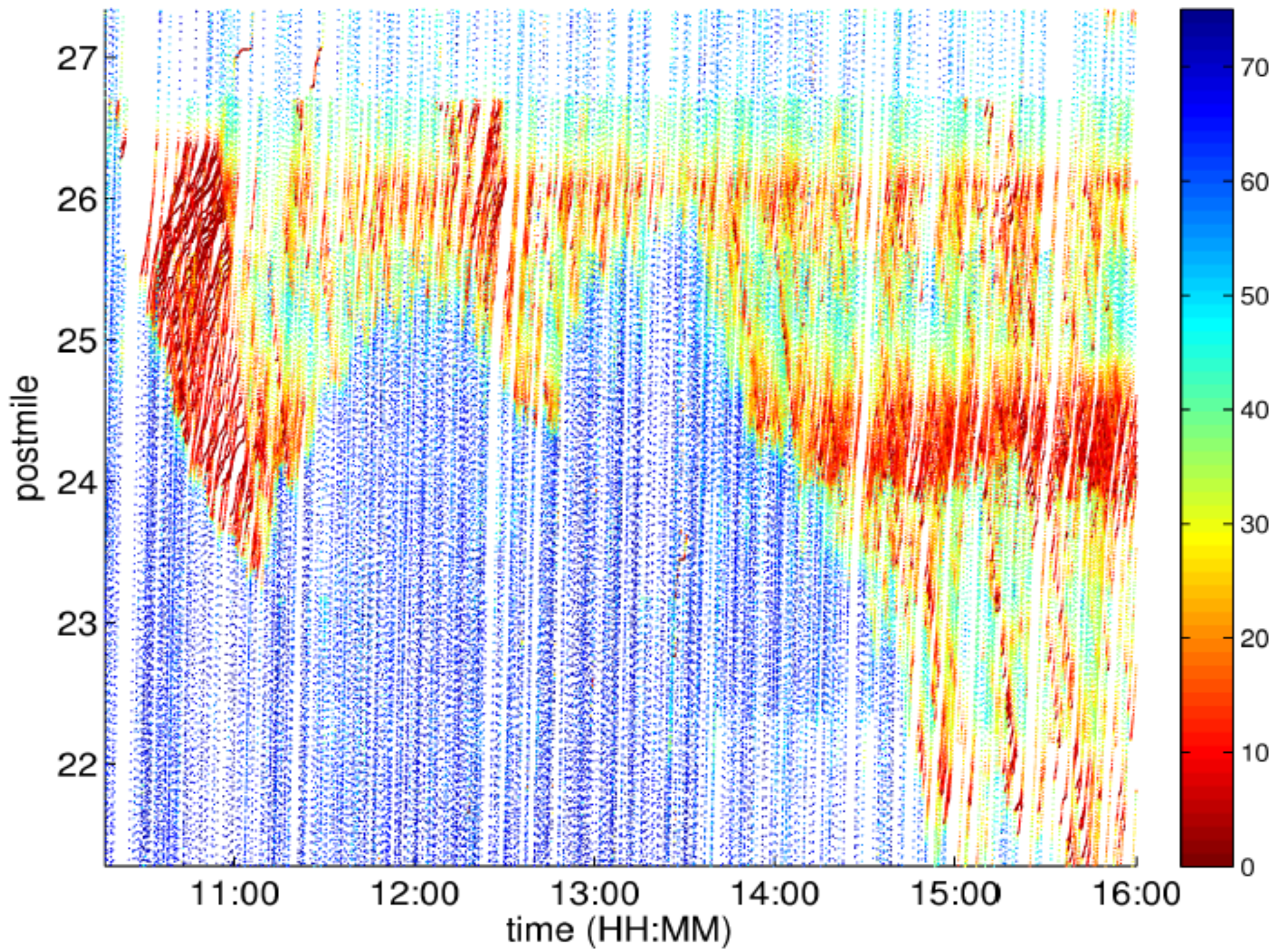
How to incorporate Lagrangian (trajectory based) and Eulerian (control volume based) measurements in a flow model.



Granularity of the data (GPS data)

Physical model and data assimilation enable state estimation

- **Works even with low penetration rate**
- **Interpolation will just not do the job**



Flow reconstruction (inverse modeling)

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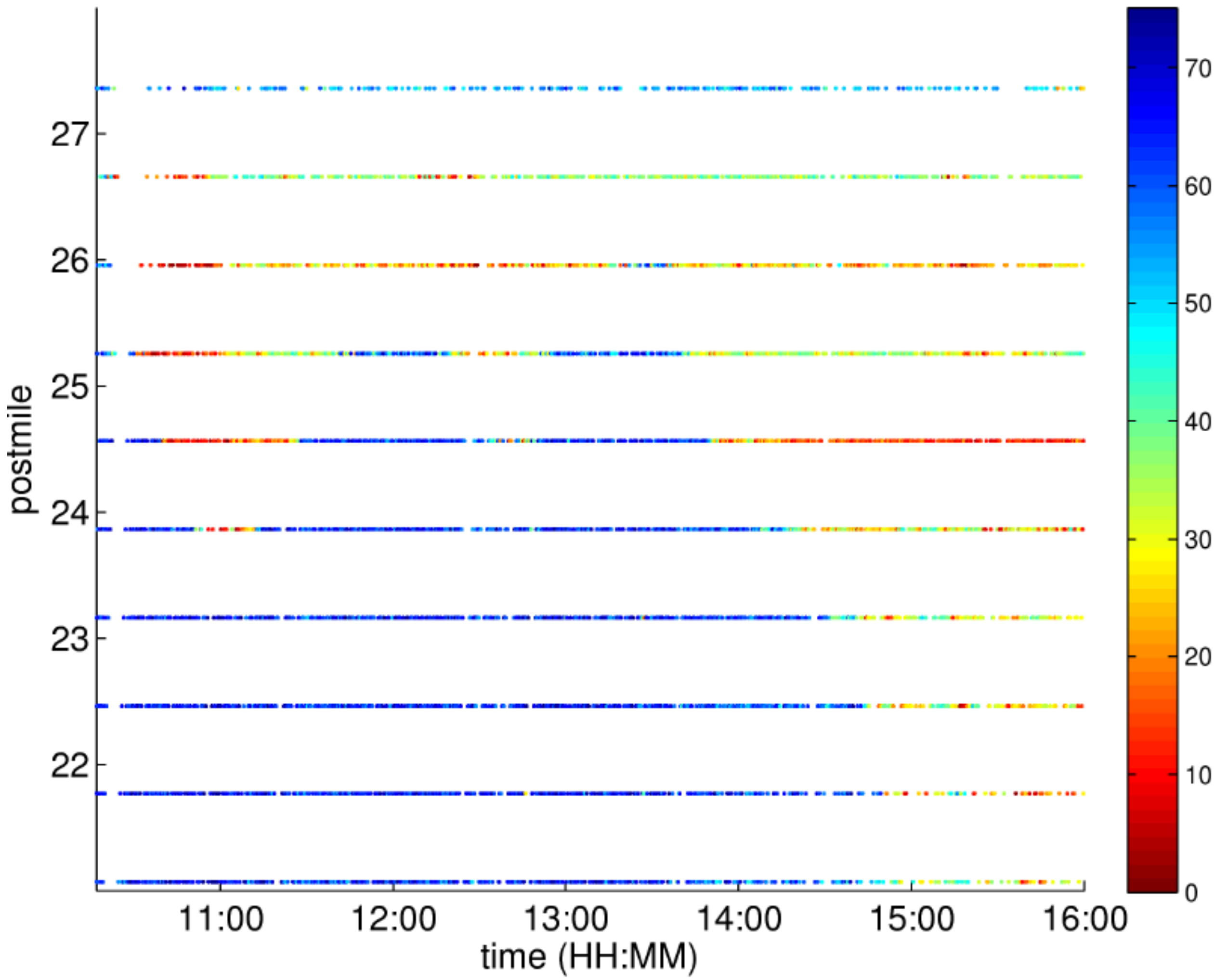
10 measurement points



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